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A Model of International Asset Pricing with a Constraint on the Foreign Equity Ownership

CHEOL S. EUN and S. JANAKIRAMANAN*

ABSTRACT

This paper derives a closed-form valuation model in a two-country world in which the domestic investors are constrained to own at most a fraction, δ , of the number of shares outstanding of the foreign firms. When the “ δ constraint” is binding, two different prices rule in the foreign securities market, reflecting the premium offered by the domestic investors over the price under no constraints and the discount demanded by the foreign investors. The premium is shown to be a multiple of the discount, the multiple being the ratio of the aggregate risk aversion of the domestic and foreign investors. Given the aggregate risk-aversion parameters, the equilibrium premium and discount are determined by the severity of the δ constraint and the “pure” foreign market risk.

IN ADDITION TO THE expanded opportunity set facing investors, international investment entails two unique dimensions which would not particularly matter in domestic investment—namely, the problem of exchange risk and the problem of market segmentation. For this reason, an “international” asset pricing model is expected to address at least one of these two international problems. Since Solnik’s [15] pioneering work appeared, various researchers, such as Grauer-Litzenberger-Stehle [11], Kouri [13], Fama-Farber [8], Stulz [19], and Adler-Dumas [2], have derived alternative versions of the international asset pricing model. Apart from the effect of a widened choice universe, these studies are primarily concerned with the effect of exchange risk on portfolio behavior and on equilibrium asset pricing. Owing to the aforementioned studies, we now have a deeper understanding of the nature of exchange risk and its effect on the capital market equilibrium. In contrast, research in the area of market segmentation is still in its nascent stage, leaving much to be done.

Barriers to international investment may take many forms such as exchange and capital controls by governments, which restrict the access of foreigners to the local capital markets, reduce their freedom to repatriate capital and dividends, and limit the fraction of a local firm’s equity that foreigners may own. Individuals may be inhibited by a lack of information, the fear of expropriation, or more generally, by discriminatory taxation. The existence of such barriers will constrain the portfolio choice of the individuals, and hence the resulting equilibrium may very well be different from that under no barriers. As previously mentioned,

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there have been relatively few works addressing the effect of barriers to international investment. Using numerical analysis, Stapleton and Subrahmanyam [17] examine the effect of various forms of barriers on asset pricing. Although their numerical analysis yields useful insights, they do not derive explicit closed-form valuation models.

Black [3] and Stulz [18] study the barrier in terms of a cost associated with holding foreign securities in the portfolio. This cost may represent the transaction cost, information cost, or differential taxation. Both of these papers assume that this cost can be represented as proportional taxation and use a two-country, single-period model for analysis. In the Black model, the tax is on an investor's net holdings (long minus short) of risky foreign assets while in the Stulz model, the tax on both long and short positions is positive. But both models show that the world market portfolio will not be efficient for any investor in either country. Stulz also shows that some of the foreign securities may not be held at all in the domestic investor's portfolio. Errunza and Losq [5], on the other hand, study the barrier in the form of legal restriction imposed on the investors of the domestic country by the foreign government. In their model, the domestic country investors are completely precluded from investing in the foreign country securities, whereas the foreign country investors have unlimited access to the domestic country securities. In this setting, they show that the return demanded by the foreign investors on the foreign securities will be higher as compared to the return under no such restrictions.

Given a perplexing variety of barriers to international investment, it seems to be difficult, if not impossible, to model these barriers in a catch-all manner. The challenge for researchers is thus to isolate and quantify an important barrier and then investigate its impact on portfolio behavior and on the asset pricing relationship. The purpose of this paper is to derive a closed-form asset valuation model under a particular type of barrier to international investment, and to thereby contribute to the understanding of the asset pricing mechanism in an international setting. Specifically, we analyze the effect of legal restrictions imposed by the government on the fraction of equities of local firms that can be held by foreigners.

It is well known that governments in both developed and developing countries often impose these restrictions as a means of ensuring domestic control of local firms, especially those firms that are regarded as strategically important to national interests. In France and Sweden, for instance, foreigners are allowed to purchase at most 20% of the total number of outstanding shares of a local firm. In countries like India and Mexico, the limit is 49%. In Switzerland, a local firm can issue two different types of shares, i.e., bearer shares and registered shares. Foreigners are only allowed to hold bearer shares, thereby effectively controlling the maximum proportionate ownership of Swiss shares by foreigners.¹ Table I presents data on the restrictions placed by various governments on equity

¹ A cursory examination of the stock prices of several Swiss firms shows that the bearer shares are almost always selling at a higher price than the registered shares. This seems to indicate that the restriction is "binding."

Table I
Restrictions Imposed on Foreign Equity Holdings in Various Countries

Country	Restrictions on Foreign Ownership
Australia	10% in banks, 25% in Uranium mining, 20% in broadcasting, and 50% in new mining ventures.
Burma	Investment is not allowed.
Canada	20% in broadcasting, and 25% in banks and insurance companies.
Finland	Limited to 20%.
France	Limited to 20%.
India	Maximum of 49%.
Indonesia	Maximum of 49%.
Japan	Maximum of 25–50% in a group of 11 major firms. Acquisition of over 10% of the shares of a single firm requires approval of the Ministry of Finance.
South Korea	Maximum of 15% of the major firms eligible to foreigners for investment.
Malaysia	20% in banks, 30% in natural resources, and a maximum of 70% in other firms.
Mexico	Maximum of 49%.
Netherlands	No restrictions in listed securities. Special permission needed if investment is in unlisted securities.
Norway	10% in banking industry, 20% in industrial or oil shares, 50% in shipping industry, and 0% in pulp, paper, and mining.
Spain	Maximum of 50% with no investment in defense and public information.
Sweden	20% of voting shares and 40% of total share capital.
Switzerland	A local firm can issue either bearer shares or registered shares. Foreigners can hold only bearer shares.

Data source: George and Giddy [10], ABD Securities [1], Esslen [6], and various publications of Price-Waterhouse [14].

holdings of foreigners in the local firms. From this table, it can be seen that these restrictions can be of several types:

- (i) The fraction of equity that can be held by foreigners is uniform across all firms.
- (ii) The fraction varies across different industries with some industries closed to investment by foreigners.
- (iii) Foreign investment is banned completely.

This paper will be mainly concerned with the effect of the uniform restrictions described in (i) above. The restriction of type (iii), in fact, can be viewed as a special case of (i) where the proportionate ownership by foreigners is constrained to be zero.

We assume that there are only two countries in the world, one domestic and one foreign. There are no restrictions imposed upon investors of the foreign country who invest in domestic country firms. But the foreign country restricts investment by investors from the domestic country in firms of the foreign country. The proportion of the number of shares outstanding of the i^{th} foreign firm is restricted to be at most δ_i , and this proportion is uniform across all firms, i.e., $\delta_i = \delta_j = \delta$. This we term as the “ δ constraint.” Using the assumptions of the

standard CAPM model, a closed-form valuation model is developed for pricing the securities.²

The organization of the paper is as follows. In Section I, we briefly describe the economy. Section II presents the investor's choice problem and derives the demand for asset functions. In Section III, we derive the equilibrium asset pricing model and interpret it. In Section IV, numerical analysis is conducted to supplement the theoretical analysis. Section V offers conclusions and a summary.

I. The Economy

To simplify the analysis, it is assumed that there are only two countries in the world—the domestic country, D , and the foreign country, F . Since we are not concerned with the effect of exchange risk in this paper, we use a framework which is devoid of exchange risk. Specifically, it is assumed that the two countries maintain a fixed exchange rate regime. It is also assumed that the risk-free interest rate expressed in any currency, either domestic or foreign, is identical.

In the domestic country D , there are M_D risky assets available and in the foreign country F , there are M_F risky assets. There are no restrictions placed by country D on foreigners investing in the M_D risky assets. But country F restricts investment by country D investors in the M_F risky assets to an aggregate proportion of the number of shares outstanding of any foreign firm no greater than δ .³ In addition, the domestic country investors cannot hold an aggregate short position in any of the risky assets of the foreign country. There are no restrictions on short sales by foreign investors either in the domestic country or in the foreign country.

The following additional assumptions are made throughout the paper.

- A1: The capital market is perfectly competitive in each country.
- A2: There exist neither transaction costs, information costs nor differential taxation.
- A3: Investors, both domestic and foreign, have homogeneous expectations as to the risk and return characteristics of all assets.
- A4: The security prices have a joint-normal Gaussian distribution with finite first and second moments.
- A5: Investors, both domestic and foreign, can borrow or lend at the risk-free rate of $r - 1$.

Notation is defined as follows:

$N_D = \{N_{iD}\}$, i.e., the M_D -dimensional vector of the number of shares of the domestic securities outstanding.

$N_F = \{N_{iF}\}$, i.e., the M_F -dimensional vector of the number of shares of the foreign securities outstanding.

² It is noted that Errunza-Losq [5] address the issue in (iii) where foreign investment is completely barred in all of the firms. Since the restriction of type (iii) is, in fact, a special case of (i) in which δ is equal to zero, their model can be considered as a special case of the model to be developed in this paper.

³ In this paper, the δ value is assumed to be given exogeneously. Consequently, no attempt will be made to explain how it is determined.

n_D^k, n_F^k = vectors of the number of shares of domestic and foreign securities held by the k^{th} investor, respectively, $k \in D, F$.

P_D, P_F = vectors of the current prices of the domestic and foreign securities, respectively.

\tilde{P}_D, \tilde{P}_F = vectors of the random end-of-period prices of the domestic and foreign securities, respectively.

μ_D, μ_F = vectors of the expected value of the end-of-period prices of the domestic and foreign securities, respectively.

$\Gamma_D = M_D \times M_D$ covariance matrix of the prices of the domestic securities.

$\Gamma_F = M_F \times M_F$ covariance matrix of the prices of the foreign securities.

$\Gamma_{DF} = M_D \times M_F$ covariance matrix of the prices of the domestic securities and the prices of the foreign securities.

W^k = the investable wealth of the investor k at time 0.

\tilde{W}^k = the random end-of-period wealth of investor k .

In case asset i is a domestic (foreign) asset, it is denoted by $i \in D$ ($i \in F$).

II. The Investor's Choice Problem

Each investor k allocates his or her investment funds W^k over the $(M_D + M_F)$ risky assets and the risk-free asset in order to maximize the expected utility of his or her end-of-period wealth. It is assumed that each investor k treats his or her Arrow-Pratt measure of risk aversion $A^k = -U''/U'$ as a constant during his or her investment horizon. Therefore, each investor k can be assumed to act in terms of a negative exponential utility function,

$$U^k(\tilde{W}^k) = -\exp[-A^k \tilde{W}^k], \quad A^k > 0. \tag{1}$$

Each investor chooses that portfolio of risky assets and the riskless asset which will maximize his or her expected utility of end-of-period wealth, $E[U^k(\tilde{W}^k)]$. Since \tilde{W}^k is normal, as the prices of the securities are joint-normally distributed, the expected utility can be expressed as,

$$E[U^k(\tilde{W}^k)] = -\exp\left\{-A^k \left[\bar{W}^k - \frac{A^k}{2} \text{Var}(\tilde{W}^k)\right]\right\}, \tag{2}$$

where \bar{W} and $\text{Var}(\tilde{W})$ are the expected value and the variance of end-of-period wealth. The investor's optimal investment is, therefore, the one that maximizes the certainty equivalent (CEQ) of his or her end-of-period wealth,

$$CEQ^k = \bar{W}^k - \frac{A^k}{2} \text{Var}(\tilde{W}^k) \tag{3}$$

for any possible vector $\{n_i^k\}_{i \in D, F}$ of the number of shares of each security held at a given set of market prices $\{P_i\}_{i \in D, F}$.

The investor will allocate his or her total wealth, W^k , among risky and riskless assets such that the budget constraint will be satisfied. The budget constraint is given by

$$W^k = n_D^{k'} P_D + n_F^{k'} P_F + L^k, \tag{4}$$

where L^k is the amount invested in the risk-free asset. The holding period rate of return on the risk-free asset is $r - 1$. Therefore, the expected end-of-period wealth and its variance are, respectively, given by

$$\bar{W}^k = n_D^{k'}(\mu_D - P_D r) + n_F^{k'}(\mu_F - P_F r) + W^k r, \tag{5}$$

$$\text{Var}(\tilde{W}^k) = n_D^{k'} \Gamma_D n_D^k + 2n_D^{k'} \Gamma_{DF} n_F^k + n_F^{k'} \Gamma_F n_F^k. \tag{6}$$

Substituting equations (5) and (6) into equation (3), we obtain the following expression for the certainty equivalent:

$$\begin{aligned} CEQ^k &= n_D^{k'}(\mu_D - P_D r) + n_F^{k'}(\mu_F - P_F r) + W^k r \\ &\quad - \frac{A^k}{2} [n_D^{k'} \Gamma_D n_D^k + 2n_D^{k'} \Gamma_{DF} n_F^k + n_F^{k'} \Gamma_F n_F^k]. \end{aligned} \tag{7}$$

A. Portfolio Selection by Domestic Investors

Although the aggregate holdings by domestic investors are subject to the δ constraint, individual investors face no such restrictions as long holdings in excess of δ by some may be offset by short positions held by others. Therefore, domestic investor k will solve his or her portfolio problem as

$$\begin{aligned} \max_{\{n_D^k, n_F^k\}} CEQ^k &= n_D^{k'}(\mu_D - P_D r) + n_F^{k'}(\mu_F - P_F r) + W^k r \\ &\quad - \frac{A^k}{2} [n_D^{k'} \Gamma_D n_D^k + 2n_D^{k'} \Gamma_{DF} n_F^k + n_F^{k'} \Gamma_F n_F^k]. \end{aligned} \tag{8}$$

The maximization yields the following first-order conditions:

$$(\mu_D - P_D r) - A^k [\Gamma_D n_D^k + \Gamma_{DF} n_F^k] = 0, \tag{9a}$$

$$(\mu_F - P_F r) - A^k [\Gamma'_{DF} n_D^k + \Gamma_F n_F^k] = 0. \tag{9b}$$

Rewriting (9a) and (9b) in terms of demand for assets, we obtain

$$\begin{bmatrix} n_D^k \\ n_F^k \end{bmatrix} = \frac{1}{A^k} \begin{bmatrix} \Gamma_D & \Gamma_{DF} \\ \Gamma'_{DF} & \Gamma_F \end{bmatrix}^{-1} \begin{bmatrix} \mu_D - P_D r \\ \mu_F - P_F r \end{bmatrix}. \tag{10}$$

Define:

$$\begin{bmatrix} \Gamma_D & \Gamma_{DF} \\ \Gamma'_{DF} & \Gamma_F \end{bmatrix}^{-1} = V = \begin{bmatrix} V_D & V_{DF} \\ V'_{DF} & V_F \end{bmatrix}. \tag{11}$$

Then the demand equations for investor k of the domestic country can be rewritten as follows:

$$n_D^k = \frac{1}{A^k} \{V_D[\mu_D - P_D r] + V_{DF}[\mu_F - P_F r]\}, \tag{12}$$

$$n_F^k = \frac{1}{A^k} \{V'_{DF}[\mu_D - P_D r] + V_F[\mu_F - P_F r]\}. \tag{13}$$

Aggregating the demand over all investors of the domestic country, we obtain

the following aggregate demand functions of the domestic country:

$$n_D^d = \frac{1}{A^D} \{V_D[\mu_D - P_D r] + V_{DF}[\mu_F - P_F r]\}, \quad (14)$$

$$n_F^d = \frac{1}{A^D} \{V'_{DF}[\mu_D - P_D r] + V_F[\mu_F - P_F r]\}, \quad (15)$$

where

$$n_D^d = \sum_{k \in D} n_D^k; \quad n_F^d = \sum_{k \in D} n_F^k; \quad \text{and} \quad \frac{1}{A^D} = \sum_{k \in D} \frac{1}{A^k}.$$

B. Portfolio Selection by Foreign Investors

The foreign investor q faces a problem similar to that of the domestic investor, and thus, the demand equations can be written as

$$n_D^q = \frac{1}{A^q} \{V_D[\mu_D - P_D r] + V_{DF}[\mu_F - P_F r]\}, \quad (16)$$

$$n_F^q = \frac{1}{A^q} \{V'_{DF}[\mu_D - P_D r] + V_F[\mu_F - P_F r]\}. \quad (17)$$

Define $n_D^f = \sum_{q \in F} n_D^q$ and $n_F^f = \sum_{q \in F} n_F^q$ as the aggregate demand vectors of the foreign country, and $\frac{1}{A^F} = \sum_{q \in F} \frac{1}{A^q}$ as the aggregate risk tolerance of the foreign country. Then, the aggregate demand functions of the foreign country is given by

$$n_D^f = \frac{1}{A^F} \{V_D[\mu_D - P_D r] + V_{DF}[\mu_F - P_F r]\}, \quad (18)$$

$$n_F^f = \frac{1}{A^F} \{V'_{DF}[\mu_D - P_D r] + V_F[\mu_F - P_F r]\}. \quad (19)$$

III. Equilibrium Asset Pricing

To arrive at equilibrium asset prices, we can aggregate demand across the two countries and apply the market-clearing conditions. The market-clearing conditions require that

$$n_i^d + n_i^f = N_i, \quad i \in D \quad (20)$$

$$n_i^d \leq \delta N_i \quad \text{and} \quad n_i^f = N_i - n_i^d, \quad i \in F \quad (21)$$

$$n_i^d \geq 0, \quad i \in F. \quad (22)$$

The condition in (20) states that the total demand for domestic securities should equal the supply. The conditions in (21) are the δ constraint, while condition (22) is the short-sale restriction. Depending upon the binding nature of the δ constraint, the asset pricing relationship will change. We consider the case where

the δ constraint in (21) is binding on all securities of the foreign country.⁴ Since the δ constraint is binding, it is clear that foreign securities will be held long, in aggregate, by the domestic investors. Therefore, the short-sale restriction need not be considered.

When the δ constraint is binding on the domestic investors, it implies that their demand for the foreign security i is higher than δN_i . Because of the constraint, they are supplied with only δN_i . Since the demand exceeds the supply, the domestic investors will be willing to pay a price higher than what they would have paid under no restrictions. Similarly, for the foreign investors, their demand would have been less than $(1 - \delta)N_i$ for the foreign security i but the supply is $(1 - \delta)N_i$. Since the demand is less than the supply, the securities will be selling at a discount for the foreign investors. Thus, there will be two different prices for the foreign securities. In order to preclude arbitrage opportunities, it is assumed that foreign investors cannot purchase the foreign security at a lower price and sell it at a higher price to the domestic investors. This is not an unreasonable assumption since, with the δ constraint in force, the government generally has other restrictions on the foreigners who are trading in the local market, which act to eliminate such arbitrage opportunities.⁵

A. The Relationship between Premiums and Discounts

Let the price of foreign securities under no restrictions be P_F^* . Let π and λ be, respectively, the premium paid by the domestic investors over P_F^* and the discount from P_F^* demanded by the foreign investors. Then the foreign asset prices for domestic and foreign investors, P_F^d and P_F^f , are given by

$$P_F^d = P_F^* + \pi, \quad (23)$$

$$P_F^f = P_F^* - \lambda. \quad (24)$$

Since there are no restrictions on trading in domestic securities, domestic security prices will be the same for both domestic and foreign investors and will equal the prices under no restrictions. In what follows, we will be mainly concerned with the determination of π and λ .

To determine π and λ , it is necessary first to derive the relationship between these two quantities.

PROPOSITION 1. *The premium offered by the domestic investors over the price under no constraints is a multiple of the discount demanded by the foreign investors,*

⁴ The alternative cases will be examined once we solve this case.

⁵ An example of the controls imposed by the government can be seen in Sweden. In case the Swedish securities are already owned by the foreigners, they can be purchased using the "switch currency," which is the proceeds of a sale of Swedish securities by a foreign investor. Unless the stock is already owned by the foreigners, its purchase must be approved by the Sveriges Riksbank, the central bank. Foreign ownership in most Swedish companies is usually limited to 20% of the voting shares or to 40% of the total share capital. This restriction is accomplished by issuing four types of shares—Class A "restricted" and "free" and Class B "restricted" and "free." The foreigners can only buy free shares. Class B shares have a lower voting right than Class A shares. Usually 80% or more of the Class A shares issued are "restricted." For more details, refer to ABD Securities [1] and Esslen [6].

the multiple being the ratio of the aggregate risk aversion of the domestic and foreign investors.

Proof: Aggregating demand across the two countries and using equations (23) and (24), we obtain the aggregate world demand functions:

$$\begin{bmatrix} N_D \\ N_F \end{bmatrix} = \frac{1}{A^W} \begin{bmatrix} V_D & V_{DF} \\ V'_{DF} & V_F \end{bmatrix} \begin{bmatrix} \mu_D - P_D r \\ \mu_F - P_F^* r \end{bmatrix} - \frac{1}{A^D} \begin{bmatrix} V_D & V_{DF} \\ V'_{DF} & V_F \end{bmatrix} \begin{bmatrix} 0 \\ \pi r \end{bmatrix} + \frac{1}{A^F} \begin{bmatrix} V_D & V_{DF} \\ V'_{DF} & V_F \end{bmatrix} \begin{bmatrix} 0 \\ \lambda r \end{bmatrix}, \quad (25)$$

where

$$N_D = n_D^d + n_D^f; \quad N_F = n_F^d + n_F^f; \quad \text{and} \quad \frac{1}{A^W} = \frac{1}{A^D} + \frac{1}{A^F}.$$

Rewriting equation (25) in terms of price, we obtain

$$\begin{bmatrix} P_D \\ P_F^* \end{bmatrix} = \frac{1}{r} \left\{ \begin{bmatrix} \mu_D \\ \mu_F \end{bmatrix} - A^W \begin{bmatrix} \Gamma_D & \Gamma_{DF} \\ \Gamma'_{DF} & \Gamma_F \end{bmatrix} \begin{bmatrix} N_D \\ N_F \end{bmatrix} + \frac{A^W}{A^D} \begin{bmatrix} 0 \\ \pi \end{bmatrix} - \frac{A^W}{A^F} \begin{bmatrix} 0 \\ \lambda \end{bmatrix} \right\}. \quad (26)$$

Under no δ constraints, the pricing relationship is given by

$$\begin{bmatrix} P_D \\ P_F^* \end{bmatrix} = \frac{1}{r} \left\{ \begin{bmatrix} \mu_D \\ \mu_F \end{bmatrix} - A^W \begin{bmatrix} \Gamma_D & \Gamma_{DF} \\ \Gamma'_{DF} & \Gamma_F \end{bmatrix} \begin{bmatrix} N_D \\ N_F \end{bmatrix} \right\}. \quad (27)$$

From equations (26) and (27), we obtain $(A^W/A^D)\pi = (A^W/A^F)\lambda$. This implies that

$$\pi = \frac{A^D}{A^F} \lambda, \quad (28)$$

which is Proposition 1. Q.E.D.

The above result shows that the premium the domestic investors are willing to pay is proportional to their aggregate risk aversion measure, *ceteris paribus*. This follows logically from the fact that the more risk averse the domestic investors are collectively, the higher the premium they would be willing to pay for foreign securities in order to avoid diversification loss.

B. The International Asset Pricing Relationships

Invoking the market-clearing conditions for foreign securities under the δ constraint, i.e., $n_i^d = \delta N_i$, $n_i^f = (1 - \delta)N_i$, $i \in F$, and substituting $P_F^d = P_F^* + \pi$ and $P_F^f = P_F^* - \lambda$, we can rewrite equations (15) and (19) as follows:

$$A^D \delta N_F = \{V'_{DF}[\mu_D - P_D r] + V_F[\mu_F - P_F^* r]\} - V_F \pi r, \quad (29)$$

$$A^F (1 - \delta) N_F = \{V'_{DF}[\mu_D - P_D r] + V_F[\mu_F - P_F^* r]\} + V_F \lambda r. \quad (30)$$

Subtracting equation (30) from equation (29), we obtain

$$[A^F - (A^D + A^F)\delta]N_F = V_F(\pi + \lambda)r. \quad (31)$$

Substituting equation (28) for π in equation (31) and rearranging, we have

$$\lambda = \frac{1}{r} [A^F(1 - \delta) - A^W] V_F^{-1} N_F. \tag{32}$$

Finally, noting that⁶

$$V_F^{-1} = [\Gamma_F - \Gamma'_{DF} \Gamma_D^{-1} \Gamma_{DF}], \tag{33}$$

we obtain the following expression for the discount demanded by the foreign investors:

$$\lambda = \frac{1}{r} [A^F(1 - \delta) - A^W] [\Gamma_F - \Gamma'_{DF} \Gamma_D^{-1} \Gamma_{DF}] N_F. \tag{34}$$

Substituting equation (34) into equation (28) and rearranging, we obtain the following expression for the premium offered by the domestic investors:

$$\pi = \frac{1}{r} [A^W - A^D \delta] [\Gamma_F - \Gamma'_{DF} \Gamma_D^{-1} \Gamma_{DF}] N_F. \tag{35}$$

The equilibrium asset pricing relationships can then be written as,

$$P_D = \frac{1}{r} \{ \mu_D - A^W [\Gamma_D N_D + \Gamma_{DF} N_F] \}, \tag{36}$$

$$P_F^d = \frac{1}{r} \{ \mu_F - A^W [\Gamma'_{DF} N_D + \Gamma_F N_F] + (A^W - A^D \delta) [\Gamma_F - \Gamma'_{DF} \Gamma_D^{-1} \Gamma_{DF}] N_F \}, \tag{37}$$

$$P_F^f = \frac{1}{r} \{ \mu_F - A^W [\Gamma'_{DF} N_D + \Gamma_F N_F] - (A^F(1 - \delta) - A^W) [\Gamma_F - \Gamma'_{DF} \Gamma_D^{-1} \Gamma_{DF}] N_F \}. \tag{38}$$

Equations (36) to (38) provide the basic asset pricing relationships under the δ constraint. The domestic securities are priced as if there were no δ constraint. Subjected to δ constraints, however, the foreign securities are priced differently in the domestic and foreign countries, reflecting the premia offered by the domestic investors and discounts demanded by the foreign investors. From the equilibrium asset pricing relationships presented above follows Proposition 2.

PROPOSITION 2. *Given the aggregate risk-aversion parameters, the equilibrium premium and discount on a foreign security critically depend on (i) the severity of the δ constraint and (ii) the “pure” foreign market risk, defined as the covariance of the security with the foreign market portfolio minus the covariance of that security with the “adjustment” portfolio which is the portfolio of domestic securities most highly correlated with the foreign market portfolio.*

Although Proposition 2 is implicit in the pricing relationships (37) and (38), it can be seen more clearly if we rewrite them in scalar terms:

$$P_i = \frac{1}{r} \{ \mu_i - A^W \text{cov}(P_i, V_M) \}, \quad i \in D \tag{39}$$

⁶ For the derivation of equation (33), refer to Hadley [12].

$$P_i^d = \frac{1}{r} \{ \mu_i - A^W \text{cov}(P_i, V_M) + (A^W - A^D \delta) [\text{cov}(P_i, V_F) - \text{cov}(P_i, V_A)] \}, \quad i \in F \quad (40)$$

$$P_i^f = \frac{1}{r} \{ \mu_i - A^W \text{cov}(P_i, V_M) - (A^F(1 - \delta) - A^W) [\text{cov}(P_i, V_F) - \text{cov}(P_i, V_A)] \}, \quad i \in F \quad (41)$$

where V_M , V_F , and V_A are, respectively, the value of the world market portfolio, the value of the foreign market portfolio, and the value of the adjustment portfolio.⁷ Equations (40) and (41) provide the pricing relationships of the foreign securities across a continuum of δ values for $0 < \delta < A^W/A^D$.⁸ It is clear from equations (40) and (41) that as the restriction becomes tighter, i.e., δ decreases, both the premium (π) and the discount (λ) increase.⁹

In the special case where an asset's covariance with the foreign market portfolio is equal to that with the adjustment portfolio, i.e., $\text{cov}(P_i, V_F) = \text{cov}(P_i, V_A)$, there will be neither premium nor discount on the foreign security, i.e., $\pi = \lambda = 0$. This means that the security will be selling at the same price, P_i^* , in both the domestic and foreign countries. The two covariances will be equal to each other when the adjustment portfolio is perfectly correlated with the foreign market portfolio. When such perfect correlation exists, domestic investors can achieve the same international diversification by holding the home-made adjustment portfolio as by holding foreign securities. The domestic investors thus would not offer premia to hold foreign securities, since these securities are redundant in terms of spanning the investment opportunity set. In the absence of such perfect correlation, holding the adjustment portfolio cannot be a perfect substitute for holding foreign securities and, as a result, the domestic investors would offer a premium to avoid the diversification loss.¹⁰

⁷ Note that the adjustment portfolio A , the composition of which is given by $\Gamma_D^{-1} \Gamma_{DF} N_F$, is the one the returns of which replicate the returns to the foreign market portfolio F more closely than any other feasible domestic portfolio.

⁸ The δ constraint is no longer binding for $\delta \geq \frac{A^W}{A^D}$, as the portfolio holding of domestic investors under no restrictions is given by $n_i^d = \frac{A^W}{A^D} N_i, i \in F$.

⁹ In the special case of $\delta = 0$, where the domestic investors are not allowed to invest in the foreign securities at all, P_i^d becomes irrelevant. The price of the foreign security i facing the foreign investors, P_i^f , however, will be reduced to $P_i^f = (1/r) \{ \mu_i - A^W \text{cov}(P_i, V_M) - (A^F - A^W) [\text{cov}(P_i, V_F) - \text{cov}(P_i, V_A)] \}, i \in F$ which is essentially identical to the pricing relationship derived by Errunza-Losq [5].

¹⁰ In deriving the portfolio choices and the asset pricing model, individuals were assumed to have exponential utility functions. The key results concerning the asset pricing as well as the portfolio holdings, however, do not hinge on this assumption. Under the assumptions of normally distributed security returns and concave utility functions, "rational" investors would maximize the expected utility (EU) which can be written solely in terms of the mean (\bar{W}) and the variance (V_w) of the future wealth. Then, for a representative investor k , the optimal portfolio rule can be generally written as follows:

$$n^k = - \frac{(\partial EU^k / \partial \bar{W}^k)}{(\partial EU^k / \partial V_w^k)} \Gamma^{-1} (\mu - Pr).$$

Due to the fact that foreign securities are in short supply and thus available only at a premium, domestic investors would try to economize on them. They do so by holding a portfolio of domestic securities as a (partial) substitute for foreign securities. This consideration gives rise to the following proposition:

PROPOSITION 3. *The domestic investors' demand function for domestic securities can be viewed as consisting of two portfolios: one that they would hold under no restrictions, i.e., the domestic market portfolio, and the other, an adjustment portfolio, that is held to adjust for the δ constraint. The more severe the δ constraint, the more of the adjustment portfolio they hold.*

Proof: Substituting equations (37) and (38) into the aggregate demand functions (14) and (18) and rearranging the result, we obtain the proposition:

$$n_D^d = \frac{A^W}{A^D} N_D + \left(\frac{A^W}{A^D} - \delta \right) [\Gamma_D^{-1} \Gamma_{DF} N_F], \quad (42)$$

$$n_D^f = \frac{A^W}{A^F} N_D - \left(\frac{A^W}{A^D} - \delta \right) [\Gamma_D^{-1} \Gamma_{DF} N_F], \quad (43)$$

where the first term on the RHS of each equation represents the aggregate demand for domestic securities under no δ constraints. As can be seen in equations (42) and (43), the foreign investors, in effect, supply the adjustment portfolio to the domestic investors. Given that the adjustment portfolio comprises only domestic securities, the existence of δ constraints introduces "home-bias" in portfolio holdings of foreign as well as domestic investors.

In this model, we have considered the case where the δ constraint is binding on all foreign securities. The other types of binding constraints can also be included in the model. If the δ constraint is non-binding, but a short-sale restriction is binding on all securities, then it is equivalent to a self-imposed restriction that δ be zero. If the δ constraint is binding on only some securities, then those securities for which the δ constraint is non-binding can be included in the portfolio of domestic (unrestricted) securities for the purpose of asset pricing. If δ varies across securities instead of being uniform for all securities, the model should be modified to account for varying δ . Finally, Table II gives the comparative statics of the asset pricing and portfolio composition under complete segmentation, under complete integration, and under a δ constraint.

The preceding equation shows the well-known separation theorem stating that investors would hold the same portfolio, with the utility function only affecting the dollar amount to be invested in the portfolio. In other words, the utility function will only affect the scale factor of the portfolio demand via its effects on the marginal rate of substitution between risk and return (MRS). When individuals have exponential utility functions, the MRS is constant and invariant to wealth. But for logarithmic as well as quadratic utility functions, the MRS is a function of expected wealth, \bar{W}^k . Since the expected wealth itself is a function of the portfolio demands, n^k , the portfolio demand function is not closed. This is one of the reasons why we chose to work with the exponential utility function. To recapitulate, the utility function affects the scale factor of the portfolio demand, but not the composition of the optimal portfolio and, consequently, neither relevant risk measures in asset pricing nor the basic structure thereof is dependent upon the particular utility function assumed.

Table II
Asset Pricing and Portfolio Composition under Alternative Market Structures

	Complete Segmentation	Complete Integration ^a ($\delta \geq A^W/A^D$)	δ -Constraint ($0 < \delta < A^W/A^D$)
Asset pricing			
P_D	$\frac{1}{r} \{ \mu_D - A^D [\Gamma_D N_D] \}$	$\frac{1}{r} \{ \mu_D - A^W [\Gamma_D N_D + \Gamma_{DF} N_F] \}$	$\frac{1}{r} \{ \mu_D - A^W [\Gamma_D N_D + \Gamma_{DF} N_F] \}$
P_D^f	NA ^b	$\frac{1}{r} \{ \{ \mu_F - A^W [\Gamma_{DF} N_D + \Gamma_F N_F] \}$	$\frac{1}{r} \{ \mu_F - A^W [\Gamma_{DF} N_D + \Gamma_F N_F] + (A^W - A^D \delta) [\Gamma_F - \Gamma_{DF} \Gamma_D^{-1} \Gamma_{DF}] N_F \}$
P_F	$\frac{1}{r} \{ \mu_F - A^F [\Gamma_F N_F] \}$	$\frac{1}{r} \{ \{ \mu_F - A^W [\Gamma_{DF} N_D + \Gamma_F N_F] \}$	$\frac{1}{r} \{ \mu_F - A^W [\Gamma_{DF} N_D + \Gamma_F N_F] - [A^F (1 - \delta) - A^W] [\Gamma_F - \Gamma_{DF} \Gamma_D^{-1} \Gamma_{DF}] N_F \}$
Portfolio rules			
n_D^d	N_D	$\frac{A^W}{A^D} N_D$	$\frac{A^W}{A^D} N_D + \left(\frac{A^W}{A^D} - \delta \right) \left[\Gamma_D^{-1} \Gamma_{DF} N_F \right]$
N_D^b	0	$\frac{A^W}{A^F} N_D$	$\frac{A^W}{A^F} N_D - \left(\frac{A^W}{A^D} - \delta \right) \left[\Gamma_D^{-1} \Gamma_{DF} N_F \right]$
N_F^f	0	$\frac{A^W}{A^D} N_F$	δN_F
n_f	N_F	$\frac{A^W}{A^F} N_F$	$(1 - \delta) N_F$

^a This case clearly encompasses the situation where there exists no δ -constraint, i.e., $\delta = 1$.

^b NA, not applicable.

IV. Numerical Analysis

To supplement the theoretical analysis presented in the earlier section, a numerical analysis is conducted in a 20-person, 8-firm, 2-country world, to arrive at the equilibrium prices of the securities for varying δ constraints. These prices will then be compared with those which would have been obtained under complete integration as well as under complete segmentation.

The description of the model economy is given in Table III. Firms 1 to 4 belong to the domestic country and firms 5 to 8 belong to the foreign country. The details of the number of shares outstanding for each firm, the expected value of the firm, \bar{V}_i , at the end of the period, the standard deviation, σ_{V_i} , of the value of the firm and the correlation coefficients are given in the upper part of the table. The correlation matrix conforms to the well-known fact that securities are less positively correlated across countries than within a country. The lower part of the table gives the constant absolute risk aversion (CARA) parameters for the 10 domestic investors and the 10 foreign investors.¹¹

The equilibrium asset price, the expected rate of return for each firm, and the aggregate portfolio holdings for the domestic and foreign investors are given in Table IV under alternative market structures, namely,

- (i) complete segmentation in which the investors of any one country can invest only in their own country firms;
- (ii) complete integration in which there are no restrictions placed on international investment; and
- (iii) a δ constraint in which investors of the domestic country are restricted to hold a fraction of the number of shares outstanding of the foreign firms not more than δ .

In this economy, we find that the optimal portfolio holding for domestic investors under no restrictions is 560 shares in each of the foreign firms or that they demand 56% of the outstanding shares. If $\delta < 56\%$, then the constraint will be binding on them. We have considered three different δ values ($\delta = 40\%$, $\delta = 20\%$, and $\delta = 0\%$) in computing the prices, rates of return, and portfolio holdings.

From Table IV, it can be seen that the prices and the rates of return for the domestic firms do not change even if there are δ constraints. But the prices of the foreign firms for domestic investors are different from those for foreign investors. As δ decreases or as the restriction becomes tighter, the premium paid by domestic investors increases, and so does the discount demanded by foreign investors. Accordingly, the expected rates of return on foreign securities decrease (increase) for the domestic (foreign) investors as δ decreases. It can also be seen from Table IV that foreign security prices reach their lowest levels when domestic investors are not allowed to invest in foreign securities at all, i.e., when $\delta = 0$.

The portfolio composition also changes under δ constraints as compared to complete integration. Under complete integration, domestic investors hold equal number of shares of the domestic firms. Under δ constraints, they hold a

¹¹ The model economy presented in Table III is identical to that of Stapleton and Subrahmanyam [17]. This will make it possible to compare the δ constraint with various other forms of market imperfections considered by them in terms of portfolio choices and asset pricing implications.

Table III
Description of the Model Economy^a

Firms	No. of Shares (N_i)	$\bar{V}_i = \mu_i N_i$ (\$)	$\sigma_{V_i} = \sigma_i N_i$	Correlation Matrix					
1	1,000	100,000	18,000	1.0	0.7	0.9	0.1	0.1	0.1
2	1,000	100,000	22,000	0.7	1.0	0.7	0.9	0.1	0.1
3	1,000	100,000	18,000	0.1	0.1	1.0	0.7	0.1	0.1
4	1,000	100,000	22,000	0.1	0.1	0.1	1.0	0.1	0.1
5	1,000	100,000	25,000	0.1	0.1	0.1	0.1	1.0	0.7
6	1,000	100,000	30,000	0.1	0.1	0.1	0.1	0.7	0.9
7	1,000	100,000	25,000	0.1	0.1	0.1	0.1	1.0	0.7
8	1,000	100,000	30,000	0.1	0.1	0.1	0.1	0.7	1.0
Domestic Investors									
			CARA Parameter ^b			Foreign Investors			CARA Parameter ^b
1			7,600			11			6,000
2			7,800			12			6,200
3			8,000			13			6,400
4			8,200			14			6,500
5			8,400			15			6,600
6			8,500			16			6,800
7			8,800			17			7,000
8			9,000			18			7,200
9			9,500			19			7,400
10			10,000			20			7,500

^a Initial wealth is \$17,000 for each investor, and the risk-free interest rate is 8%.

^b The numerical value measures the individual investor's absolute risk tolerance, i.e., $1/A^k$.

Table IV
Equilibrium in International Capital Market

Assets	Complete Segmentation	Complete Integration	δ -constraints ^a		
			$\delta = 40\%$	$\delta = 20\%$	$\delta = 0\%$
Equilibrium Asset Prices					
1	79.96	84.34	84.34	84.34	84.34
2	76.69	82.23	82.23	82.23	82.23
3	79.96	84.34	84.34	84.34	84.34
4	76.69	82.23	82.23	82.23	82.23
5	61.96	77.90	81.68/73.11	86.44/67.06	61.02
6	54.80	74.50	79.16/68.58	85.03/61.13	53.68
7	61.96	77.90	81.68/73.11	86.44/67.06	61.02
8	54.80	74.50	79.16/68.58	85.03/61.13	53.68
Expected Rates of Return (%)					
1	25.06	18.57	18.57	18.57	18.57
2	30.40	21.61	21.61	21.61	21.61
3	25.06	18.57	18.57	18.57	18.57
4	30.40	21.61	21.61	21.61	21.61
5	61.39	28.37	22.43/36.78	15.69/49.12	63.88
6	82.48	34.23	26.32/45.81	17.60/63.58	86.29
7	61.39	28.37	22.43/36.78	15.69/49.12	64.88
8	82.48	34.23	26.32/45.81	17.60/63.58	86.29
Portfolio Composition ^b					
1	1000/0	560/440	589/411	626/374	663/337
2	1000/0	560/440	584/416	613/387	644/356
3	1000/0	560/440	589/411	626/374	663/337
4	1000/0	560/440	584/416	613/387	644/356
5	0/1000	560/440	400/600	200/800	0/1000
6	0/1000	560/440	400/600	200/800	0/1000
7	0/1000	560/440	400/600	200/800	0/1000
8	0/1000	560/440	400/600	200/800	0/1000

^a The two figures for assets 5 to 8 indicate the prices in the first panel (rate of return in the second panel) for the domestic investors and for the foreign investors, respectively.

^b The two figures show the aggregate number of shares held by the domestic/foreign country investors.

higher number of shares of domestic firms 1 and 3 than of firms 2 and 4. This is because firms 1 and 3 provide greater diversification than firms 2 and 4. The results in Table IV also show that domestic (foreign) investors will hold a larger (smaller) share of each of the domestic firms under a δ constraint than under complete integration.

V. Conclusions and Summary

In this paper, we derived optimal portfolio choices and equilibrium asset pricing relationships under a specific form of barrier to international investment—

namely, the δ constraint. For the sake of analytical simplicity, we assumed that there are only two countries in the world—one domestic and one foreign, and that the domestic investors are constrained to own a fraction of the outstanding shares of the foreign firms not greater than δ , while the foreign investors do not face such restrictions on their investment in domestic firms. The major findings of the paper can be summarized as follows.

First, for foreign securities to which the δ constraint applies, there are two ruling prices, a higher one for domestic investors and a lower one for foreign investors. This two-tier pricing relationship reflects a premium offered by domestic investors over the equilibrium price with no constraints and a discount demanded by foreign investors. The premium is a multiple of the discount, the multiple being the ratio of the aggregate risk aversion of the domestic investors to that of the foreign investors.

Second, both the equilibrium premium and the discount are determined by the severity of the δ constraint on the one hand and the “pure” foreign market risk on the other. The more severe the δ constraint, and, at the same time, the higher the pure foreign market risk, the higher premium (discount) the domestic (foreign) investors pay. The domestic securities to which no such constraints apply are priced as if there were no constraints at all.

Third, in order to minimize the diversification loss under the δ constraint, the domestic investors hold an “adjustment” portfolio of domestic securities that is the most highly correlated with the foreign market portfolio. The more severe the δ constraint, the more of the adjustment portfolio they hold.

In this paper, we made a number of simplifying assumptions, including (i) that δ constraints apply to foreign securities, but not to domestic securities, and (ii) that the value of δ is uniform across all securities. The present paper may be extended by relaxing any of these assumptions.

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