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Dynamic Macroeconomics. Final December 9, 2013.
(4 questions.)

1. (30%) Consider an economy with a constant population of identical agents who maximize

$$\Sigma_{t=0}^{\infty} \beta^t u(c_t) \ ,$$

with $0 < \beta < 1$ and $u(c) = \frac{1}{1-\sigma} c^{1-\sigma}$, who have access to a constant-returns-to-scale production function

$$F(K, X) = X * f(\hat{K}) \ , \quad \text{where} \quad \hat{K} = K/X \ .$$

K is physical capital with a depreciation rate of δ and there is one good so output can be consumed or used as capital. X is the contribution of labor. We assume the first order derivatives are positive and second order derivatives negative and the usual Inada conditions are satisfied.

a) Write down the Euler equation for investment in physical capital.

b) If labor input is constant at $X_t = L$ find an expression for the steady state capita-labor ratio and consumption.

c) Now assume labor-augmenting technological change at the gross rate $(1 + \mu)$, where

$$X_t = A_t \, L, \ \text{with} \ \ A_t = (1 + \mu) \, A_{t-1} \ .$$

Find an expression for the optimal steady state capital labor ration.

2. (20%) a) Explain why a dynamic panel regression with cross sectional fixed effects of form

$$y_{it} = \mu_i + \alpha \, y_{i,t-1} + u_{it} \ ,$$

gives a biased estimate of α for fixed (small) time dimension T.

b) What is the order of the magnitude of the bias?

3. (25%) Bellman equations.
a) Write down the Bellman equation. (Explain the definition of all terms.)
b) State (best to derive, but you will full points if you state if correctly) the Benveniste-Scheinkman equation (assuming all functions are differentiable).
c) Find the Euler equation implied by the Bellman equation and the Benveniste-Scheinkman formula under a certain assumption (explain which).

4. (15%) For a structural VAR model

$$A_0 y_t = C + A_1 y_{t-1} + ... + A_k y_{t-k} + u_t \ ,$$

give 2 examples of ways to identify a the model.

5. (10%) Consider a Markov Chain with transition matrix

$$\begin{bmatrix} .7 & .3 & 0 \\ 0 & .8 & .2 \\ 0 & .2 & .8 \end{bmatrix} .$$

Find the unique stationary (invariant distribution). (Hint: you can try and find a certain eigenvector, but if you look at the transition matrix for a while you should be able to guess/deduce the invariant distribution. Even if you guess it, you need to verify it.)