VC 12311 135

CBM003 ADD/CHANGE FORM

APPROVED APR 24 2013

☒ **Undergraduate Council** ☐ **New Course** ☒ **Course Change** **Core Category:** **Math/Reason** **Effective Fall** ~~2013~~ 2014	or	☐ **Graduate/Professional Studies Council** ☐ **New Course** ☐ **Course Change** **Effective Fall 2013**

1. Department: Mathematics College: NSM
2. Faculty Contact Person: Charles Peters Telephone: 743-3516 Email: charles@math.uh.edu
3. Course Information on New/Revised course:
   - Instructional Area / Course Number / Long Course Title: MATH / 1432 / Calculus II

     RECEIVED APR - 4 2013
   - Instructional Area / Course Number / Short Course Title (30 characters max.) MATH / 1432 / CALCULUS II
   - SCH: 4.00 Level: FR CIP Code: 27.0101.00.01 Lect Hrs: 4 Lab Hrs: 0
4. Justification for adding/changing course: **To meet core curriculum requirements**
5. Was the proposed/revised course previously offered as a special topics course? ☐ Yes ☒ No

   If Yes, please complete:
   - Instructional Area / Course Number / Long Course Title: ____ / ____ / _____
   - Course ID: _____ Effective Date (currently active row): _____
6. Authorized Degree Program(s): _____
   - Does this course affect major/minor requirements in the College/Department? ☐ Yes ☒ No
   - Does this course affect major/minor requirements in other Colleges/Departments? ☐ Yes ☒ No
   - Can the course be repeated for credit? ☐ Yes ☒ No (if yes, include in course description)
7. Grade Option: Letter (A, B, C ...) Instruction Type: lecture ONLY (Note: Lect/Lab info. must match item 3, above.)
8. If this form involves a change to an existing course, please obtain the following information from the course inventory: Instructional Area / Course Number / Long Course Title MATH / 1432 / Calculus II
   - Course ID: 31105 Effective Date (currently active row): 8272012
9. Proposed Catalog Description: (If there are no prerequisites, type in "none".)

   Cr: 4. (4-0). Prerequisites: MATH 1431. Description (30 words max.): Calculus of transcendental functions: additional techniques and applications of integration, indeterminate forms, improper integrals, Taylor's formula, and infinite series.
10. Dean's Signature: ______________________________ Date:__________

    Print/Type Name: _____

REQUEST FOR COURSES IN THE CORE CURRICULUM

Originating Department or College: Department of Mathematics
Person Making Request: Charles Peters Telephone: 713-743-3516
Email: charles@math.uh.edu
Dean's Signature: ____________________________ Date: 2/13/2013

Course Number and Title: MATH 1432: Calculus II
Please attach in separate documents:

☒ Completed CBM003 Add/Change Form with Catalog Description

☒ Syllabus

List the student learning outcomes for the course (Statements of what students will know and be able to do as a result of taking this course. See appended hints for constructing these statements):
Students will extend their understanding of the ideas and techniques of differentiation and integration to the transcendental functions. They will develop proficiency in techniques of integration and apply them to stated problems of growth and decay, area, length, volume, and work. Students will be able to make appropriate use of polar or rectangular coordinates depending on the geometry of a given problem. They will be able to describe plane curves parametrically when appropriate and analyze their properties. They will understand sequences and series and be able to apply several tests for convergence. Students will understand Taylor expansions and be able to work with error bounds for the Taylor approximation.

Component Area for which the course is being proposed (check one):
***Note:** If you check the Component Area Option, you would need to also check a Foundational Component Area.

☐ Communication
☐ Mathematics
☐ Language, Philosophy, & Culture
☐ Creative Arts
☐ Life & Physical Sciences
☐ American History
☐ Government/Political Science
☐ Social & Behavioral Science
☒ Component Area Option

Competency areas addressed by the course (refer to appended chart for competencies that are required and optional in each component area):

- ☒ Critical Thinking
- ☒ Communication Skills
- ☒ Empirical & Quantitative Skills
- ☐ Teamwork
- ☐ Social Responsibility
- ☐ Personal Responsibility

Because we will be assessing student learning outcomes across multiple core courses, assessments assigned in your course must include assessments of the core competencies. For each competency checked above, indicated the specific course assignment(s) which, when completed by students, will provide evidence of the competency. Provide detailed information, such as copies of the paper or project assignment, copies of individual test items, etc. A single assignment may be used to provide data for multiple competencies.

Critical Thinking:
Several examples of exercises and assignments addressing critical thinking competencies are attached.

Communication Skills:
See attached.

Empirical & Quantitative Skills:
See attached.

Teamwork:
Click here to enter text.

Social Responsibility:
Click here to enter text.

Personal Responsibility:
Click here to enter text.

Will the syllabus vary across multiple section of the course? ☐ Yes ☒ No

If yes, list the assignments that will be constant across sections:
Click here to enter text.

Inclusion in the core is contingent upon the course being offered and taught at least once every other academic year. Courses will be reviewed for renewal every 5 years.

The department understands that instructors will be expected to provide student work and to participate in university-wide assessments of student work. This could include, but may not be limited to, designing instruments such as rubrics, and scoring work by students in this or other courses. In addition, instructors of core courses may be asked to include brief assessment activities in their course.

Dept. Signature: ____

The following courses have been reviewed and approved by the NSM Curriculum Committee to meet the new core requirements. Given the length of the individual submissions I have elected to submit these requests by electronic means only.

Natural Sciences: Core Courses

BIOL 1309 – Human Genetics and Society

BIOL 1310 – General Biology

BIOL 1320 – General Biology

BIOL 1361 - Introduction to Biological Science I

BIOL 1362 - Introduction to Biological Science II

CHEM 1301 – Foundations of Chemistry

CHEM 1331 – Fundamentals of Chemistry I

CHEM 1332 – Fundamentals of Chemistry II

GEOL 1302 - Introduction to Global Climate Change

GEOL 1330 - Physical Geology

GEOL 1340 - Introduction to Earth Systems

GEOL 1350 - Introduction to Meteorology

GEOL 1360 - Introduction to Oceanography

GEOL 1376 - Historical Geology

PHYS 1301 - Introductory General Physics I

PHYS 1302 - Introductory General Physics II

PHYS 1321 - University Physics I

PHYS 1322 - University Physics II

Mathematics: Core Courses

MATH 1310 – College Algebra

MATH 1311 – Elementary Mathematical Modeling

Math/Reasoning: Core Courses

COSC 1306 – Computer Science and Programming

MATH 1330 - Precalculus

MATH 1431 - Calculus I

MATH 1432 – Calculus II

MATH 2311 – Introduction to Probability and Statistics

Writing in the Disciplines: Core Courses

BCHS Biochemistry Lab II

BIOL 3311 – Genetics Lab

PHYS 3313 – Advanced Lab I

Ian Evans

Associate Dean

4/4/13

Math 1432

Section 13209
MWF 10:00-11:00, 100 SEC

Instructor: Dr. Jeff Morgan, 651 PGH, jmorgan@math.uh.edu.

Office Hours: 11:00 - Noon MWF or by appointment.

Course Homepage: http://www.math.uh.edu/~jmorgan/Math1432

Course Learning Materials: The textbook, online quizzes, EMCF assignments, and additional help materials will be made available by logging into *CourseWare* at http://www.casa.uh.edu. The first portion of these materials are freely available for the first two weeks of class. All students must purchase a ***Course Access Code*** and enter it on *CourseWare* by the first day of the third week of class to continue accessing the course learning materials. A ***Course Access Code*** can be purchased for $47.35 from the University Bookstore. If you want a physical copy of the text for the course, then purchase *CALCULUS, 9th edition.* Authors: Salas/Hille/Etgen. Publisher: John Wiley & Sons, Inc. **Note: You do not need to purchase a physical copy of this text.** You will have access to the text electronically on *CourseWare* once you enter your ***Course Access Code***. Also, even if you purchase a physical copy of the text, you will still need the ***Course Access Code*** to access the additional learning materials, including the online electronic quizzes and EMCF assignments.

Additional Learning Materials: Lecture notes, videos and other materials will be posted on the course homepage.

The material covered in the course is listed below:

Chapter 7. THE TRANSCENDENTAL FUNCTIONS

Section 7.1. One-to-One Functions;Inverses
Section 7.2-3. The Logarithm Function
Section 7.4. The Exponential Function
Section 7.5. Arbitrary Powers; Other Bases; Estimating e
Section 7.6. Exponential Growth and Decay
Section 7.7. The Inverse Trigonometric Functions
Section 7.8. The Hyperbolic Sine and Cosine Functions

Chapter 8. TECHNIQUES OF INTEGRATION

Section 8.2. Integration by Parts
Section 8.3. Powers and Products of Trigonometric Functions
Section 8.4. Trigonometric Substitutions
Section 8.5. Partial Fractions

Section 8.7. Numerical Integration

Chapter 9. POLAR COORDINATES; PARAMETRIC EQUATIONS

Section 9.3. Polar Coordinates
Section 9.4. Graphing in Polar Coordinates
Section 9.5. Area in Polar Coordinates
Section 9.6. Curves Given Parametrically
Section 9.7. Tangents to Curves Given Parametrically
Section 9.8 Arc Length and Speed

Chapter 10. SEQUENCES; INDETERMINATE FORMS; IMPROPER INTEGRALS

Section 10.1-2. The Least Upper Bound Axiom; Sequences of Real Numbers
Section 10.3-4. Limit of a Sequence; Some Important Limits
Section 10.5. The Indeterminate Form (0/0)
Section 10.6. The Indeterminate Form (∞/∞); Other Indeterminate Forms
Section 10.7. Improper Integrals

Chapter 11. INFINITE SERIES

Section 11.1. Infinite Series
Section 11.2. The Integral Test; Comparison Theorems
Section 11.3. The Root Test; The Ratio Test
Section 11.4. Absolute and Conditional Convergence; Alternating Series
Section 11.5. Taylor Polynomials in x; Taylor Series in x.
Section 11.6. Taylor Polynomials in x-a; Taylor Series in x-a.
Section 11.7. Power Series
Section 11.8. Differentiation and Integration of Power Series

Weekly Homework: Homework will be collected on Monday in recitation, starting the second week of class, except on weeks when recitation does not meet on Monday. In these cases, additional information will be given. A list of problems will be posted on the course home page along with instructions. Many of the homework sets will come in part from this URL.

Daily Poppers: Daily grades will be given in lecture beginning the first day of the third week of class. You need to purchase a course packet of Popper Forms for Math 1432 section 13209 from the **BOOK STORE**. You must bring one of these forms to class every day beginning week 3. No other form will be accepted. Questions will be asked in lecture at random times. You will mark your answers on your form and drop the form in a box at the end of class. Your forms will not be returned.

EMCF: "EMCF" stands for "Electronic Multiple Choice Form". EMCF assignments are answered on *CourseWare* using the EMCF tab. The EMCF assignment questions will be posted on the course home page. EMCF assignments will typically be due each Monday, Wednesday and Friday of the semester. Please see the course calendar page for more information.

Written Quizzes: Written quizzes will be given every Friday in recitation. Quizzes will be returned in recitation. If you do not attempt all of the questions on the written homework that is due the following week, or you do not follow the instructions, then your grade on the written quiz will automatically become a 0.

Online Quizzes: At least one online quiz will be given each week. You can attempt these quizzes up to 20 times, and the highest grade will be used for your score. If you fail to reach 70% during three weeks of the semester, **I have the option** to drop you from the course. You can access the quizzes by logging into *CourseWare* at http://www.casa.uh.edu.

Exams: All sections of Math 1431 take common exams. Four regular exams will be given during the semester. The first exam is an online exam that will be available by the first day of class at http://www.casa.uh.edu. The other three exams will be given in CASA (located on the second floor of Garrison). You can access the scheduler for these exams by logging into *CourseWare* at http://www.casa.uh.edu. The exams given in CASA will consist of both multiple choice and written questions. The multiple choice questions will be machine graded. The written questions will be graded by the instructors and teaching assistants for all sections of Math 1432, and they will be returned in lab. The scheduler will be available 2 weeks prior to the start of the exam cycle.

Final Exam: A comprehensive final exam will be given in CASA. You can access the scheduler for this exam by logging into *CourseWare* at http://www.casa.uh.edu.

Grades:

400 points determined by exams 1, 2, 3 and 4 (100 points each)
100 points determined by weekly written quizzes and online quizzes (equally weighted).
100 points determined by Daily Grades and Daily EMCF (equally weighted).
200 points determined by the final exam
800 points total

Note: The percentage grade on the final exam can be used to replace your lowest test score.

90% and above - A
at least 80% and below 90%- B
at least 70% and below 80% - C
at least 60% and below 70% - D

below 60% - F

Attendance is Mandatory!! Attendance will be taken in lab, and the daily poppers will be used to determine your attendance in lecture. I will allow you a total of 3 unexcused absences from lecture and lab (total). You will lose 1% of your grade for every unexcused absence from lecture or lab after the third. Documented University of Houston excused absences will be permitted.

Whenever possible, and in accordance with 504/ADA guidelines, we will attempt to provide reasonable academic accommodations to students who request and require them.

Math 1432 Calculus II – Topics List

THE TRANSCENDENTAL FUNCTIONS
One-to-One Functions;Inverses
The Logarithm Function
The Exponential Function
Arbitrary Powers; Other Bases; Estimating e
Exponential Growth and Decay
The Inverse Trigonometric Functions
The Hyperbolic Sine and Cosine Functions

TECHNIQUES OF INTEGRATION
Integration by Parts
Powers and Products of Trigonometric Functions
Trigonometric Substitutions
Partial Fractions
Numerical Integration

POLAR COORDINATES; PARAMETRIC EQUATIONS
Polar Coordinates
Graphing in Polar Coordinates
Area in Polar Coordinates
Curves Given Parametrically
Tangents to Curves Given Parametrically
Arc Length and Speed

SEQUENCES; INDETERMINATE FORMS; IMPROPER INTEGRALS
The Least Upper Bound Axiom; Sequences of Real Numbers
Limit of a Sequence; Some Important Limits
The Indeterminate Form (0/0)
The Indeterminate Form (∞/∞); Other Indeterminate Forms
Improper Integrals

INFINITE SERIES
Infinite Series
The Integral Test; Comparison Theorems
The Root Test; The Ratio Test
Absolute and Conditional Convergence; Alternating Series
Taylor Polynomials in x; Taylor Series in x.
Taylor Polynomials in x-a; Taylor Series in x-a.
Power Series
Differentiation and Integration of Power Series

Below are some representative questions from various assessment pieces that demonstrate our commitment to the three objectives. We begin with Test 2 as Test 1 is a readiness test over prerequisite materials

Part A: Critical Thinking Skills

Developing and deepening **Critical Thinking Skills** is a large part of a student's successful completion of Math 1432. To this end, when we assess a student's work, we use both multiple choice questions and free response questions.

Test 2 – Question 1 – Multiple Choice

Find the slope of the normal line to the graph of $f(x) = 2 \ln(\sec(x))$ at the point where $x = \pi/6$.

This question tests the student's understanding of the concept of slope of the normal line as it relates to the derivative of the function. The student must evaluate the derivative and then combine this with prior knowledge of the relationship between the tangent line and the normal line to properly answer the question.

Test 2 – Question 7 – Free Response

Part a: Solve ${}^{dy}\!/_{dx} = -2y$ given that $y(0) = 2$.

Part b: Suppose that a bacteria's population doubles every 3 hours. If there are 10^6 bacteria in the population at present, what was the size of the population 4 hours ago?

In part a, the student must evaluate the given information and determine the appropriate method to solve the differential equation. In part b, the student extends knowledge of the process to a different setting.

Test 2 – Question 8 – Free Response

Compute the following:

$$\int \frac{\cosh(x)}{\sqrt{36 - \sinh^2(x)}}\,dx$$

In computing this integral, the student must recognize the presence of both the function sinh(x) and it's derivative and use this knowledge to evaluate the integral using an appropriate change of variables.

Test 2 – Question 9 – Free Response

Compute:

$$\int \tan^4(x)\sec^4(x)\,dx$$

This question tests the student's ability to create and innovate. The student must decide among numerous methods involving different changes of variable to select the appropriate one, and must combine this with knowledge of trigonometric identities to appropriately evaluate the integral.

Test 3 – Question 8 – Free Response

Part a: Set up the integral needed to compute

$$\int \frac{1}{x^2\sqrt{49 - x^2}}\,dx$$

using trigonometric substitution. DO NOT EVALUATE THE INTEGRAL.

Part b: Suppose that the trigonometric substitution $x = 3\sin u$ is used to compute an integral and the answer to the integral is

$$\frac{1}{2}u - \frac{1}{2}\sin(u)\cos(u) + C$$

Finish the problem by rewriting the answer in terms of x.

This question requires synthesis on information given to the student. In particular, part b requires to student to combine knowledge of a given change of variables with previous knowledge of trigonometric identities to simplify the expression.

Test 4 – Question 9 – Parts a, b, c – Free Response

Part a: For which of the following limits can L'Hopital's Rule be applied? DO NOT COMPUTE THE LIMIT.

(A) $\lim_{x\to 9} \frac{\sqrt{x} - 3}{x - 9}$

(B) $\lim_{x\to 0} \frac{\cos(x)}{(3x)}$

(C) $\lim_{x\to \infty} \frac{x^2}{3 - x^2}$

(D) $\lim_{x\to 0} \frac{\cos(x) - \cos(3x)}{\sin(x^2)}$

(E) $\lim_{x\to 1} \frac{\ln(x)^3}{x}$

Part b: For each of your answers in part a in which L'Hopital's Rule can be applied, state the indeterminant form. DO NOT COMPUTE THE LIMIT.

Part c: Compute:

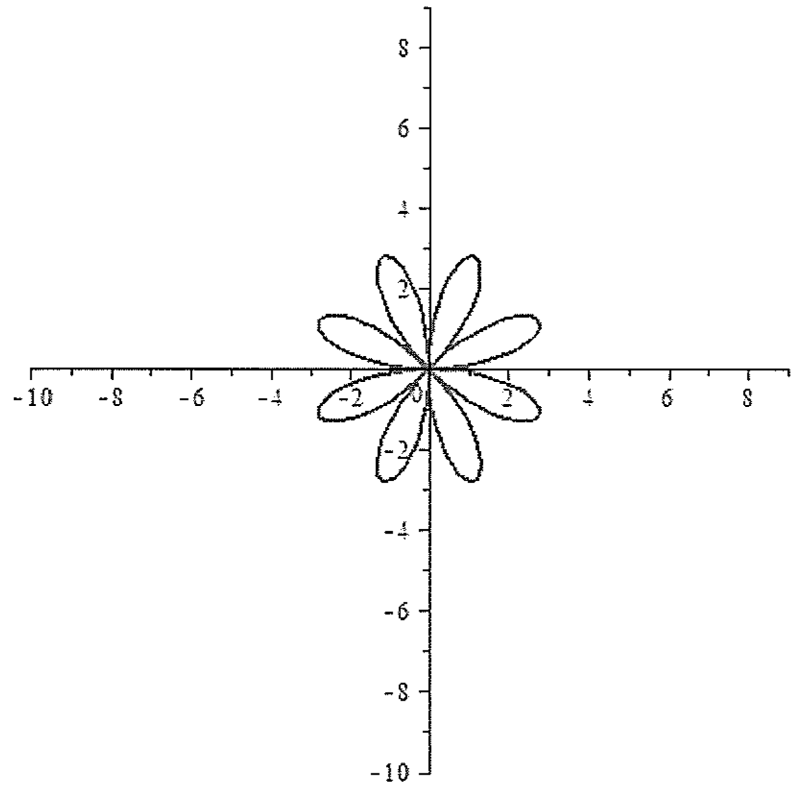
$$\lim_{x\to\infty} \left(x^2 + 1\right)^{\frac{3}{x}}$$

This question requires the student to think inquisitively about each of the limits presented to determine which limit meet the appropriate criteria for the application of L'Hopital's Rule.

Part c requires use of complex techniques in analysis to evaluate the limit using L'Hopital's Rule. The limit in part c is not straightforward, the student must first consider the limit of the exponentiated form of the function, and then use natural log to relate this limit to the original limit.

Final Exam – Question 8 – Multiple Choice

Which of the following represents the area inside **one** petal of $r = 3\sin(4\theta)$?



a) ○ $\displaystyle\int_0^{\frac{1}{8}\pi} \frac{1}{2}(3\sin(4\theta))^2\,d\theta$

b) ○ $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1}{2}(3\sin(4\theta))^2\,d\theta$

c) ○ $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{1}{2}(3\sin(4\theta))\,d\theta$

d) ○ $\displaystyle\int_0^{\pi} \frac{1}{2}\left(3\sin(4\theta)\right)^2 d\theta$

e) ○ $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{1}{2}\left(3\sin(4\theta)\right)^2 d\theta$

This question requires the student to think critically about the graph and identify the appropriate subset of the domain, and appropriate integral for the desired area.

Final Exam – Question 17 – Free Response

Compute:

$$\int_0^8 x^{\left(-\frac{2}{3}\right)}\, dx$$

Be sure to use proper limit notation.

Here the student must pause to identify the integral as improper and use analysis techniques involving limits to rewrite the integral before beginning to integrate.

Final Exam – Question 18 – Free Response

Given:

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{2^k (k+1)}$$

Part a: Find the radius of convergence.
Part b: Find the interval of convergence.
Part c: Give the antiderivative, F, of the power series so that F(0) = 0.

The student must analyze the given power series to determine the radius of convergence. For part b, the interval of convergence requires the student to ask questions about the behavior of the power series at the endpoint of the inverval. Part c requires students to combine knowledge of power series with prior knowledge of integration techniques.

Part B: Communication Skills

Communication Skills are important in this course. We consistently require the student to express the *solution* to a problem, not simply give the answer. In many exam questions, the correct final answer is worth no points unless the entire solution is appropriately communicated. You can find examples of these types of questions below.

Test 2 – Question 6 – Free Response

Part a: Solve $dy/dx = -2y$ given that $y(0) = 4$.

Part b: A 100 liter tank initially full of water develops a leak at the bottom. Given that 20% of the water leaks out in the first 5 minutes, find the amount of water left in the tank t minutes after the leak develops if the water drains off at a rate that is proportional to the amount of water present.

Here the student must communicate the *solution* in part a, little credit is given for the final answer unless the solution presented is logical and mathematically sound. Also, in part b, the student must provide a mathematical formula that provides the amount of water present at an arbitrary time.

Test 3 – Question 10 – Free Response

For each of the following, state whether the sequence converges or diverges. If the sequence converges, find the limit. If the sequence diverges, explain why.

Part a: $\left\{\dfrac{6n^4+5}{4n^3+4n^2+3}\right\}$

Part b: $\left\{\ln\left(\dfrac{6n}{3n+1}\right)\right\}$

Part c: $\left\{\tan\left(\dfrac{n\pi}{4n+6}\right)\right\}$

In each of the parts, the correct answer is worth very little. The student must effectively communicate the method that they used to reach their conclusion and explain their reasoning.

Test 4 – Question 10 – Free Response

Part a: Determine whether or not each of the following integrals are improper. Give a reason for each of your answers. DO NOT COMPUTE THE INTEGRAL.

(A) $\displaystyle\int_0^1 \frac{1}{(1-x)^{2/3}}\,dx$

(B) $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

(C) $\displaystyle\int_1^\infty \frac{1}{x}\,dx$

(D) $\displaystyle\int_{-1}^1 e^x\,dx$

Part b: Using proper limit notation, compute the following improper integral:

$$\int_0^\infty \frac{1}{1+x^2}\,dx$$

For this question, the student *must* explain their reasoning. If the integral is improper, the student must explain why. For part b, the student must communicate the solution using proper mathematical notation.

Test 3 – Question 11 – Free Response

Determine if each of the following series converges or diverges. State which test you use and show all of your work.

(A) $\displaystyle\sum \frac{2^{3k}}{k^k}$

(B) $\displaystyle\sum \frac{3k-1}{2k^3+k}$

(C) $\displaystyle\sum \frac{(-1)^k}{2k+1}$

Again, the correct answer (converge/diverge) is worth very few points. The student must correctly explain their reasoning, including which method/series test was used and why they reach their conclusion.

Part C: Empirical and Quantitative Skills

Empirical and Quantitative Skills are very important in Calculus II. We have many questions that focus on manipulation of data and facts.

Test 2 – Question 4 – Free Response

Part a: If $f(x)$ is differentiable and invertible, $f'(x)$ is nonzero, and $f(a) = b$, give a formula for $(f^{-1})'(b)$

Part b: Given

$$f(x) = -4x^3 - x - 9$$

verify that $f(x)$ is invertible.

Part c: Using the function in part b, note that $f(-2) = 25$. Find $(f^{-1})'(25)$.

Part d: Find the equation of the tangent line to $f^{-1}(x)$ at the point where $x = 25$.

Here the student is given a function along with particular data about the function and is asked to process this information to obtain useful results about the derivative of the inverse of the function as well as the equation of the line tangent to the graph of the inverse function.

Test 3 – Question 3 – Multiple Choice

Find a parameterization, $x = x(t)$, $y = y(t)$, $t \in [0, 1]$, for the line segment from (2, -3) to (-2, 5).

The student is given two data points and asked to provide a parameterization of the segment connecting these two points.

Test 3 – Question 6 – Free Response

Given

x	$f(x)$
0	-8
0.5	5
1	2
1.5	-4
2	9
2.5	-10
3	-3
3.5	8
4	-7
4.5	-10
5	4

and the integral

$$\int_0^3 f(x)\,dx$$

Part a: Use the trapezoid method with $n = 3$ to approximate the integral.

Part b: Use the midpoint method with $n = 3$ to approximate the integral.

Part c: Use Simpson's rule with $n = 3$ to approximate the integral.

This is a classic example of data analysis. The student is given bits of data about a function and asked to approximate the integral of the function over the given interval using 3 different methods.

Test 4 – Question 12 – Free Response

Let f be a function that has derivatives of all orders on the interval (-1, 1).
Part a: Use the values in the table below and the formula for Taylor polynomials to give the 5th degree Taylor polynomial for f centered at $x = 0$.

$f(0)$	$f'(0)$	$f''(0)$	$f'''(0)$	$f^{(4)}(0)$	$f^{(5)}(0)$
-2	-2	4	-5	1	5

Part b: Assume that $|f^{(6)}(x)| \leq 20$ for all x in the interval (0, 1). If you used your answer to part (a) to estimate $f(0.1)$, what is the maximum possible error?
You do not have to simplify your answer.

For this question, the student analyzes several bits of information about the function and its first 5 derivatives to generate the Taylor polynomial. Then, in part b, the student must estimate the error if this Taylor polynomial were used to approximate a function value.

Final Exam – Question 1 – Multiple Choice

Suppose $f(x)$ is an invertible differentiable function and $f(1) = 5$, $f(5) = -3$, $f'(1) = 3$, $f'(5) = 2$.
Evaluate the derivative of the inverse of f at 5.

Here the student has no knowledge of f apart from the given data points and the fact that f is both invertible and differentiable. The student must using this given data to give information about the derivative of the inverse of f.

Final Exam – Question 19 – Free Response

Part a: Give the Taylor series for

$$f(x) = e^{4x}$$

centered at $x = 0$.

Part b: Find $f^{(7)}(0)$ for the series in part a.

Part c: Let

$$f(x) = \sin(x)$$

Use the Lagrange formula to find the smallest value of n so that the n^{th} degree Taylor polynomial centered at $x = 0$ approximates f at $x = 1$ with an error of no more than 0.001.
You may use the following table to help with your calculations:

5!	6!	7!	8!	9!	10!	11!	12!
120	720	5,040	40,320	362,880	3,628,800	39,916,800	479,001,600

Analysis of data is again an important concept. For this question the student is asked to guarantee that their approximation meets a certain tolerance.