Density-stable yield-stress displacement flow of immiscible fluids in inclined pipes

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A B S T R A C T

We experimentally study the displacement flow of a non-Newtonian yield-stress fluid by an immiscible Newtonian one in an inclined pipe. The pipe has a small diameter-to-length ratio. The less dense displacing fluid is placed above the denser displaced fluid i.e. a density-stable configuration. The displacing and displaced solutions are oil and water-based respectively. The flow between the Newtonian and non-Newtonian fluids has been studied over a wide range of controlling parameters, namely imposed flow rate, inclination angle, density difference, yield stress, viscosity, and surface tension. Compared to the previously studied miscible limit, we observe novel fracturing behavior at the interface between the two fluids; this fracture pattern is preserved in the non-Newtonian gel as the advancing oil flows through. Instead of both slump and center-type flows observed in the miscible case, we primarily observe center-type flows, which maintain similar flow profiles across mean imposed flow rates $\dot{V}_o$ in the study. Similarly to the miscible case, the inclination angle $\beta$ is not found to have an effect on the velocity of the advancing frontal region of the displacing fluid, $\dot{V}_g$. The average thickness of the residual yield-stress layer in this study was found to be 0.14 of the radius, decreasing to 0.12 of the radius when higher viscosity oils are used as the displacing fluid. The residual layer thickness slightly increases with Reynolds number, $Re$, but shows no dependency on the density difference between the two fluids captured by the Atwood number, $At$. Finally, the unevenness in the yield-stress residual layer for immiscible fluids is found to be significantly less than that of the miscible case.

1. Introduction

The displacement of a gelled material from a pipe has several industrial applications, including food processing [1], water treatment [2], personal care [3], and oil and gas [4]. The latter has significantly motivated the present study. During the cementing phase of a conventional oil and gas well, a series of water-based fluids are often pumped into the wellbore, which can be at diverse tilt angles from the vertical direction [4]. However, for deep water and unconventional shale wells, high performance oil-based mud (OBM) can instead be used in order to reduce swelling problems and drilling risks [5,6]. The existence of both water and oil-based phases during removal or cleaning operations introduces immiscibility effects which, in turn, can significantly increase the complexity of the problem.

In the absence of an imposed flow i.e. exchange configuration, the interpenetration of miscible Bingham fluids has been studied by Frigaard and Scherzer [7]. A flow regime was identified, in which, an unyielded cement slurry core slumped within a ring of yielded drilling mud due to buoyancy. Vargas et al. [8] has recently studied immiscible buoyancy-driven exchange flows with a more-dense yield-stress fluid placed above a less-dense Newtonian oil in a vertical pipe. Three flow regimes have been identified namely unstable, quasi-stable, and stable. A core-annular flow was observed in the unstable regime, a plug flow which began after a time delay was observed in the quasi-stable regime, and no flow was observed under the stable regime.

In the present manuscript, we investigate the effect on an imposed flow which relates most directly to two most recent studies in the literature: Alba and Frigaard [9] and Oladosu et al. [10]. The former considers a density-unstable configuration for a miscible Newtonian/yield-stress fluid pair, whereas the latter explores a density-stable configuration of a Newtonian/Newtonian fluid pair in the immiscible limit. In Alba and Frigaard [9] two major flow regimes were observed: 1) Center-type flows, in which the displacing fluid cores out a path through the center of the (carbopol) gel, and 2) slump-type flows, in which the displacing fluid slumps underneath the lighter yield-stress layer. The flows in the displacing layer were found to be mostly in the laminar regime [9]. Higher displacement efficiencies are observed for the center-type flows compared to the slump-type ones. Oladosu et al. [10] on the other hand reported that immiscible fluids in a density-stable configuration demonstrate a novel effect where the oil-water interface remains "pinned" to

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the entrance of the pipe due to differences in wetting properties of fluids at the wall. It was further observed that efficiency of removal of displaced fluid can be reduced by as much as 14% in the immiscible case compared to the miscible one, particularly at close-to-horizontal tilt angles and high imposed flow rates.

The significant contribution of our study is to analyze the combination of density-stable, immiscible, and yield stress flow effects over a wide range of flow rates and pipe inclination angles for the first time. As discussed in this manuscript, the interplay between these three effect results in original flow regimes with significant application in cementing of wells drilled using OBM. The paper is organized as follows: Section 2 discusses the experimental set-up as well as the range of dimensional and dimensionless governing parameters used in this study. Section 3 investigates the effects of variation of governing parameters on flow regime. A brief summary is provided towards the end in Section 4.

2. Experimental set-up

2.1. Apparatus & measurement

Our experiments have been conducted in a 2-m-long acrylic pipe of inner diameter \( D = 9.53 \text{ mm} \), the same one used in the previous experimental study [11]; see Fig. 1 for a schematic representation. In this paper, dimensional/dimensionless parameters are denoted with/without symbol. The light fluid density is denoted by \( \rho_L \); while that of the heavy fluid is labeled \( \rho_H \). The displaced fluid in the experiments is a carbolipin solution (Carbomer 940, Making cosmetics Co.) densified by glycerol (Natural Essentials Inc.) in the range of 0–23% by volume, resulting in a density of \( \rho = 999, 1068 \text{ kg m}^{-3} \). The less dense displacing fluid is an oil; in most experiments, silicone oil of density \( \rho_L = 918 \text{ kg m}^{-3} \) and viscosity \( \mu_L = 0.005 \text{ Pa s} \). The surface tension between the fluids was measured with a Sigma 701 Force Tensiometer from Biobin Scientific Inc. The device has been successfully calibrated against air-water (\( \gamma = 72.9 \text{ mN m}^{-1} \)) and air-silicone oil (\( \gamma = 22.1 \text{ mN m}^{-1} \)) [11]. To measure the surface tension between the gel-oil interface, a Du-Nouy ring is immersed in the gel and pulled gradually upward toward the less dense oil. The maximum force required for the ring to break free of the gel surface is then used to calculate the interfacial tension as described in [12]. Details of the preparation procedure and rheology of the yield-stress gel (flow sweep, frequency sweep, and creep tests) are given in Appendices A, Appendix B respectively, with the result of the Herschel-Bulkley parameters (\( \dot{\gamma} = \dot{\gamma}_r + \dot{\gamma}_S \)) given in Table 1. Here, \( \dot{\gamma}_r \) is the applied shear stress, \( \dot{\gamma}_S \) the shear rate, \( \dot{\gamma}_y \) the yield stress, and \( \dot{\gamma}_r \) and \( \dot{\gamma}_S \) are the power-law and flow consistency indices respectively [13–15].

2.2. Experiment procedure

During an experiment sequence, the angle of the pipe is fixed and several experiments are run at that angle using different flow rates and fluids density differences (total of 93). Prior to each experiment, the pipe is filled above the gate valve with the displacing fluid and below the gate valve with the displaced fluid. At the start of each experiment, the gate valve is opened, and the flow moves under the influence of gravity towards the drain (Fig. 1). The volumetric flow rate, \( Q = \pi D^2 V_0 / 4 \), is controlled by the adjustment of a needle valve located before the drain and is measured using a beaker and a stopwatch. This method, which is accurately calibrated in our test experiments, showed superiority over rotameter and magnetic flow meter measurements due to existence of oil and water mixtures with non-standard viscosity (\( \mu \neq 0.001 \text{ Pa s} \)) [10]. Here, \( V_0 \) is the mean imposed velocity. The experiments are captured using a high-speed black-and-white digital camera (Basler Ace acA2040-90um CMOS, 2048\(^2\) pixels) with 4096 gray scale levels that facilitate analysis of a wide range of concentrations. The camera captures the entire 2-m length of the pipe within its field of view using a high-resolution lens (16 mm F/1.8. C-mount) and records images at a rate of 12–71 Hz, depending on the imposed flow rate. A well-tested MATLAB image processing code is used to convert gray scale images from the camera into color pictures for presentation and analysis purposes [11]. Following [10], details of a soap-water cleaning procedure between the experiments is provided in Appendix C.

2.3. Parameter range

An analysis of the flow in this study reveals several relevant dimensional and dimensionless parameters. Displacements in which the yield stress dominates the characteristic viscous stress of the Newtonian fluid are considered, i.e.:

\[
\dot{\mu} \dot{V}_0 / D = \dot{\gamma}_r \ll \dot{\gamma}_S,
\]

where \( \dot{\mu} \) is the viscosity of the Newtonian fluid, \( \dot{V}_0 \) is the mean imposed velocity, and \( D \) is the diameter of the pipe [14]. Otherwise formulated, this condition states

\[
B_N \gg 1, \quad B_N = \frac{\dot{\gamma}_S D}{\mu_0 V_0}.
\]

where \( B_N \) is the Bingham number. This condition also gives rise to the Herschel-Bulkley dimensionless parameter \( HBY \), a ratio of the yield

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<th>Table 1</th>
<th>Rheological properties of different carbolipin solutions used as the displaced fluid in our experiments.</th>
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Fig. 1. Schematic view of the experimental set-up used. Interface shape is illustrative only.
stress to the non-Newtonian viscous stress:

\[
H_{BN} = \frac{\hat{\tau}_x}{\hat{\tau}_y + k(\hat{V}_y/D)\hat{\mu}}.
\]  

(3)

This parameter is analogous to the Bingham number \(B_N = \frac{\hat{\tau}_x \hat{\mu}}{\hat{\tau}}\) and employs a Herschel-Bulkley model for representing the viscous stress instead. To capture the ratio of the viscous stress of the yield-stress fluid to the viscous stress of the Newtonian fluid, a viscous stress ratio \(N_v\) is calculated as:

\[
N_v = \frac{\hat{\tau}_x + k(\hat{V}_y/D)\hat{\mu}}{\hat{\mu}(\hat{V}_y/D)}.
\]  

(4)

The pipe inclination angle, \(\beta\), ranges from vertical (\(\beta = 0^\circ\)) to near-horizontal directions (\(\beta = 85^\circ\)). Due to the relevance to cementing and cleaning operations in industry, experiments are conducted in the \(\delta \ll 1\) range, where \(\delta = \hat{D}/\hat{L}\) represents the aspect ratio of the pipe \([11,16]\). The Atwood number, \(At = (\hat{\rho}_L - \hat{\rho}_H)/(\hat{\rho}_L + \hat{\rho}_H)\), is a measure of the density difference of the light and heavy fluids. For this study, \(\approx 0.075 < At < -0.042\), as we focus on the density-stable configuration. Due to the magnitude of At in our study, both Boussinesq and weakly non-Boussinesq effects may be relevant \([11]\). The Reynolds number is defined as \(Re = \hat{V}_y \hat{D}/\hat{\nu}\), where \(\hat{\nu}\) is the kinematic viscosity based on the average density of the fluids (\(\hat{\nu} = (\hat{\rho}_L + \hat{\rho}_H)/2\)) and viscosity of the light oil, \(\hat{\mu}\). The densimetric Froude number, \(Fr = \hat{V}_y \sqrt{\hat{g} \hat{D}}\), captures the relationship between the inertial and buoyancy forces. The effect of immiscibility in the system is captured via the Capillary number, \(Ca = \hat{\mu} \hat{V}_y \hat{\rho}_L / \hat{g}\) (ratio of viscous to surface tension stresses) arises from the immiscibility considered in the flow \([11]\). The range of dimensional and dimensionless parameters is shown in Table 2. Both are provided in figure captions for convenience.

3. Results

3.1. Flow characterisation

Firstly, we present the main features of immiscible density-stable yield-stress displacement flows. Fig. 2a shows a typical experiment in which silicone oil displaces carbopol gel. The interface between the oil and the gel is smooth, similar to the Newtonian case (Fig. 2b), though the “pinning” effect associated with an elongated front \([10]\) is absent in the yield-stress case. Further, a region of fracturing occurs near the gate valve (pictured in green) in which the oil cuts irregularly through the gel before proceeding in a more uniform flow profile. This region remains static over time (the gel is not fully swept away) even after the frontal region of the imposing light fluid has moved further down the pipe. Lastly, the color of the fluid within the pipe region is shown to be of a concentration \(C\) that is between 0 and 1 (\(C \approx 0.85\)), in comparison with \(C = 0\) for silicone oil and \(C = 1\) for carbopol gel, indicating that a combination of the two fluids is present in the pipe region. This is in agreement with the miscible density-unstable displacement flow of \([9]\) shown in Fig. 2c, where \(C \approx 0.65\) shows that the invading heavy salt water does not fully evacuate the carbopol gel. The difference between miscible and immiscible yield-stress flows is evident from the concentration profiles shown in Fig. 2a and c.

Spatio-temporal diagrams of the depth-averaged concentration field, \(\hat{C}_i(x, t)\), for the same experiments as Fig. 2 are shown in Fig. 3. These figures further elucidate the contrast between the concentration differences (as shown by the color) in the Newtonian (Fig. 3b) and the yield-stress cases, an effect that is due to the yield stress of the displaced fluid. It can be seen that in Fig. 2a, the slope created by the oil-carbopol interface steepens towards the end of the experiment. The gel leaves the pipe over time, bringing head loss down and allowing for a higher flow rate. Nevertheless, instabilities in the flow, marked by a light yellow region near the gate valve, can be seen to maintain their position over time. The slight decrease in head loss is inevitable over long-time yield-stress experiments due to continuous evacuation of the yield-stress fluid. The reported imposed velocities, \(\hat{V}_y\), in this paper represent an average value throughout the experiment. Compared to the orange color of the immiscible case, the light green of the miscible case in Fig. 3c indicates that the fluid in the pipe is closer to displaced yield-stress gel (in blue) than it is to displacing salt water (in red). This indicates that the thickness of the static gel layer left behind in the pipe is larger in the miscible than in the immiscible case, an effect that will be quantified in Section 3.3.

3.2. Fracturing of yield-stress layer

In the classification regime presented in \([13,14]\), slump-type displacements have two frontal regions: a faster-moving thin layer front near the bottom of the pipe and a slower-moving thicker front that follows. Center-type displacement flows feature residual fluid layers of non-uniform thickness along the length of the pipe. The flows in this study are center-type, with regions of instabilities that approach the exotic nature of corkscrew and helical-type flows reported in \([14]\). Fig. 4 shows an example of a common fracturing or ripple instability observed across inclination angles in our experiments. The mechanism behind this pattern is explored here. Snapshots of the displacement flow and the corresponding spatiotemporal diagram are presented in Fig. 4a and b respectively. An initial region of fracturing near the gate valve extends until \(\hat{x} \approx 300\) mm, covering that distance within the first 2 s of the flow. The fractured region then begins to travel downstream slowly between 2.5 s until the end of the experiment (\(\approx 6s\)). It can be observed that the fracturing instabilities emerge during a period of high shear in the flow when the invading oil imposes a velocity on the stationary gel in a short time. This causes a pattern very similar to the Kelvin-Helmholtz (KH) instabilities observed in the study of immiscible density-unstable Newtonian flows by Hasnain et al. \([11]\). The appearance of KH instabilities in \([11]\) has been quantitatively captured via the Richardson number,
Fig. 2. (a) Immiscible density-stable yield-stress, (b) immiscible density-stable Newtonian (taken from Oladosu et al. [10]), and (c) miscible density-unstable yield-stress displacements (taken from Alba and Frigaard [9]). Fig. 2b and 2c were obtained from previous works by the authors. The bottommost image in each panel is a colorbar of concentration $C$, with 0 and 1 referring to the displaced and displacing fluids respectively. (a) Snapshots of the displacement flow for Solution D, $\hat{\tau}_Y = 7.49$ Pa, $\beta = 45^\circ$, $\hat{\nu}_L = 139$ mm s$^{-1}$, $\hat{\rho}_H = 999$ kg m$^{-3}$ and $\hat{\rho}_L = 918$ kg m$^{-3}$, at times $\hat{t} =$[0.54, 1.42, 2.30, ..., 5.82, 6.7] s ($At = -0.042$, $Re = 254$, $Fr = 2.21$, $BN_H = 102.69$, $HB_N = 0.54$, $N_\eta = 191$, $Ca = 0.01$). The field of view is $1746 \times 9.53$ mm$^2$. (b) Snapshots of the displacement flow for $\beta = 45^\circ$, $\hat{\nu}_L = 456.9$ mm s$^{-1}$, $\hat{\rho}_H = 1181$ kg m$^{-3}$ and $\hat{\rho}_L = 918$ kg m$^{-3}$, at times $\hat{t} =$[0.42, 0.84, 1.26, ..., 2.94, 3.36] s ($At = -0.125$, $Re = 914$, $Fr = 4.22$, $BN_H = 0$, $N_\eta = 0$, $Ca = 0.65$, $\theta = 56^\circ$). The field of view is $1732 \times 9.53$ mm$^2$. (c) Snapshots of the displacement flow for $\hat{\tau}_Y = 14.89$ Pa, $\beta = 60^\circ$, $\hat{\nu}_L = 77.4$ mm s$^{-1}$, $\hat{\rho}_H = 1007$ kg m$^{-3}$ and $\hat{\rho}_L = 1000$ kg m$^{-3}$, at times $\hat{t} =$[8, 8.5, ..., 10, 10.5] s ($At = 0.0035$, $Re = 1485$, $Fr = 3.01$, $BN_H = 3606.0$, $Ca = \infty$, $\theta = 90^\circ$). The field of view is $488 \times 19$ mm$^2$, taken 1733 mm below the gate valve. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 3. Spatio-temporal diagrams of depth-averaged concentration field, $\bar{C}_y(\hat{x}, \hat{t})$, obtained for the same experiments as shown in Fig. 2. Fig. 3b and 3c were obtained from previous works by the authors.
\( R_i \), which is the ratio of stabilizing potential energy of gravity and surface tension to destabilizing kinetic energy at the interface. The interesting difference between the current yield-stress case and Newtonian flow of Hasmann et al. [11] is that once these instabilities form, they become “frozen” in the fluid due to yield stress. After the flow settles into a more stable condition i.e. reduced shearing, as evidenced by a less steep slope from \( \approx 1 \) s onward in Fig. 4b, it progresses in a smooth center-type displacement observed in the study [14]. Note that at very high mean imposed velocities, the fracturing instabilities can persist throughout the length of the pipe, as shown in Fig. 4c. The fracturing patterns observed in our experiments are moreover investigated in relation to the work of Hickox [17] on linear stability analysis of immiscible core-annular flow. For the range of approximated viscous stress ratio, \( N_a \), and displacing area values, \( \alpha \) (see Section 3.3), studied among fracturing flows, we have found that Hickox’s theory predicts an unstable regime which is in agreement with our experimental observation.

Looking further into factors that drive the extent of the fracturing instability, a combination of high mean imposed velocity, \( \dot{V}_m > 318 \text{ mm s}^{-1} \), and low yield stress, \( \tau_y < 4.5 \text{ Pa} \), was observed to result in 9 experiments of 10 experiencing a fracture whose length extended to the end of the pipe, as in Fig. 4c. All of the experiments that experienced the pipe-length fracture were performed at \( A_t = -0.042 \); however the effect of density difference alone cannot be isolated, as the yield stresses at the higher \( A_t \approx -0.075 \) were not in the range \( \tau_y < 4.5 \text{ Pa} \). Other parameters in the problem, such as \( B_W, H_B, \beta, \text{ Re, Ca, Pr, and Wi} \) were similarly found to not significantly affect the fracturing behavior.

3.3. Displacing fluid area

In Alba and Frigaard [9]’s study, a quantitative method of interpreting the occupying areas of the displacing/displaced fluids across the duct is presented. Briefly, in this method the depth-averaged value of the concentration profile \( C \) (e.g. Fig. 5a) can be used to calculate a dimensionless area fraction, \( \alpha \), that represents the portion of the pipe cross-section that is “cored out” of the carbopol solution by the invading light oil [9].

The \( \alpha \) parameter is calculated as \( \alpha (C) \), where \( C \) is the depth and length averaged concentration in the pipe after the flow is fully developed. The standard deviation \( \sigma (C) \) of \( C \) can also be calculated for use in measuring an effective pipe roughness due to the residual layers of carbopol within the pipe. A schematic representation of \( \alpha \) is presented in Fig. 5b.

Fig. 5 presents depth-averaged concentration profiles for the experimental snapshots shown in Fig. 2a. A value of 1 represents a section of the pipe that contains only displacing silicone oil while a value of 0 represents a section that contains only displaced carbopol gel. Accordingly, a section with \( C \approx 0.7 \) represents the displacement of most but not all of the carbopol gel. Values of \( \alpha \) calculated from the concentration profiles of all yield-stress experiments performed with silicone oil are presented in Fig. 5c.

Fig. 4. (a) Snapshots of the displacement flow for \( \dot{V}_m = 4.44 \text{ Pa}, \beta = 0 \cdot \dot{V}_m = 173 \text{ mm s}^{-1}, \rho = 999 \text{ kg m}^{-3} \), and \( \rho = 918 \text{ kg m}^{-3} \), at times \( t = 0, 0.8, 1.6, \ldots, 4.8, 5.6 \) s \((A_t = -0.042, \text{ Re } = 316, \text{ Fr } = 2.74, B_W = 48.8, H_B = 0.44, N_a = 110, \text{ Ca } = 0.01)\). The field of view is \( 1743 \times 9.53 \text{ mm}^2 \). (b) Spatio-temporal diagram for the same experiment. (c) Snapshots of the displacement flow for \( \dot{V}_m = 3.95 \text{ Pa}, \beta = 85 \cdot \dot{V}_m = 3754 \text{ mm s}^{-1}, \rho = 999 \text{ kg m}^{-3} \), and \( \rho = 918 \text{ kg m}^{-3} \), at times \( t = 0, 0.03, 0.06, \ldots, 0.22, 0.25 \) s \((A_t = -0.042, \text{ Re } = 6899, \text{ Fr } = 60.14, B_W = 2.00, H_B = 0.15, N_a = 13, \text{ Ca } = 0.29)\). The field of view is \( 1743 \times 9.53 \text{ mm}^2 \). The bottommost image in panels A and C and the rightmost image in panel B is a colorbar of concentration \( C \), with 0 and 1 referring to the displaced and displacing fluids respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Fig. 5. (a) Experimental profiles of depth averaged concentration $C$ corresponding to the snapshots shown in Fig. 2a. (b) Schematic representation of dimensionless displacing fluid area $a$, defined as the area of displacing fluid to the total cross-sectional area of the pipe. (c) Variation of $a$ with $\beta$ for silicone oil and carboxol gel. The colorbar corresponds to values of the viscous stress ratio $N_a$. (d) Variation of $\alpha$ with $Re$ for silicone oil and carboxol gel. The colorbar corresponds to values of the Herschel-Bulkley number $HB$. Squares represent $At = -0.042$ (■) while diamonds represent $At = -0.075$ (●). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 6. Dependence of the viscous stress ratio $N_a$ on the displacing front velocity normalized by the mean imposed velocity, $\dot{V}_f/\dot{V}_o$. The colorbar corresponds to values of the Bingham number $B_N$. Squares represent $At = -0.042$ (■), diamonds represent $At = -0.075$ (●), circles represent $At = -0.110$ (○), and triangles represent $At = -0.104$ (▲). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$V_f$ (higher $\alpha$) corresponds with higher $N_a$ as well as higher $B_N$. For $N_a$, this effect is due to competing forces of non-Newtonian and Newtonian viscous stresses. From Fig. 5d, it was observed that increased $\alpha$ corresponds with increased $HB$, and the role of a reduced non-Newtonian viscous stress was highlighted for increased yield-stress gel removal. A higher $N_a$ corresponds with a higher non-Newtonian viscous stress, so it might be expected that lower $\alpha$ (higher $V_f$) would be observed with increased $N_a$. However, Fig. 6 shows the opposite trend. This indicates that the effect of reduced Newtonian viscous stresses is dominating increased non-Newtonian stresses to produce an overall correlation of $N_a$ with lower $V_f$ (higher $\alpha$). This effect of Newtonian viscous stress also results in the correlation of increased $B_N$ with decreased $V_f$ (increased $\alpha$), as shown by the colormap in Fig. 6. Further, when $\dot{V}_f$ is plotted against $\dot{V}_o$ (not shown), the best-fit of $\dot{V}_f = 1.45\dot{V}_o$ is constant across both Atwood numbers. This best-fit gives rise to a displacement efficiency of 69% ($\phi = \dot{V}_f/\dot{V}_o \approx 69\%$ [18]), which is higher compared to 62% ($\dot{V}_f = 1.62\dot{V}_o$) in the miscible density-unstable case [9].

3.4. Relative roughness in yield-stress layer

In order to assess the dynamics of the flow within the Newtonian layer, a measure of the relative roughness of the yield-stress layer can be attained using the displacing fluid area $a$ [9]. This roughness in the yield-stress layer represents the roughness of the almost-rigid “pipe”
through which the Newtonian fluid flows, and it can hypothetically be used with a Moody Chart to classify flows as *laminar*, *transition*, or *turbulent*. To begin, the hydraulic diameter is defined as \( D_h = 4A/P \), where \( A \) is the cross-sectional area and \( P \) is the wetted perimeter. For center-type flows, \( D_h = \sqrt{A/D} \) [9]. The hydraulic Reynolds number is then calculated as \( Re_h = \Re \sqrt{D/D} \). \( \Re \) here represents the Reynolds number of the fluid flowing within the hydraulic diameter, \( D_h \), or the Reynolds number of the Newtonian fluid within the yield-stress layer; see Fig. 7 for a schematic representation of these described parameters. In this study, \( Re_h \) increases with \( Re \) in both cases with smaller and larger density difference (not shown). Subsequently, the constricted flow diameter and hydraulic diameter are then defined taking a to be \( C + \sigma(C) \) and \( C - \sigma(C) \) for \( D_{cf} \) and \( D_h \) respectively, where \( C \) is the previously defined depth-averaged concentration across the pipe and \( \sigma \) is its standard deviation. From these parameters, the roughness can be calculated as \( \varepsilon = (D_h - D_{cf})/2 \) and the relative roughness as \( \varepsilon/D_h \) [19].

The values of the relative roughness for this study are presented in Fig. 8. Fig. 8a shows that at lower \( At \), an increased Bingham number, \( B_w \), has the effect of slightly reducing the relative roughness. This indicates that the combination of lower yield stresses and higher incoming Newtonian fluid velocity is more likely to create irregular channels within the yield-stress material in comparison to higher yield stresses and a slower-moving fluid (higher \( B_w \)), which results in a smoother path. In the miscible case studied by Gabard and Hulin [20], irregular channels were also observed in yield-stress materials, though with higher velocity displacing fluids carving out a smoother path through the yield-stress gel. This is likely because interfacial mixing in the miscible case takes place over a longer time period and is more extensive in a slower-moving flow, contributing to roughness of the path. The result in Fig. 8b shows that for \( At = 0.075 \), the range and average magnitude of relative roughness values decreases (from 0.014-0.068 to 0.016-0.043), indicating that a greater density difference has the effect of creating a smoother channel within the displaced fluid through which the displacing fluid flows. Accordingly, the \( At = 0.075 \) values of relative roughness are relatively constant across \( B_w \), demonstrating a reduced influence of yield stress and mean imposed velocity at higher density differences. \( Fr \) and \( Ca \) both increase with relative roughness at lower \( At \) (not shown); however their effect on flow behavior is minimal in comparison to other dimensionless parameters. Similar to the miscible case, \( \varepsilon/D_h \) has no monotonic dependence on \( \beta \), as shown in Fig. 8c. However, it is interesting to note that in this study, smaller roughness values are found at the high inclination of \( \beta = 15^\circ \) in both the aqueous and the aqueous-glycerol cases, while lower inclinations (\( \beta = 85^\circ \)) produce larger roughness values in both types of solutions as well. The effect of \( Re \) on the relative roughness is also not distinct, though in Fig. 8c it can be seen that higher Reynolds numbers tend to increase the relative roughness of the yield-stress layer. Compared to the miscible case, relative roughness for yield-stress flows is significantly reduced, in the range of \( 0.01 \leq \varepsilon/D_h \leq 0.07 \) across all experiments to compared to \( 0.03 \leq \varepsilon/D_h \leq 0.25 \) in the miscible case [9]. The effects of mixing and dissolving in the miscible case likely create irregular surfaces at the interface of the fluids while the absence of these types of interfacial interactions in the immiscible cause the oil to slide more smoothly over the surface of the yield-stress gel. On a standard Moody chart (valid up to relative roughness \( \approx 0.05 \), well capturing the current data), the range of Reynolds numbers (see Table 2) suggest that the Newtonian layer likely experiences *laminar* flow at lower Reynolds numbers (\( Re \leq 2000 \)) and *transition* flow at the upper range of the Reynolds numbers (\( Re \leq 8000 \)) in the study.

### 3.5. Other carbopol-oil solutions

The flow patterns described so far were observed for experiments with silicone oil as the displacing fluid. In order to evaluate the consistency of the reported experimental results when different oils are employed, tests were also performed with other couples of immiscible fluids, using light viscosity (\( \mu = 853 \text{ kg m}^{-1} \text{s} \), \( \delta = 46.9 \text{ mN m}^{-1} \)) and heavy viscosity mineral oils (\( \mu = 864 \text{ kg m}^{-1} \text{s} \), \( \delta = 0.129 \text{ Pa s} \), \( \delta = 69.0 \text{ mN m}^{-1} \)). Aqueous glycerol solutions L and M are used as the displaced fluids.

As the results in Fig. 9 depict, primarily center-type displacements are observed in other oil-carbopol solutions as well. Both increased viscosity oils show comparable values of \( \alpha \) to those of silicone oil (\( \alpha \approx 0.73 \pm 0.05 \)) at \( a \approx 0.74 \pm 0.03 \) and \( a \approx 0.78 \pm 0.02 \) for light and heavy viscosity mineral oil, respectively. The residual layer thicknesses of the light and heavy mineral oil are 0.14 ± 0.02 and 0.12 ± 0.01. This suggests that larger amounts of the yield-stress material can be removed by imposing fluids of increasing viscosity, in part due to the slower speeds of higher viscosity fluids and corresponding lower \( Re \), which has the effect of increasing \( \Delta \) slightly, as described above (see Fig. 5d). Instabilities in the flow, observed as fracturing of the gel layer, are confirmed with higher viscosity oils as well.

### 3.6. UDV profile analysis

In this study, Ultrasonic Doppler Velocimetry (UDV) is used to gain insight into the internal dynamics of the flow. The probe captures profiles every 30 ms, and these profiles are averaged over \( \approx 1.2 \text{ s} \), selected to achieve a balance of resolution fine enough to capture the flow features while reducing the effects of instantaneous variations in velocity. Fig. 10 presents the results for a typical experiment in which carbopol solution J is displaced by silicone oil. Due to refraction errors, velocities at the lower part of the pipe are difficult to measure, as reported in Hasnain et al. [11] and Oladosu et al. [10]. Guidelines are provided on the figures to mark the shape of the flow in the presence of refraction errors.

Snapshots of the displacement flow are presented in Fig. 10a, showing the approach and exit of the imposing oil on the UDV probe region (marked by a dashed yellow line) over \( \approx 5 \text{ s} \). The letters at the end of each snapshot (B-E) correspond to the subfigure that displays the UDV profile at the probe location for that snapshot. A plug flow region due to the yield stress of the carbopol is observed at the start of the flow, as shown by a relatively flattened velocity profile at the tip of the curve in Fig. 10b and c. As the oil crosses the boundary of the probe in Fig. 10d, it can be seen that the plug profile begins to round out, eventually sharpening to a tip under the influence of the invading Newtonian fluid. This velocity profile continues to evolve through Fig. 10d before approaching a Poiseuille-type flow with a thin layer of gel near the wall Fig. 10e.

### 4. Summary

The displacement flow of two immiscible fluids in a density-stable configuration, a light displacing Newtonian oil and a heavy displaced
Fig. 8. Dependence of the interface relative roughness, $\hat{\varepsilon}/\hat{D}_h$, on $B_N$ for (a) $\Delta t = -0.042$ and (b) $\Delta t = -0.075$. The colorbar in each figure corresponds to values of the Herschel-Bulkley number, $HB_N$. Dependence of the interface relative roughness, $\hat{\varepsilon}/\hat{D}_h$, on $Re$, and inclination angle $\beta$ for (c) $\Delta t = -0.042$ and (d) $\Delta t = -0.075$. The colorbar in each figure corresponds to the values of the viscous stress ratio, $N_\eta$.

Fig. 9. Snapshots of change in displacement flow with $\beta$ for immiscible yield-stress fluids in a density-stable configuration. (a) Silicone oil displacing carbopol gel with $\hat{\rho}_H=1067 \text{ kg m}^{-3}$, $\hat{\rho}_L=918 \text{ kg m}^{-3}$, $\hat{\mu} = 0.005 \text{ Pa s}$, $\hat{\sigma} = 65.6 \text{ mN m}^{-1}$, $\hat{\tau}_Y$ $\in [5.59, 9.17]$ Pa ($\Delta t = -0.075$, $Re \in [106, 112]$, $Fr \in [0.67, 0.72]$, $B_N \in [179, 302]$, $HB_N \in [0.50, 0.58]$, $N_\eta \in [360, 518]$, $\mathcal{C}_a = 0.02$). From left to right $\hat{V}_0 = 59, 56, 58 \text{ mm s}^{-1}$ and $\hat{t} = 12.9, 16.3, 15.5 \text{ s}$. (b) light viscosity mineral oil displacing carbopol gel with $\hat{\rho}_H=1067 \text{ kg m}^{-3}$, $\hat{\rho}_L=855 \text{ kg m}^{-3}$, $\hat{\mu} = 0.018 \text{ Pa s}$, $\hat{\sigma} = 46.9 \text{ mN m}^{-1}$, $\hat{\tau}_Y$ $= 8.56$ Pa ($\Delta t = -0.110$, $Re \in [8, 14]$, $Fr \in [0.16, 0.27]$, $B_N \in [168, 286]$, $HB_N \in [0.67, 0.73]$, $N_\eta \in [249, 392]$, $\mathcal{C}_a = 0.01$). From left to right $\hat{V}_0 = 16, 25, 27 \text{ mm s}^{-1}$ and $\hat{t} = 48.5, 38.2, 32.3 \text{ s}$. (c) heavy viscosity mineral oil displacing carbopol gel with $\hat{\rho}_H=1067 \text{ kg m}^{-3}$, $\hat{\rho}_L=866 \text{ kg m}^{-3}$, $\hat{\mu} = 0.129 \text{ Pa s}$, $\hat{\sigma} = 69.0 \text{ mN m}^{-1}$, $\hat{\tau}_Y$ $= 9.54$ Pa ($\Delta t = -0.104$, $Re \in [3, 4]$, $Fr \in [0.42, 0.51]$, $B_N \in [13.8, 17.0]$, $HB_N \in [0.60, 0.62]$, $N_\eta \in [23, 27]$, $\mathcal{C}_a \in [0.08, 0.10]$). From left to right $\hat{V}_0 = 51, 41, 48 \text{ mm s}^{-1}$ and $\hat{t} = 17.0, 20.6, 20.4 \text{ s}$. The field of view is 1689 x 9.53 mm².
yield-stress gel, has been investigated experimentally in an inclined pipe, with applications to cementing of wells drilled using an oil-based mud (OBM). Our experiments have covered a broad range of governing dimensional and dimensionless parameters (Re, β, At, Fr, Ca, Bw, HBw, Np), and their major effects are now summarized. A reduction in Re was found to correspond to higher values of α and increased removal of the yield-stress material, a result in line with observations of lower Re resulting in increased displacement efficiency for Newtonian fluids [10]. Similar to the miscible case [9], the core-annular flow patterns in this study were observed to be largely independent of the inclination angle β. An increased density difference, At = −0.075, was found to have a homogenizing effect on the flow compared to the lower At = −0.042, reducing the effects of other governing parameters. For example, while Bw and Re were found to decrease and increase the relative roughness, ̇c/Dbw, respectively at the lower At = −0.042 case, these effects were not present in the higher At case. While increased Fr and Ca were found to correlate to decreased HBw, their influence on the flow behavior was not significant, due to the dominance of other parameters such as At and Re. In addition, a higher HBw was found to correlate with increased yield-stress material removal (higher α), as was increased Np. A novel fracturing pattern has been observed in the flow, also at lower At, and its mechanisms have been explored. For the range of Np and α studied, the flows were found to be unstable according to the classification laid out by Hickox [17]. In contrast to the miscible density-stable case of center-type displacements, a slight improvement in removal of gel, as measured by the dimensionless area parameter α, has been observed (α ≈ 0.73 compared to previous reported value of 0.55), especially at lower Reynolds numbers. This is in agreement with the improved displacement efficiency at lower Reynolds numbers in the Newtonian immiscible-density-stable case [10]. The inclination angle, β, was not found to significantly affect the gel removal as well as the speed of the frontal region of displacing fluid (̇Vf = 1.45 ̇Vg, corresponding to displacement efficiency, φ ≈ 69%). A lower density difference, At, of the fluids was shown to slightly increase the relative roughness of the yield-stress layer when combined with high Re. Overall, the relative roughness of the flow was characterized to be in the range 0.01 ≤ ̇c/Dbw ≤ 0.07, a significant reduction from the relative roughness of the miscible case [9]. The residual layer thickness is calculated to be 0.12 to 0.14 of the pipe radius which is substantially lower than that of miscible fluids (≈ 0.25 radius). Internal dynamics of the flow were observed using Ultrasonic Doppler Velocimetry, revealing plug flow regions within the carboxymethyl gel layer.
The Appendix for experiments.

Acknowledgments

Declaration of Competing Interest

None to report.

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Appendix A. Fluid preparation

Carbopol is a powder that forms a clear hydrogel frequently used in the pharmaceutical and cosmetic industries [21,22]. It forms an acidic solution in water that results in a gel with its highest viscosity when neutralized to a pH of about 5–7 [23]. Solutions used in this study are carefully prepared by slowly adding carbopol to water in appropriate quantities for a concentration of 0.18% wt/wt. To form a solution with higher density, glycerol at a ratio of 0.3:1 (≈ 23% by volume) is added to water before adding carbopol. An IKA F2M03GLA stirrer mixes the carbopol solution at 300 rpm for 30 min, with the impeller placed near the bottom of the container in order to avoid incorporating bubbles. Subsequently, the appropriate mass of sodium hydroxide required for neutralization, 0.04% wt/wt, is mixed with water to generate a 20% aqueous solution and then added to the carbopol. The resulting gel is mixed at 400 rpm for five min to ensure homogeneity. The pH is taken with a Hanna Instruments pHep Pocket Sized pH Meter. Black dye (ink) with a concentration of 1200 mg L\(^{-1}\) is added to the displaced fluid in order to measure concentration via optical absorption [24]. The low concentration of dye used does not change the fluid properties. The solution is mixed for another five minutes at 400 rpm throughout the whole container. A sample of the gel is then taken for rheometry purposes.
Appendix B. Rheology

The rheological measurements are carried out using a TA Instruments Discovery Hybrid Rheometer 3 (DHR-3) 10 min after the mixing of the solution. A 40-mm-diameter parallel plate geometry at 1 mm gap is used in our tests, with 400 grit sandpaper attached to the geometry and the Peltier Plate in order to avoid wall slip effects [9,13,14,22]. Sample flow curves for solutions without (aqueous) and with glycerol (aqueous glycerol) are presented in Fig. 11a [8]. These flow curves are well described by the Herschel-Bulkley rheological model:

\[ \dot{e} = \dot{\varepsilon}_y + \kappa \dot{\gamma}^n \]  

(5)

The Herschel-Bulkley model incorporates Newtonian and Bingham models using yield stress \( \dot{\varepsilon}_y \), the power law index \( n \), and the fluid consistency index \( \kappa \). Table 1 presents rheological properties of carbopol solutions A - M used in our experiments after fitting a Herschel-Bulkley model where the yield stress \( \dot{\varepsilon}_y \) is taken to be the stress at a shear rate of 0.01 s\(^{-1}\) [13,14].

To characterize the viscoelastic nature of the gel, oscillatory amplitude sweep tests are performed at \( \dot{\omega} = 1 \) rad s\(^{-1}\) over the amplitude range \( \dot{\omega} = 0.001 \) to 30 Pa [25] for solutions E-M. Fig. 11b presents the results for carbopol solution F (\( \dot{\varepsilon}_y = 3.95 \) Pa, \( \kappa = 1.36 \) Pa s\(^n\), \( n = 0.47 \)). A linear viscoelastic region can be observed, in which the storage and loss moduli, \( G' \) and \( G'' \) respectively, remain relatively constant, signifying that in this region, the material functions depend only on the frequency [26]. The variation of these moduli in short and long time scales was observed via a frequency sweep performed at amplitude \( \dot{\omega} = 0.06 \) Pa, a stress amplitude selected as it is within the linear viscoelastic range; as shown in Fig. 11b, consistent values of stress and loss moduli are produced at this amplitude, but it remains well below the yielding point of the gel at 3.95 Pa [26]. The frequency sweep was performed over the range \( \dot{\omega} = 0.01 \) to 100 rad s\(^{-1}\) for the same solution, and is presented in Fig. 11c. At high values of \( \dot{\omega} \), the inertia of the geometry can tend to dominate the measurement signal, as indicated by a raw phase oscillation angle (a property measured directly by the rheometer) greater than 175° [27]. These values are plotted with hollow (instead of filled) circles. At low values of \( \dot{\omega} \), (\( \dot{\omega} < 0.1 \) rad s\(^{-1}\)), the time required to collect data can be extensive, and evaporation effects can also become significant; accordingly less data points were taken in this region [27]. To quantify the relative strength of elastic to viscous properties, the ratio \( \tan \delta = G''/G' \) can be used. When tan \( \delta \) is greater than 1, the loss modulus \( G'' \) is dominant, revealing more viscous behavior in the gel. As tan \( \delta \) is below 1, the elastic effects are important for this solution. Values of tan \( \delta \) for all experiments ranged from 0.03 - 0.20 over frequencies at which the response of the moduli is approximately linear (\( \dot{\omega} < 1 \) rad s\(^{-1}\)).

The dominance of elastic effects in the carbopol solutions was confirmed by the calculation of the Weissenberg number, \( Wi \), which is also a measure of elastic to viscous properties of flow. \( Wi \) is defined as Poole [28]:

\[ Wi = \dot{\gamma} \frac{\dot{\gamma}}{D} \]  

(6)

where \( \dot{\gamma} \) is the relaxation time and \( \dot{\gamma} \) is the characteristic shear rate, which, in the case of simple shear, is given by the characteristic velocity and length scales of the flow; \( \dot{\gamma} \) and \( D \) respectively. The relaxation time of a viscoelastic material can be measured using an oscillatory shear frequency sweep performed in the linear viscoelastic region as follows. For steady shear flow, both the first (\( N_1 \)) and second (\( N_2 \)) normal differences are proportional to the first (\( \dot{\psi}_1 \)) and second (\( \dot{\psi}_2 \)) normal coefficients respectively by the square of shear rate (\( \dot{\gamma} \)) [29]. The shear rate is related to the viscosity (\( \dot{\eta} \)) and shear stress (\( \dot{\eta} \)) of the material:

\[ \delta(\dot{\gamma}) = \dot{\eta}(\dot{\gamma}) \dot{\gamma}^2 \]  

(7)

\[ \dot{N}_1(\dot{\gamma}) = \dot{\psi}_1(\dot{\gamma}) \dot{\gamma}^2 \]  

(8)

\[ \dot{N}_2(\dot{\gamma}) = \dot{\psi}_2(\dot{\gamma}) \dot{\gamma}^2 \]  

(9)

The first normal coefficient and viscosity are used to calculate the relaxation time as [29,30]:

\[ \dot{\psi}_1(\dot{\gamma}) = \frac{\dot{\eta}(\dot{\gamma})}{2\dot{\gamma}} \]  

(10)

The experiments in this study were conducted using a parallel-plate geometry, meaning that calculation of \( \dot{\psi}_1 \) could not be performed using direct measurements of \( \dot{N}_1 \), instead, the first normal coefficient was derived from oscillatory shear data using the following relations: [29,31,32]

\[ \dot{\eta}(\dot{\gamma}) = \dot{\eta}_0 \frac{\dot{G}''(\dot{\omega})}{\dot{\omega}}. \]  

(11)

\[ \dot{\psi}_1(\dot{\gamma}) = \dot{\psi}_1(0) = \lim_{\dot{\omega} \to 0} \frac{\dot{G}''(\dot{\omega})}{\dot{\omega}^2}. \]  

(12)

To determine the storage \( G' \) and loss \( G'' \) moduli, an amplitude sweep test was performed on the materials to locate the linear viscoelastic region, and subsequently, a frequency sweep test was performed on the material at a stress within the linear viscoelastic region. Values of \( G' \) and \( G'' \) at \( \dot{\omega} = 0.1 \) rad s\(^{-1}\) were taken as proxies for the constants they approach as \( \dot{\omega} \to 0 \). This is because, below \( \dot{\omega} = 0.1 \) rad s\(^{-1}\), extended time is required to gather measurements and sample evaporation begins to become relevant. For Solution F, these values are \( G' = 30.41 \) Pa and \( G'' = 3.68 \) Pa, and they are used in combination with the non-Newtonian viscosity, \( \dot{\eta}(\dot{\gamma}) \), and the first normal stress coefficient, \( \dot{\psi}_1(\dot{\gamma}) \), to calculate the relaxation time as given in Eq. 13.

\[ \dot{\psi}_1(\dot{\gamma}) = \frac{\dot{\psi}_1(0)}{2\dot{\omega}_0}. \]  

(13)

Relaxation times in this study are calculated to be in the range 69 - 137 s. The calculated Weissenberg numbers are in the range of 0.015 - 6.4 (\( \times 10^3 \)), indicating that elastic forces tend to dominate viscous forces in the flows. Note that if values of \( \dot{\omega} \) (0.01 rad s\(^{-1}\)) are used in calculations, ranges of relaxation times and Weissenberg number are \( \dot{\psi} \in [0.2, 612] \) s and \( Wi \in [0.49, 9.4 \times 10^3] \).

A creep test is finally performed as an alternate means of measuring the yield stress of the carbopol [8]. A creep test for a yield-stress fluid measures its strain response \( \dot{\gamma} \) to a constant applied shear stress \( \dot{\gamma} \). In order to ensure a reproducible initial state, a strong pre-shear \( \dot{\gamma}_{pre-shear} = 100 \) s\(^{-1}\) is applied for 60 s, after which the carbopol is allowed to recover for a period of 100 s under zero applied shear stress [33,34]. The creep test begins immediately after this rest time. The tests are performed for 1000 s each in random order of applied stress creep to avoid systematic errors [33].

Fig. 11d shows the results for creep tests run on solution K. There exists a period of initial oscillation where the applied stress is not yet a constant due to the inertia of the geometry [33]. These oscillations, shown by a curved peak, become damped within a few seconds, after which the creep response is less transient. For gels such as carbopol, an applied creep stress lower than the yield stress (\( \dot{\gamma} < \dot{\gamma}_y \)) results in a creep regime in which the shear rate tends towards zero (see Fig. 11d, \( \dot{\gamma} = 2.85 \) Pa) [8]. However, for experiments where \( \dot{\gamma} > \dot{\gamma}_y \), a flow regime can instead be observed, where the shear rate increases with time (see Fig. 11d, \( \dot{\gamma} = 9.14 \) Pa). This transition from creep regime to flow regime takes longer the closer the applied stress is to the yield stress of the solution [33]. It can be seen from Fig. 11d that the transition from the creep to the flow regime happens between 8.5 and 9 Pa, in good agreement with the calculated Herschel-Bulkley parameter of \( \dot{\gamma}_y = 9.17 \) Pa for solution K (see Table 1).

Appendix C. Cleaning procedure using soap water

Prior to each experiment, the pipe is filled above the gate valve with the displacing fluid and below the gate valve with the displaced one.
During the experiment, the oil-based displacing fluid fills the length of the pipe, flushing out the water-based displaced fluid. At the time of next experiment, a thin layer of oil can still remain in the bottom portion of the pipe, below the gate valve, which can introduce an error into our study. Following on our previous study [10], a meticulous cleaning procedure involving soap-water is therefore employed to remove this unwanted layer of oil. This would ensure that the oil-based solution does not contaminate the water introduced into the lower portion of the pipe for the next experiment set. Liquid soap is added to water at a concentration of 2.2 g L⁻¹. This solution is pumped through the pipe for 60 sec at high velocity. The authors recognize that the soap can possibly introduce small quantities of residual surfactant molecules into the pipe. However, these surfactant molecules are not hypothesized to significantly influence observed flow regimes, as a series of experiments performed on fluids with comparable surface tension values revealed the same flow regimes observed with and without the soap-cleaning procedure [10]. In order to minimize the effect of any such molecules introduced into the pipe by the soap, the pipe is allowed to empty, and subsequently water is pumped through the pipe at a high velocity for 60 sec to flush all the in-situ fluids out, ensuring a clean, oil free pipe. This cleaning procedure is employed for each experiment (total of 93).

References


