CONVOLUTION

DSP for Scientists

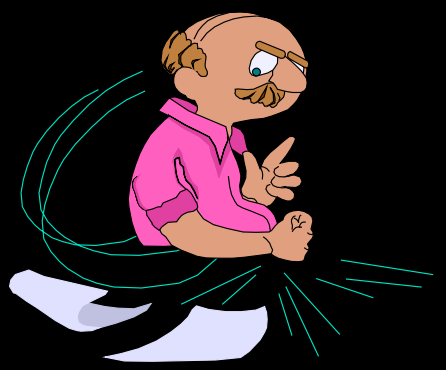
Department of Physics

University of Houston

Most Important in DSP

- Combining two (or more) signals to form a third output. (Superposition)

- Signal Filtering (Estimation)
  Function Approximation
- Interpolation
- Prediction (extrapolation)



CONCEPTS

- Delta Function
- Unit Impulse $\delta[n]$

$\delta[n] = 1$, when $n = 0$;

$\delta[n] = 0$, when $n \neq 0$.

Standard Function

- Standard Function $u[n]$

- $u[n] = 1$, when $n \geq 0$
- $u[n] = 0$, when $n < 0$

$$\delta[n] = u[n] - u[n - 1]$$

$$u[n] = \sum_{k = 0, \infty} \delta[n - k]$$

Impulse Response

- Impulse response $h[n]$ of linear system S
  - The signal that exits a system when a Delta function (unit impulse) is the input.

$$h[n]=S(\delta[n])$$

Delta Decomposition

- Any discrete sequence $x[n]$ can be represented by weighted Delta function

$$x[n] = \sum_{k=-\infty,\infty} x[k]\delta[n-k]$$

$$x[n] = \sum_{k=-\infty,\infty} x[n-k]\delta[k]$$

Periodic Signal

- $x[n] = x[n + N]$, N: period

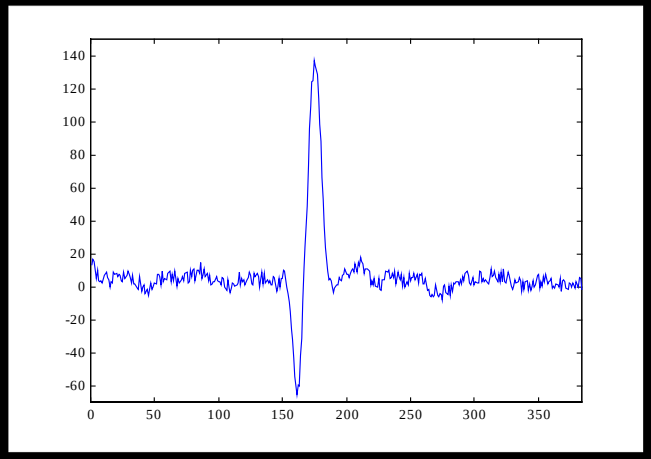
- *Example: When* $f \in Z$
- $sin[2\pi f(n + 1)] = sin[2\pi fn]$
- f is normalized frequency,
- minimum period is 1.

Convolution

- $y[n] = h[n] * x[n]$

- $x[n]$ → Linear System $h[n]$ → $y[n]$
-
- The *impulse response* of the system
- How the relation between input and output.

Example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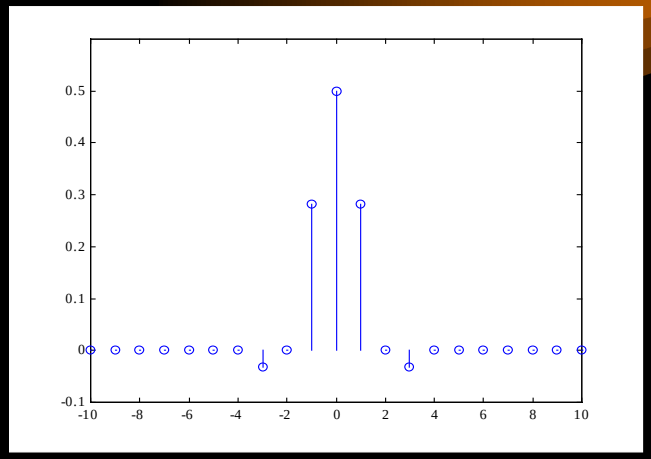
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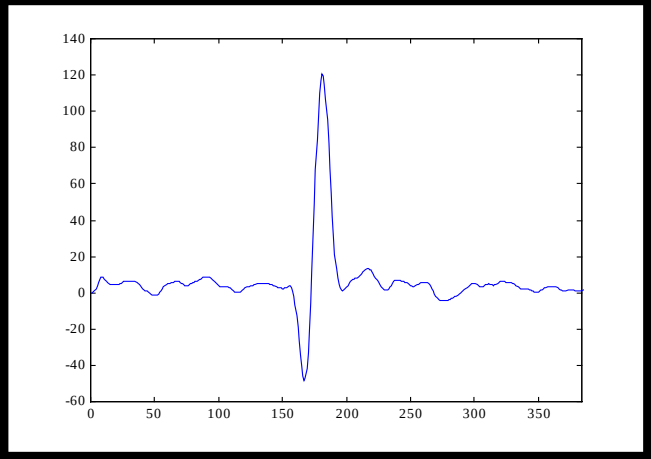
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Convolution Sum (Linear)

- $y[i] = \sum_{j = -\infty,\infty} h[j]x[i-j]$
- $y[i] = \sum_{j= -\infty,\infty} x[j]h[i-j]$
- $y[n] = h[n] * x[n] = x[n] * h[n]$

Convolution Calculation

- Keep x[i] invariant
- Symmetry of $h[j]$

  $g[j] \Rightarrow h[-j]$
- $y[0] = \sum_{j=-\infty,\infty} x[j]g[j]$

  $= \sum_{j=-\infty,\infty} x[j]h[-j]$
- Shift g[j] Left to obtain y[i < 0]
- Shift g[j] Right to obtain y[i > 0]