

Syllabus: Topology Preliminary Examination

Topics:

Basic definitions and elementary results from topology, basis and subbasis, basic constructions such as initial and final topologies, the product topology and quotient spaces (e.g. surfaces from quotient spaces of polygonal domains). Nets and convergence, continuity and its properties, topological and hereditary properties. Compactness, local compactness, spaces of functions (e.g., continuous, compactly supported, vanishing at infinity), one point and Stone-Čech compactifications; the characterization of separable metric spaces (Urysohn's metrization theorem), embeddings of (compact) manifolds, partitions of unity. The Tychonoff theorem. Connectedness and path connectedness, connected components, Urysohn's lemma and Tietze's extension theorem.

Introduction to algebraic topology: simply connected spaces, homotopy and the fundamental group, covering spaces, liftings and the lifting correspondence theorem, examples, universal covering spaces.

References:

With small exceptions (such as nets) these topics are covered well in "Topology" by James R. Munkres.

A presentation of nets can be found in "A Taste of Topology" by Volker Runde.