Syllabus for Optimization Theory Preliminary Examination 01 2022

It is expected that any student taking the Optimization preliminary examination will have a thorough working knowledge of finite dimensional differential real analysis. This includes

- The metric topology of and norms on R^n.
- The metric topology and properties of R := [−∞, ∞].
- infs, sups, lim infs and lim sups of sets in R or R.
- Conjugate functions.
- Dual norms.
- The analysis and properties of real valued functions defined on subsets of R^n, including Gateaux derivatives, gradients and Hessians.
- Matrix calculus (derivatives, gradients, and Hessians in finite dimensional spaces)

Topics in optimization to be covered are:

1. Weierstrass existence theorem and its extension to coercive functions on closed subsets of R^n
2. Necessary and sufficient conditions for a local minimizer of a function (1-d and n-d).
3. Criteria for the existence of a local minimizer and error estimates.
4. Definitions, examples, properties and operations on convex sets.
5. Generalized inequalities, separating and supporting hyperplanes, and dual cones.
6. Definitions, examples, and properties of convex functions on subsets of R and R^n.
7. Definitions, examples, and properties of convex functions on subsets of R and R^n.
8. Operations that preserve convexity.
9. Definitions, examples, and properties of quasiconvex functions on subsets of R and R^n.
10. Analysis of minimization of convex functions on convex sets in R^n.
11. Convex optimization problems:
    a. Convex optimization problems.
    b. Linear optimization problems.
    c. Quadratic optimization problems.
    d. Geometric programming.
    e. Vector optimization and Pareto optimality.
12. Algorithms for convex minimization, including steepest descent and Newton’s method.
13. Stopping criteria and convergence analysis.
14. KKT theory, Lagrange multipliers and criteria satisfied by minimizers of differentiable functions subject to equality and inequality constraints.
15. Perturbation and sensitivity analysis.
16. Lagrangians, Lagrange dual functions, and dual optimization problems.
17. Saddle points and the existence of solutions of dual optimization problems.
18. Theory and algorithms for unconstrained optimization:
    a. Newton and Quasi-Newton methods.
    b. Conjugate gradient methods.
    c. Linesearch and Trust Region methods.
19. Theory and algorithms for constrained optimization:

a. Linear programming.
b. Equality and inequality linear constraints.
c. Barrier and augmented Lagrangian methods.
d. Sequential quadratic programming.
e. Infeasible start Newton method.
f. Interior-point methods (inequality constrained minimization; barrier method; primal-dual interior point methods)

References:

1. L. D. Berkovitz, Convexity and Optimization in R^n, Wiley 2002.
2. A. L. Peressini, F. E. Sullivan & J. J. Uhl, Jr, The Mathematics of Nonlinear Programming, Springer 1988.
3. J. Nocedal and S. J. Wright, Numerical Optimization, Springer 1999.
4. S. Boyd and L. Vandenberghe, Convex Optimization, Cambridge University Press 2004.
5. A. Beck, Introduction to Nonlinear Optimization, SIAM 2014.