UH Mathematics Department Preliminary Examination
Syllabus for Optimization Theory.

It is expected that any student taking the Optimization preliminary examination will have a thorough working knowledge of finite dimensional differential real analysis. This includes

- the metric topology of, and norms on, $\mathbb{R}^n$.
- the metric topology and properties of $\overline{\mathbb{R}} := [-\infty, \infty]$.
- infs, sups, Lim infs and Lim sups of sets in $\mathbb{R}$ or $\overline{\mathbb{R}}$.
- the analysis and properties of real valued functions defined on subsets of $\mathbb{R}^n$, including Gateaux derivatives, gradients and Hessians.

Topics in Optimization to be covered are

1. Weierstrass existence theorem and its extension to coercive functions on closed subsets of $\mathbb{R}^n$.
2. Necessary and sufficient conditions for a local minimizer of a function. (1-d and n-d).
3. Criteria for the existence of a local minimizer and error estimates.
4. Definitions, examples, properties and operations on convex sets
5. Definitions, examples and properties oc convex functions on subsets of $\mathbb{R}$ and $\mathbb{R}^n$.
6. Analysis of minimization of convex functions on convex sets in $\mathbb{R}^n$.
7. Algorithms for convex minimization, including steepest descent and conjugate gradient methods. Stopping criteria and convergence.
8. KKT theory and criteria satisfied by minimizers of differentiable functions subject to inequality constraints.
9. Lagrange multipliers and criteria satisfied by minimizers of differentiable functions subject to equality constraints.
10. Lagrangians and dual optimization problems.
11. Saddle points and the existence of solutions of dual optimization problems.
12. Theory and algorithms for unconstrained optimization:

   (a) Newton and Quasi-Newton methods,
   (b) Conjugate Gradient methods,
   (c) Linesearch and Trust Region methods.

13. Theory and algorithms for constrained optimization:
   (a) Linear Programming (Interior Point methods, Simplex methods),
   (b) Equality and inequality linear constraints,
   (c) Barrier and Augmented Lagrangian methods,
   (d) Sequential Quadratic Programming.

References:

1. L.D. Berkovitz, *Convexity and Optimization in* R^n, Wiley 2002.
2. A.L. Peressini, F.E. Sullivan & J.J. Uhl, Jr, *The Mathematics of Nonlinear Programming*, Springer 1988.
3. J. Nocedal and S. J. Wright, Numerical Optimization, Springer 1999.

Lecture notes for M6366, Fall 2004 are available on the web at
www.math.uh.edu/ giles/PDFpapers/Lects.pdf