Real Analysis Preliminary Examination Syllabus

Department of Mathematics, University of Houston

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TOPOLOGY: Open and compact sets, Hausdorff spaces, Urysohn's lemma, Tietze extension theorem.

METRIC SPACES: Metrics, metric topology, continuity, the Cauchy condition and completeness.

MEASURES Sigma-algebras, Borel sets, outer measures, monotone class theorem, Riesz representation theorem, Borel measures, regularity properties, Lebesgue measure on $\mathbb{R}^n$, signed and complex measures, Jordan decomposition, total variation of a measure, absolute continuity, product measures.

INTEGRATION: Measurable functions, simple functions, approximation properties, integration of non-negative functions, integration of complex and real-valued functions, convergence theorems for integrals (e.g., Fatou, monotone, dominated convergence), almost everywhere convergence, Lusin's theorem, Egoroff's theorem, the Fubini-Tonelli theorem.

DIFFERENTIATION: Bounded variation, absolute continuity, Lebesgue and Radon-Nikodym theorem, differentiation, fundamental theorem of calculus.

BASICS of FUNCTIONAL ANALYSIS and L^p SPACES: Normed and Banach spaces, Hahn-Banach theorem, Hilbert spaces, Baire Category theorem, Open Mapping Theorem, Closed Graph Theorem, Principle of Uniform Boundedness, Applications to Fourier series, L^p spaces and their duals, completeness, convergence, density, C(X) spaces and their duals.

FOURIER TRANSFORM: Convolutions, Inversion, Plancherel's identity.

References:

- W. Rudin, Real and Complex Analysis, 3rd edition, McGraw-Hill, New York, 1986
- P. R. Halmos, Measure Theory, Springer-Verlag, Berlin, 1979.
- G. B. Folland, Real Analysis, 2nd edition, Wiley Interscience, Somerset, NJ, 1999
- H. L. Royden, Real Analysis, Prentice-Hall, Englewood Cliffs, NJ, 1988