

The Maya and Their Mathematical Exploits

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INTRODUCTION

Throughout recorded history, ancient civilizations have developed unique and varied achievements in language, arts, math and science, architecture, and astronomy. These wonderful achievements were documented and investigated. The early Egyptians built great pyramids, the early Chinese built their great wall, and the early Europeans built their Stonehenge and astronomical observatories. All of these developments required complex measurement and calculations, suggesting that no great ancient civilization went without some degree of mathematical applications. The early civilization of Central America was no exception. These people not only built magnificent buildings and pyramids, but also independently developed a concept of zero and uses a complex yet a powerful number system, which is base 20 (vigesimal system). This Central American civilization was called the Mayan civilization. Their vast territories covers of what is now present day nations of Guatemala, Western Honduras, Belize, El Salvador as well as the southern Mexican states of Chiapas, Tabasco and the Yucatan peninsula. Evidence of urban dwelling, elaborate temples and buildings can be found in the ruins of Tikal, that is hidden deep in the rainforests of Guatemala. Many archaeologist, historians and scientists believe that Mayan ancestors came from Asia through the Bering land bridge of what is now Bering Strait. They were hunter-gatherer nomadic people, however as population grows, they developed new and effective ways of farming in order to support the increasing needs of its people.

Mayan mathematics, science, and society were closely linked and in general presented an integral focus, which meant that each investigation and each specific fact had a direct relationship with Mayan society. For example their study of astronomy was closely related to different cycles for planting corn (Morley 1968). Indicates that archaeological as well as documental proof shows that the priests chose the day on which the fields should be burned in preparation for planting corn with extreme care. For example, in the city of Copan, Honduras, two pillars of inscribed stone, stelae 10 and 12, rise out of two chains of hills that bank the extremes on the west and east of the Copan valley. The distance between this two stelae is about a mile along a straight line. From stela 12, one can see the sun set directly behind stela 10 on 12 April and 7 September. The first date, 12 April, was precisely the date chosen by the priests for the commencement of the initial step in planting the corn seed. This example illustrates how the Maya combined geometry, knowledge of astronomy, and mathematical calculations, as well as construction skills, to establish the correct date for planting their most important food – corn (Alecio, Irby, and Aldana, 1998).

The Mayan independently developed the concept of zero. Symbol for zero was one of the most significant events to occur in the history of the civilization. Until zero was considered a number in its own right and given a symbol, performing more than simple calculations was impossible without a calculating device (McNeill). Tobias Dantzig sums up with the statement that “in the history of culture the discovery of zero will always stand out as one of the greatest achievement of human race” (Dantzig 1941, p.35). The numbers of the Mayan numerical system

were written from bottom to top. The historian Esparza-Hidalgo (1976) commented on how the young generation might approach older individuals to ask how to start counting—from up to down or from down to up. The older individual would likely to answer without hesitating: “well how do plants grow? Then how would we count?” given this answer, we see one more example of how the Maya were and continue to be keen observers of their natural environment (Alecio, Irby, and Aldana 1998).

The Mayans carried out astronomical measurements with remarkable accuracy yet they had no instruments other than sticks. They used two sticks in the form of a cross, viewing astronomical objects through the right angle formed by the sticks. The Caracol building in Chichén Itza is thought by many to be a Mayan observatory. Many of the windows of the building are positioned to line up with significant lines of sight such as that of the setting sun on the spring equinox of 21 March and also certain lines of sight relating to the moon. With such crude instruments the Maya were able to calculate the length of the year to be 365.242 days (the modern value is 365.242198 days). Two further remarkable calculations are of the length of the lunar month. At Copán (now on the border between Honduras and Guatemala) the Mayan astronomers found that 149 lunar months lasted 4400 days. This gives 29.5302 days as the length of the lunar month. At Palenque in Tabasco they calculated that 81 lunar months lasted 2392 days. This gives 29.5308 days as the length of the lunar month. The modern value is 29.53059 days. Was this not a remarkable achievement? (O’Connor, Robertson 2000).

This unit will be written for the 6th grade mathematics regular classes with LEP (Limited English Proficient) students and, inclusion classes. Inclusion students are students with varying learning difficulties. This unit can also be adapted for advance placement classes by varying the level of questioning and student exercises. Assessment and student exercises will follow each section. Majority of my 6th grade students in my school are Hispanics and African-Americans, and most of my Hispanics students attends ESL classes (English as Second Language), and my students seemed detach in relating mathematics to the real world situations. This unit is designed to increase students’ involvement in their learning by providing real life problems involving hands-on activities. When the materials and problems being used originate from the students’ daily life experiences or cultural heritage, we find real-world problem solving at work. In this situation, the problem solving activity can motivate the students to create mathematical models that may, in reality, become the real object of study (Bohan, Irby, and Vogel 1995). Teaching and helping my students develop interest in math and science is what I am hoping to accomplish in writing this unit.

OBJECTIVES

The objective of this unit is to give middle school students a look at mathematics from different perspectives or other vantage points. At the same time, the unit follows the TEKS objective as outlined in HISD’s 6th grade mathematics curriculum instruction.

Math.6.01.A.01 - Number, operations, and quantitative reasoning. The student will use place value to represent whole numbers. The student is expected to (A) order place value notation using vigesimal or base 20 system. (B) Compare and contrast base 20 (Mayan) and base 10 (Arabic) systems.

Math 6.04.A.02 - Use tables and symbols to represent relationships involving sequences numeric pattern of numbers. Student is expected to connect Mayan math symbols and numbers to our base 10 Arabic numbers.

Math 6.04.A.03 - Use tables and symbols to represent geometric relationships such as perimeter, area, similarity, etc. the student will construct geometric objects and then determine and

compute its area, perimeter, and volume then share and discuss geometric properties that can be found on the structure that they created.

Math.6.05.A.01 - Use letters to represent an unknown such as in an algebraic expression or equation. Student is expected to create an algebraic expression using Mayan number symbols.

Math.6.05.A.02 - Formulate an equation for a problem that is represented using concrete or pictorial models. Student is expected to generate situational problems that can be solved using Maya math and symbols.

Math.6.05.A.03 - Formulate an equation for a problem situation that is represented by the data displayed in table, chart, or as labels on a graph. The student is expected connect Mayan symbols and numbers to our base 10 Arabic numerals.

Math.6.11.A.01 - Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics. Student will construct various structures of their choice that are related to ancient civilization, and then measure the area, volume, and determine the scale factor of their created structures against the actual building or pyramid. Together with this lesson I will show to the student the movie clip about the ruins of Teotihuacan. Although Teotihuacan is not a Mayan city, evidence suggests that trades and exchange of ideas has existed between this people.

Math.6.11.B.01 - Use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness. In implementing this lesson students will use actual beans and sticks in carrying out the plan in getting the perimeter, and solving situational problems involving ancient trades of the Maya.

Math.6.12.A.01 - Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models. This curriculum unit will investigate the system of numbers that the Mayan architects and engineers employ (O' Brien, Christiansen 1986, p.136)

RATIONALE

Students will find an interest in manipulating Mayan numbers and its symbols, because most of my students or most of the students for that matter are kinesthetic learners. Therefore using manipulative in mathematics classroom will poster growth on students inquisitive mind. (National Council of Teachers of Mathematics, 2000). It looks new to them and the mathematical algorithms are pretty much straightforward. Teachers can infuse culture into the curriculum and develop students' competence and confidence by using Ethno mathematics (D'Ambrosio 1987; Massey 1989; Stigler and Baranes 1988). Ethno mathematics calls for a reconstruction of the mathematics curriculum to achieve cultural compatibility (Moll and Diaz 1987; Trueba 1988). Ethno mathematics is the relationship between math and culture. Real life math problems can be taken from many of today's student's historical or cultural environment.

One such cultural and historical event took place more than 5000 years ago in a region named Mesoamerica, where the Maya developed a numerical written system that dealt ingeniously with the relationship between numeral and number (Morales-Aldana 1994). Mayan mathematics was characterized by a positional system (Appendix C). That had as a base the number 20. The Maya created a symbol for the 0, which had a use similar to that in any other positional numerical system. Furthermore, in addition to their mathematical advances, compared with other civilizations, their system of teaching

mathematics was based on the use of concrete, semi-concrete, and representational materials. Probably that characteristic expanded the dominant structure's potential to develop numerical computations and also allowed the priests, the academicians, and the religious class of that time to carry out the great astronomical and scientific advances that are still impressive today (Alecio, Irby, and Aldana, 1998).

This unit will be taught in conjunction of the section of measurement in mathematics textbook (area, volume and perimeter). Manipulative using beans and sticks, colored markers and pencils maybe used for visual aid. Variations of this unit can be used depending of the grade level of the students. Students in an inclusion class tends to have difficulties about number sense and structure, therefore I would like to teach key concept that will address the issue. Students on grade level can benefit this lesson as well, because I will develop questions that challenge higher order thinking. I will teach this unit in collaboration with my fellow teachers in science, reading, history, art and social studies.

UNIT BACKGROUND

The principal factors that contributed to the origins of Mesoamerica's civilizations are many and debated, but most believe that began with the rise of chiefs. Prior to this, between 15,000 and 5,000 B.C., humans hunted and gathered.

Anthropologists tend to think of early tribes as egalitarian societies who restricted the accumulation of personal wealth by continually circulating food and materials through reciprocal exchange networks. But while food-sharing and gift-giving may have promoted trust and bound tribal members together, the ability to generate surpluses with plant cultivation would have created status differences. Ambitious individuals could begin capitalizing on a basic primate condition, the most successful long term leaders are not necessarily the strongest, but the most generous (Pohl).

Pohl goes on to describe how agriculture changed the human scene:

Once foods had been domesticated as staples, they would have been available to any population interested in shifting from foraging to agriculture and sedentary life. Agriculture can support large populations but it demands ever more intensive forms of cultivation. Mesoamerican people met the challenge by developing a wide variety of agricultural techniques, from terracing mountain sides to digging canals or even creating artificial wetlands. Mesoamericans domesticated dogs and turkeys, but wild animals like deer were naturally drawn to gardens where they could be easily captured and tethered. The cultivation of fruit trees attracted a wide variety of tropical birds whose colorful plumage was coveted for displaying wealth and prestige (Pohl).

Chronology: Mesoamerican timeline

It is interesting to note that surplus agriculture not only made civilization possible, but also led to the need to count or otherwise measure that surplus. There is evidence that throughout Mesoamerican history, math went hand in hand with surplus, to such an extent that Mesoamericans used chocolate beans or other small foods for many of their calculations (Koontz). In order to support the population explosion of the late Preclassic period. Maya began to develop intensive agriculture and sophisticated water management. This could have not been possible without their knowledge of astronomy and engineering. Of course astronomy and calendar calculations require mathematics, and indeed maya constructed a sophisticated number system (O'Connor, Robertson).

The following section will highlight how the Mesoamerican people evolve and develop into a complex social structure. It is important to note that these people uses mathematics to record

trades, important events such as religious ceremonies and alliances. To say the least math, science, social studies, history and art are deeply embedded on the chronology of Mesoamerican timeline, an outline of which appears below.

Mesoamerican history consists of three periods: the Preclassic or Formative period (1500 B.C. - A.D. 300), the Classic period (A.D. 300-950), and the Postclassic period (A.D. 950-1521). To help the teacher orient herself and begin to put the mathematics into an historical perspective, below is a summary of these periods, based on the work of John Pohl:

The Preclassic period

As farming became more important for Mesoamerican populations, societies developed more complex ways to organize themselves. Political specialists rose to power, probably as an outgrowth of the power of religious specialists. Eventually, these political specialists became the nobility and competed with each other for goods and followers. Increased population made possible by more efficient agriculture, along with the centralization of populations, led to the development of the first cities.

The Classic period

The Classic period was characterized by the emergence of Teotihuacán, the most complex and powerful city the Americas had yet seen, and in fact it was one of the largest cities in the world between A.D. 100-700. Other, smaller cities arose around this same time all over Mesoamerica in an explosion of cultural and economic growth. Then by A.D. 900 most of these cities collapsed, led by the fall of Teotihuacan around A.D. 650. Scholars are still not sure what caused this collapse, although it is becoming clear that there were not a single cause but many interrelated causes to what was one of the more spectacular civilizational collapses the world has yet seen. That said, there were new centers to carry on what by now was a millennial tradition of civilization.

The Postclassic period

The Postclassic period brought a new commercial orientation to Mesoamerica where trade in luxury items of all sorts had always been important. The Postclassic brought this trade and smaller scale or more regional markets to a new prominence. In the words of John Pohl: “The technology for smelting gold, silver, and copper, was introduced from Central and South America, while turquoise mined in the American southwest was exchanged for the plumage of Scarlet Macaws. Never before had the Mesoamerican economy been exposed to so many rare materials from such far away places.” It is in this environment that the Aztec people rose to prominence in the Valley of Mexico, and eventually dominated much of Mesoamerica only to lose their empire to the Spanish in 1521.

LESSON PLANS

Lesson One: Matching Mayan numbers to its corresponding symbol

Objective

Math.6.01.A.01 - Number, operations, and quantitative reasoning. The student will use place value to represent whole numbers. The student is expected to (A) order place value notation using vigesimal or base 20 system. (B) Compare and contrast base 20 (Mayan) and base 10 (Arabic) systems.

Introduction

Students will be given an opportunity to talk about the history of ancient civilizations that they already know, students may talk about the history of the United States, and some places or better if they will talk about early civilization. This section is intended to hook students by engaging

students with their prior knowledge. Once students are engaged, the Teacher, will now facilitates questioning regarding the origins of numbers, how and why we need to use numbers. At this point students will now be guided to topics regarding ancient civilizations that develop their own numbers. Subsequently, teacher will now introduce the ancient civilization, although far removed from the rest of the world during their time, but independently develop a number system that is unique and is equally superior to the number system that we are familiar with. The civilization that develops the concept of zero and using a base 20 systems is the **Mayan** civilization. Their vast territories covers of what is now present day nations of Guatemala, Western Honduras, Belize, El Salvador as well as the southern Mexican states of Chiapas, Tabasco and the Yucatan peninsula. Please see (appendix D).

Mayan people use shell for zero (0), a dot for 1, and a bar for 5; they arrange and read numbers from bottom to top. One dot means 1, two dots means 2, and the accumulation of five dots will be transcribed to a bar, and consequently accumulation of five bars will be transcribed as 20. Please see (appendix A). The Mayans may use beans or pebbles as a dot and stick or cacao pod to use as a bar.

Concept Development

Student will gather information about the history of mathematics and student will write the highlights and importance of mathematics and culture. This section enables teacher to teach the concept of integration of mathematics across curriculum (NCTM 2000).

Student Practice

Student will goes to the board and write down rows of numbers from 1 to 20, I will then write the Mayan number equivalent in the form of dots and bars right on top of the numbers. Another student will come up to the board to write the numbers 21 to 40, and again I will write the Mayan number equivalent in the form of dots and bars right on top of the numbers. After this activity a class discussion will follow. Student will then infer or try to translate Arabic numbers into Mayan numerals. Place value notation and examples will be discussed extensively here. Please see (appendix A).

Assessment

Student will be given a teacher made worksheet; student will demonstrate their understanding about the Mayan numerals by transcribing Arabic numbers into Mayan numbers or vise versa.

Closure

Student will compare and contrast similarities and difference between Arabic and Mayan numerals in their journal entry.

Resources

- Teacher made worksheet.
- Beans or pebbles, and
- Sticks or twigs (since cacao pods are not available).

Lesson Two: Base 20 addition and subtraction

Objective

Math.6.01.A.01 - Number, operations, and quantitative reasoning. The student will use place value to represent whole numbers. The student is expected to (A) order place value notation using vigesimal or base 20 system. (B) Compare and contrast base 20 (Mayan) and base 10 (Arabic) systems

Math.6.12.A.01 - Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.

Introduction

In this lesson student will develop mathematical skills in different perspective. The student will add and subtract numbers that is very different in appearance, but delivers result similar to our base 10 Arabic systems. Teacher may say “ as you may recall on our last lesson about the Mayans in Central America, they develop this numbers and its symbols, so today we are going to perform addition and subtraction of Mayan numerals, and then transcribed the result to our Arabic system”. The base 20 systems that was invented and developed by the Mayans, not only accurately predicts movements of celestial bodies and observe the 365 days year cycle.

Concept Development

Student will be given an example about addition of Mayan numerals on the board. Please see (appendix B), Conversely an example of subtraction will also be shown to the students. Class discussion will follow after the exercises on the board.

Student Practice

Student will be given a simple addition problem, sums not to exceed 19, and then class discussion will follow. Then addition problems whose sum will exceed 400 and subsequently addition whose sum will not exceed 8,000 will be given to the student for practice. Conversely an example of subtraction will be displayed on the board, and students will answer exercises on the board after each discussion. All throughout this activity student will work in pair.

Assessment

Addition, and subtraction worksheets will be passed out; student will answer the questions with the aid of their manipulative (beans and sticks), and then record and transcribed their answer in Arabic numerals.

Closure

Using Venn diagram student will make a journal entry contrasting similarities and differences between Arabic and Mayan umbers in adding and subtracting numbers.

Resources

- Addition/Subtraction worksheets
- Beans and sticks

Lesson Three: Ruins of Teotihuacan grid plotting using four quadrants grid.

Objective

Math.6.11.A.01 - Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics. Student will construct various structures of their choice that are related to ancient civilization, and then measure the area, volume, and determine the scale factor of their created structures against the actual building or pyramid. Together with this lesson I will show to the student the movie clip about the ruins of Teotihuacan. Although Teotihuacan is not a Mayan city, evidence suggests that trades and exchange of ideas has existed between this people.

Math.6.12.A.01 - Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models. This

curriculum unit will investigate the system of numbers that the Mayan architects and engineers employ (O'Brien, Christiansen 1986, p.136)

Introduction

The teacher may say, "Today we are going to see elaborate buildings and temples of the ancient Central American civilization. Last time we talked about Mayan and its numbers, and you learned how to manipulate Mayan numbers. Well today you are going to see a movie clip about one of the largest and most imposing cities in the world in its days. The city that you are about to see is the city of Teotihuacan". As you can read in oxford encyclopedia, the city of Teotihuacan flourished in highland of Central Mexico between about 150 B.C and A.D 650 to 750 it covered 20 square kilometers, with one hundred thousand or more inhabitants (Cowgill p.72). Although Teotihuacan is not a Mayan city, however, archaeological evidence suggests that trades and exchange of ideas exists among this people (Geoffrey p. 1-44).

Concept Development

After the short movie clip, student will critically review the layout of the ruins of Teotihuacan, and they will work in pairs on plotting the pyramids on the chart with grids. Student will use prior knowledge in plotting coordinates on the first quadrant. This activity will advance their skill in plotting coordinates in four quadrants.

Student Practice

Review of addition of Mayan numbers will be given, this time student will add numbers in three positional values. Please see (appendix B). This student practice is necessary because student will compute perimeters of the pyramids, and perimeters of the city ruins.

Assessment

Students in pair will present their approximate coordinates of the pyramids and other notable structures that can be found in the ruins. Layout and presentation of graphs should be made in a chart with a grid.

Closure

Student will write a journal entry about a possible scenario that might have occurred during this time. In order to have enough food for the whole village you need to have sufficient land to plant and cultivate corn (the main crop of the time). The question is, if 3 warehouses full of corn will last 1 year, how many warehouses that are full of corn you need so that your people will have enough food for 4 years.

Resources

- (A) Chart paper with grid. (B) Movie clip Teotihuacan.
- Beans and stick.
- Washable colored pens.

Lesson Four: Construction of Mayan pyramid using ratio and proportion.

Objective

- *Math 6.04.A.03* - Use tables and symbols to represent geometric relationships such as perimeter, area, similarity, etc. the student will construct geometric objects and then determine and compute its area, perimeter, and volume then share and discuss geometric properties that can be found on the structure that they created.
- *Math.6.12.A.01* - Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models. This

curriculum unit will investigate the system of numbers that the Mayan architects and engineers employ (O' Brien, Christiansen 1986, p.136)

Introduction

Student will have a chance to review the movie about the Teotihuacan ruins; this time student will particularly pay attention to the structure of the pyramids. This lesson will incorporate math, arts and geometry. The Mayan is more likely use a unit of measurement that is comparable to our 3" (inches), thus one unit of Mayan measure is more likely equal to 3"(inches) in our measurement (Cramer 1938).

Concept Development

Construction of the scale model of the pyramid will give students idea how real objects can be put into drawing boards and scale models using scale factor.

Student Practice

Student will work in pairs to make a pyramid model, and then approximate the scale factor of the model against the actual pyramid.

Assessment

Student will use their skills in Maya numerals by computing and solving the area of the base of the structure, approximate the surface areas, of their pyramid, and finally the challenge question; how to compute the volume of the pyramid.

Closure

Student will write a journal entry about geometrical properties that can be found in the structure they just finished.

Resources

- Beans and stick.
- Washable colored pens.
- Maps of Teotihuacan.
- Movie clip of the ruins of Teotihuacan.
- Cardboards – for pyramid construction

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