Geometry in Economics

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INTRODUCTION

There are many myths and stereotypes around us about geometry and economics. Very often, these stereotypes overshadow the real matter of the subject and make it look extremely difficult to understand. Some of these myths are that geometry is a set of exercises for school study without any application to real life and that economics is a set of calculations only for everyday purchases. Such myths are not true – math is neither difficult nor tedious, and economics is not just a bundle of rules on how to sell and buy. Both geometry and economics go far beyond these stereotypes.

Economics explains many things, such as why some goods are cheap and others are expensive. It also explains the many factors associated with price fluctuations and many other phenomena. The sphere of economics goes far beyond the purchases of a single customer and extends to foreign relations between countries (Gregory 4).

As for geometry, it does not deal only with polygons and solids. At present, math – with geometry as one of its branches – plays a significant role in the education of each person. The main aim of math in the high school of the 21st century is not to provide knowledge of how to *calculate* areas and volumes; rather, it is to learn how to *find* and make a *decision* for a successful solution (Polya 3). This is very important both in private life and in business. It is a well-known idea that the person who is good in math is also good in his field of activity.

Geometry provides many opportunities for different kinds of information to be easily represented (Terretti 3). Despite the development of science and technology, mankind still uses and will continue to use very old and understandable geometrical figures and images for describing numbers. For example, at present there are many devices that show time. These devices include electronic watches, cell phones, pocket PCs, and electronic tables of different kinds. Nevertheless, the geometrical image of time represented by a circle and two arrows remains a very convenient way to observe the time. People feel very comfortable with arrows rather than digits, which often look tiresome. While driving in our cars, we watch many numbers – speed, capacity of fuel, and revolutions of the motor. The gauges showing these numbers are constructed as geometric images such as circles, semicircles, sectors, or rectangles. Such geometrical representation of numbers or other information is very convenient and understandable.

The main aim of this curriculum unit is to demonstrate for students the existence of the connection between geometry and economics in life. I will show them how economic terms can be represented as well-known geometrical figures – for example, segments, triangles, or rectangles. The length of these segments or areas of rectangles can numerically describe many economic problems.

Economics and geometry intersect and enrich each other. They have an infinite number of applications around us and can also be enjoyable (Nong 149, Tesch 258). Economics itself offers new problems that can be solved through geometry. Some of these problems are forecasts of price change, estimation of amount of turnover, calculations of costs, and many others. Geometry also represents economic problems as simple and understandable pictures (Meade 5). It tries to find solutions to these problems and to give recommendations for decisions.

Moreover, without geometric images, economic problems remain very dull, and economic decisions cannot be effective or may simply be wrong (Jehele 3). When high school students begin to learn about economics or science, they face one big difficulty: often they do not have enough life experience or previous knowledge to imagine the problem under consideration. For example, when students begin to solve problems using an inclined plane, they have no previous experience with this concept. They have probably never moved real objects in their lives like moving furniture or doing housework. They do not understand why objects should be moved. A geometrical representation of a problem with the help of triangles and vectors can provide understanding and give students the opportunity to find the solution on planes of all kinds.

The same difficulty applies to economics. Most students have very little experience in economics except maybe in making simple purchases. Such terms as demand, supply, and elasticity do not often arise in everyday life. Even if they arise, they appear as a very specific problem for a given purchase. Concepts like economic agents and the economy as a whole require such a high level of abstract thought that they can intimidate the beginning student.

Students start to learn economics in the 10^{th} or 11^{th} grade. By this time, they have already finished or just begun to learn geometry. My experience shows that students understand figures of plane geometry very easily. Such objects as triangles, polygons, angles, and lines do not confuse them too much. They are also able to estimate some elements of these figures, such as areas, perimeters, and angles. As a consequence, using geometrical images in other fields, including economics, facilitates students' understanding of main ideas and helps them reach the needed level of abstract thinking. The representation of an economic term or problem as a geometrical figure helps students to understand the problem in a way that is familiar. Geometrical images and their properties such as area and length give one an opportunity to assign a numerical value to economic terms. This value could be price, amount of turnover, percent, interest, or one of many others. Such data is the basis for different decisions in business. For this reason, the application of geometry in economics is very important, and it gives the students a foundation for the study of both math and economics at the college level.

UNIT BACKGROUND

My primary intention in creating this unit is to provide an understandable method for the investigation of geometry and economics. The main focus is the explanation of economic terms via geometric images and numerical estimation of these figures. The unit consists of three topics: supply and demand and their measurements, economic equilibrium, and achieving equilibrium. Each topic contains key terms for study and a brief explanation of the theory that combines economic terms and their corresponding geometric representations. Lesson plans, placed after the narrative, describe activities with students. All topics have direct connections to geometry, economics, and Texas Essential Knowledge and Skills (TEKS) objectives (See Appendix).

SUPPLY AND DEMAND AND THEIR MEASUREMENTS

Terms for the Study

- Economics terms: supply and demand, prices, total return, the law of demand and the law of supply, total revenue
- Algebra terms: equation of the line, slope of the line
- Geometry terms: area of a rectangle

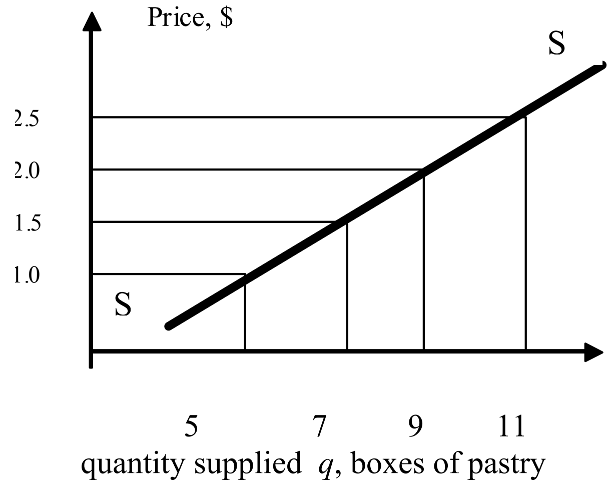
Supply and demand are the primary terms for economics. They are closely connected. One exists only together with the other. To be a successful businessperson, you need to know how to estimate the future development of the market. If you are a seller, then you need to be able to determine whether you should either increase the supply of goods you sell or stop your business and search for another field of application for your efforts and capital. Lines of demand and supply provide solutions to such concerns. Knowledge of future changes in supply and demand is essential for business strategy and future success.

Supply may be defined as a schedule of quantities that would be offered for sale at all possible prices at the market. Everyone who offers an economic product for sale is a supplier. Each supplier needs to decide how much to offer for sale and at what price. It is reasonable to predict that the higher the price, the greater the quantity the seller will offer for sale. To estimate the supply curve, you could make inquiries with manufacturers to see how much they could offer for a given price. After you accumulate enough data (or, speaking more mathematically, once you amass pairs of numbers [q and p], where p is the price and q is the amount sellers are willing to sell), you can plot your data on the same coordinate plane. The curve you will see will be the supply curve.

Consider an example of the relationship between the price and quantity supplied. Figure 1 shows a hypothetical supply schedule for a pastry you might sell in a school during lunch. If students are willing to pay $2.50 for a pastry, you can sell 11 boxes of them during lunch. If students are not willing pay more than $2.00 each, you should bring nine boxes. If they are able to pay only $1.50, then you will bring seven boxes. Finally, if the price they are willing to pay falls to $1.00, you will offer them only five boxes of pastry.

Figure 1. Supply schedule for pastry	
Price, p $	Quantity supplied, q (boxes of pastry)
1.0	5
1.5	7
2.0	9
2.5	11

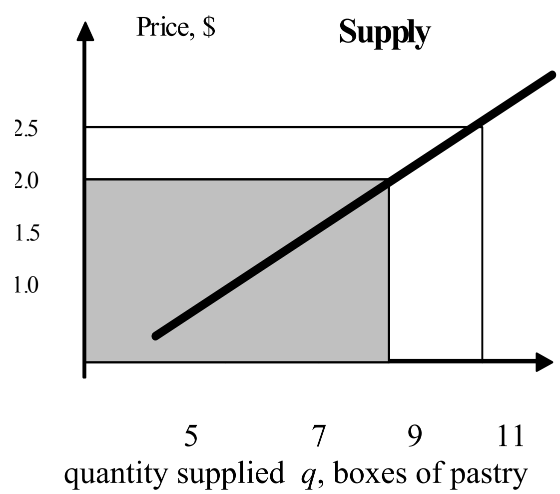
Figure 2. Line of supply



After you plot pairs (q and p) on the coordinate plane, you will see the upward line of supply. It goes from bottom-left to upper-right. Such a line is said to have a positive slope. This means that the increase of one variable causes the increase of the second variable and vice versa. In our case, if the price for one item of pastry increases from $1 to $2, then the seller will be willing to bring to lunch nine boxes of pastry instead of five boxes. So, the higher price causes an increase of goods for the seller.

This line of supply provides a strategy to forecast the amount of revenue of the seller if he or she sells his/her goods at a certain price. Let each box of pastry they sold contain 100 items. Then, if the price is $2 a piece, the total revenue will be $2×9×100=$1800. Geometrically, that means the area of the shaded rectangle in Figure 3.

Figure 3. Possible revenue



This revenue is a possible revenue rather then an actual one. It is clear from the picture that if the price increases, then the total area of the figure will also increase because the other side of the rectangle will also increase. This demonstrates the positive relationship between the price and the quantity sold.

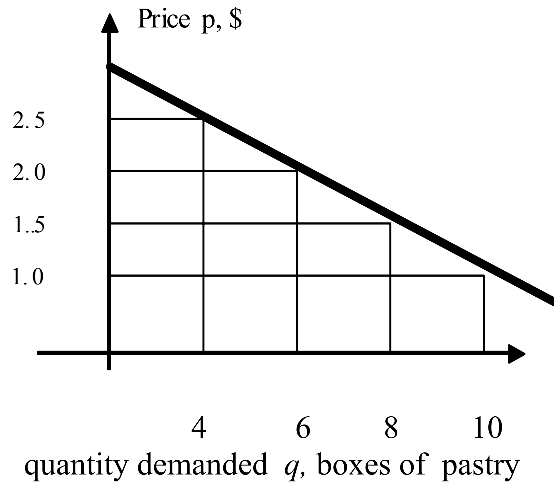
Demand is not exactly the same as willingness, although these terms are very close to each other and are often used as substitutes. Briefly, demand is the willingness and at the same time the ability to buy something. This is the economic understanding of demand. The willingness to have something without ability to purchase it is not a subject of traditional economics. It is more closely related to sociology, psychology or another behavioral science. In economics the demand of someone makes sense only if this 'someone' really wants and is really able to buy something. The amount of goods we buy depends on our desire and the price of goods. If you desire much and have little money, you buy less. If you have enough money and have no desire or necessity to have something, you also buy less. If you receive more money, then you might buy more. There are many factors that determine your desire. In our topic let us consider that the demand depends on only one factor, namely the price.

It is well known that if the price is lower, then people will buy more. If gas is cheap, people will drive more and they will need more gas. If gas becomes more expensive, people will drive less and they will need less. This is the law of demand. There is a well known geometrical technique for representation of relationships between variables. This method has existed since the time of Rene Descartes who was the first to use it in math. The idea is to prescribe each point on the plane two numbers, namely coordinates (q,p). These coordinates show the horizontal and the vertical position of a particular point from some initial point, the origin. If letter q will represent numbers of one variable and the letter p will represent the values of another variable then the whole set of points give the relationship between our variables. Let p be a price of goods on the market and q be an amount demanded at the price p. Then if we plot a pair of points (q, p) on the coordinate plane we will have some decreasing line. This line could be steeper or flatter but it will go down from the left to the right.

Consider the example discussed above. We described the behavior of the producers who sell pastry in school. Now let us consider the behavior of customers, namely students, during the lunch when they buy food in the cafeteria. Each student can be considered a customer. The firm or firms who supply food for school will be suppliers or sellers. The cafeteria will represent in our example a part of a market economy with customers and sellers or producers. If they, moreover, produce the item they sell then they can be named also as producers. So, the cafeteria will represent a competitive market with customers and suppliers. This market is competitive because there is a competition between customers (students) and suppliers. The students try to buy as much as they can for the least price. The suppliers try to sell more for a higher price and get their revenue for what they sell. There are many factors that influence the desire of customers – their tastes, advertising, customs and many others. In our unit let us imagine the demand depends on the price in a linear way. Then this curve should be a decreasing line. It will go from northwest to southeast. In algebra, such lines are said to have a negative slope. It means that the increase in one coordinate causes the decrease in another coordinate. From this graph it follows that if price decreases for one unit from $2.50 to $1.50 then the demand (purchases) will increase for 4 units, from four to eight boxes of pastry.

Figure 4. Demand schedule for pastry	
Price, *p* $	Quantity demanded, *q* (boxes of pastry)
1.0	10
1.5	8
2.0	6
2.5	4

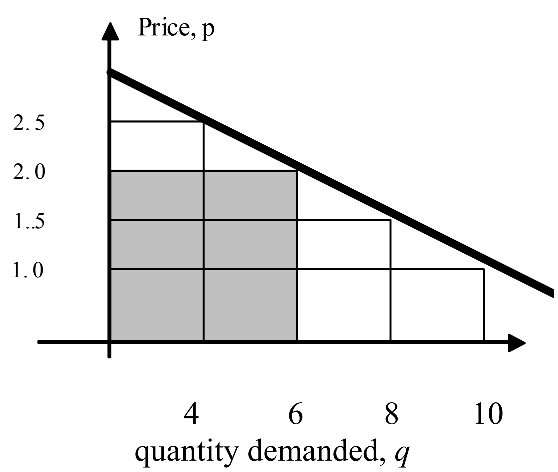
Figure 5. Line of Demand



For the customer the price is usually given. He or she cannot influence it. But every customer buys a different number of goods depending on the price. In other words, the demand depends upon the price. How could a student estimate the relationship between demand and price in real life? The hint is to consider the demand as an amount already purchased or sold. If there is no demand for some goods, then people will not buy and businesspeople could not sell. If people want to buy and moreover they can afford it, then they will purchase and the sellers will get rid of stocked goods they produced. After you plot pairs of points *P*(*amount purchased, price*) on the coordinate plane, then you will see the demand curve.

It is clear from the picture that the geometrical image of demand is the descending line. Such a line is said have a negative slope. Demand and price form the negative relationship. The higher the price, the smaller is the demand. If the price increases from $1.00 to $2.00 then students will demand only six boxes of pastry instead of ten.

Figure 6. Line of Demand

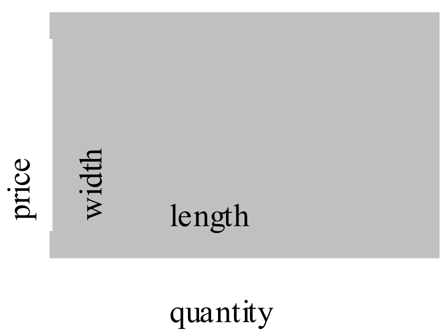


The total revenue will be $\$2 \times 6 \times 100 = \$1200$ (remember our assumption above that there are 100 pieces of pastry in the box). On the contrary, if the price goes down, for example from \$2.5 to \$1.5, then students will buy more pastry. They will be able to buy eight instead of four boxes. Here again we use the geometrical figure, the rectangle, to represent the economic term, the revenue.

How to estimate the line of supply and the line of demand is a problem of great interest and importance. Many mathematicians and economic analysts spend much time and effort in getting data and sketching these important figures, because they can answer many questions, one of which is the equilibrium price. But this is a matter for the next section.

Geometry Focus

Geometrical representation of the supply and demand as lines makes processes in the market quite understandable. From the pictures above it is clear why prices go up or down. The geometrical figure also shows that the area could have the economic sense, namely revenue.



The well-known formula for the area of a rectangle is *Area* = *length* × *width*. If we measure the distance in *feet*, then the unit of the area will be *feet* × *feet* = *square feet*. In our economic problem the sides of the rectangle are the *quantity* (the horizontal variable) and the *price* (the vertical variable). The unit of price is *money per unit of quantity* or $\frac{money}{quantity}$.

Let us put these units into the formula of the area. We will have the following expressions:

$$\text{Area} = length \times width = quantity \times price = quantity \times \frac{money}{quantity} = \text{money}$$

It follows that our area has the unit *money*. In other words, the area of the rectangle represents the total possible revenue for the amount of goods sold. The geometrical term can carry also the economic meaning.

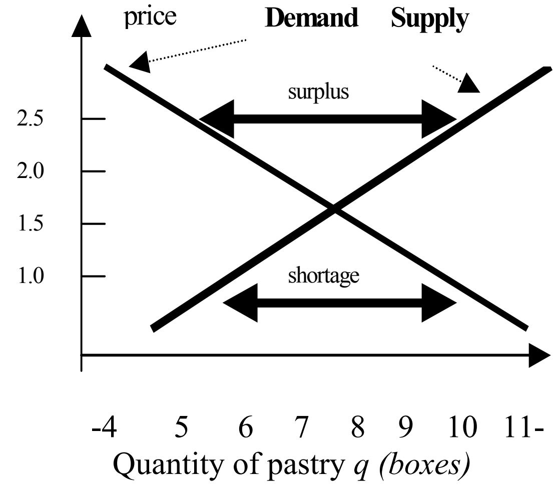
ECONOMIC EQUILIBRIUM OF SUPPLY AND DEMAND

Terms for the Study

- Geometry term: length of the segment

Suppose that the price of a pastry happened to be \$1. Figure 5 tells the same thing as Figures 2 and 4 tell separately: that at \$1/pastry, students will want to buy a total of ten boxes of pastries, and the producer will want to sell only five boxes. This discrepancy means that at \$1 each, there is a *shortage* of 10 – 5 = 5 boxes of pastry. It is a segment between 10 and 5.

Figure 7. Analysis of equilibrium



- Algebra terms: equation of the line, slope of the line
- Economics terms: market equilibrium, surplus, shortage, equilibrium price

Along a line of demand, such as the one in Figure 3 there are a lot of price/quantity combinations from which to choose the price to sell and the quantity to buy. Along the line of supply on the Figure 2, there are also many different combinations of price to buy and quantity to sell. Neither of these lines is able by itself to determine the price for the pastry and the amount sold. To answer this question we need to put both graphs together on the same coordinate plane. We should keep in mind that the line of demand means what consumers (in our case, students) are willing to buy at different prices; the line of supply indicates the amount the producer is prepared to sell at this price. These types of economic decisions are quite different. How much will be produced? How much will be purchased? How are decisions of consumers and the producer coordinated? To answer these questions, let us analyze our lines from different possible market situations in the cafeteria during lunch.

A shortage results if at the current price, the quantity demanded exceeds the quantity supplied; the price is too low to equate the quantity demanded with the quantity supplied. The increase in the price of pastry in response to the shortage will have two main effects: on the one hand, the higher price will discourage consumption; on the other hand, the higher price will stimulate the producer to bring more pastries. Thus, the increase in the price of pastries will lead both the buyers (in our case, the students) and the seller in the marketplace (that is, in the cafeteria) to make decisions that will reduce the shortage of pastries.

From our lines in Figure 5, it follows that when the price of pastry is about $1.75, the shortage of pastry disappears completely. At this price, students want to buy a total of 7.5 boxes of pastry, and the producer wants to sell the same amount. What would happen if the price rose above the equilibrium price of $1.75 per piece of pastry? At the price of $2.50 per piece, students want to buy 4 boxes, and the producer wants to sell 11 boxes. Thus, at $2.5 there is a surplus of 11 - 4 = 7 boxes on the market. It is a segment on the horizontal coordinate axis between 4 and 11. A surplus results if at the current price, the quantity supplied exceeds the quantity demanded: the price is too high to equate the quantity demanded with the quantity supplied.

The seller will be disappointed as pastry inventories pile up. He will set his/her prices lower. This fall in the price will encourage students to buy more, and the surplus of pastry will therefore disappear.

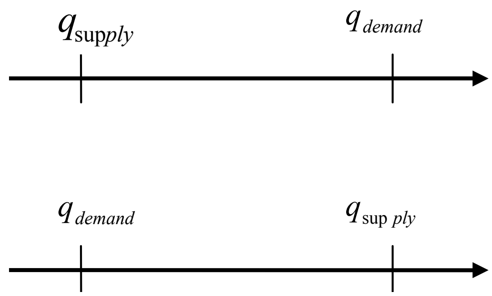
Again, we find that the price will tend toward $1.75 and the quantity will tend toward 7.5 boxes. The equilibrium point is where the supply and demand lines intersect. In Figure 5, there is no other price/quantity combination at which the quantity demanded equals the quantity supplied – any other price brings about a shortage of pastry or a surplus of pastry. The arrows in Figure 5 indicate the pressures on price above or below $1.75 and how the amount of shortage or surplus changes.

The equilibrium of supply and demand is stationary in the sense that the price will tend to remain the same once the equilibrium price is reached. Movements away from the equilibrium price will be restored by the bidding of excess buyers or excess sellers in the marketplace.

In short, in the market economy, the adjustment process moves the economy toward market equilibrium – a situation in which prices are relatively stable and the quantity demanded is approximately equal to the quantity of goods and services supplied.

Geometry Focus

These lines also explain why the surplus or shortage takes place. Geometrical terms such as segment and length give the opportunity to numerically estimate surplus or shortage and provide a decision-making strategy for students and businesspeople. Both supply and demand are scores on the horizontal coordinate line.



horizontal coordinate axis

If demand is greater than supply, then the difference between them gives the value of the shortage:

$$shortage = q_{demand} - q_{supply}$$

If, on the contrary, supply is greater than demand, then the surplus emerges which value is:

$$surplus = q_{supply} - q_{demand}$$

So, the geometrical term, segment, relates directly to economic concepts and has a meaning in addition to that connected to the measurements on the plane or earth.

ACHIEVING EQUILIBRIUM

Terms for the study

- Geometry term: cobweb model
- Algebra terms: solution of system of equations, slope of line
- Economic terms: economic lag, rate of supply, rate of demand

Equilibrium is a very important term in economics and other fields, such as science and politics. We hear this word every day. In math, there are many ways to describe equilibrium. In this unit, we choose only one geometric method, a so-called cobweb model (Samuelson 382). It gets its name because its picture looks like a spider's cobweb.

In our study, we will treat equilibrium as a state when supply and demand remain unchanged over time. We saw from the above section that equilibrium exists only if demand is equal to supply. Geometrically, it means the point of intersection of the line of supply and the line of demand. If demand is greater than supply, then a shortage will exist in the market. Customers will be able to pay more for goods and prices will go up. As for producers, they will increase the amount of goods for the market. If, on the contrary, the demand is smaller than the supply, then sellers will decrease their prices to attract customers and make them more willing to make purchases. In both cases the prices that customers are ready to pay to satisfy their wants and the prices needed by producers to provide them the profit will go towards each other until they meet. This price, when the demand is equal to the supply, is called the equilibrium price. How to estimate this equilibrium price and how to predict when (if any) it can be reached is a question of a great importance. Does the inequity of either a shortage or a surplus always eventually lead to equilibrium? Does the market economy have the wonderful feature of automatically adjusting itself to equilibrium when demand equals supply?

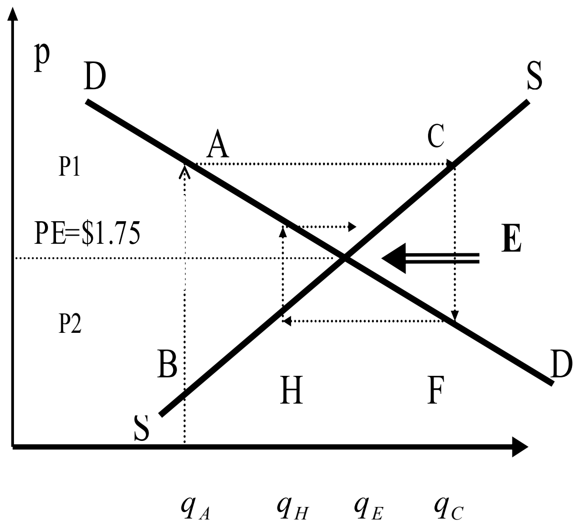
Before answering these questions, let us remark that in real life, customers and suppliers need some time to make decisions on how much to buy or sell. This time is called a time lag. This time lag or economic lag depends on many factors and is quite different for different types of business. In our study, let us assume this time lag is one day. For us, this is quite natural because students come to school every day, and their decisions depend upon their purchase from yesterday and the difference in prices today and the day before. It is worth mentioning that the laws of supply and demand work in the same way no matter how big the economic time lag is.

Consider our pastry market in the school near some equilibrium point $E\ (q_E, p_E)$. This point is the point of equilibrium because the lines of supply and demand intersect at this point. The value q_E is the equilibrium value for supply and demand, and p_E is the equilibrium price. Suppose that one day, supply dropped to q_A, which is below the equilibrium amount, q_E. In Figure 6, it corresponds with point B. A shortage will take place in our market. The price will go up to P1 at point A. But that is not the end of the story. Point A is not an equilibrium point. Price P1 is higher than the equilibrium price, $1.75. For this reason, the next day, producers will offer amount q_C. On our figure, that means taking the path rightward to point C on the supply line SS. But price P1 is too high, and students will not be able to make purchases. They are able to purchase amount q_C only at the price P2. Amount q_C will be greater than the equilibrium amount, q_E. We know from the above discussion that in this case a supply surplus will occur in our market. Demand for pastries will drop to q_H. This corresponds to the path downward from the line of supply to the line of demand. To get rid of inventories and stimulate demand, sellers will reduce prices and offer fewer pastries in the cafeteria. Geometrically, that means taking the path from point F leftward to point H on the line of demand, DD. After the first cycle, it arrives at point H, which is closer to the equilibrium point than the previous point, B. In the limit, supply and demand reach their equilibrium values. This type of equilibrium is called stable equilibrium.

So, like a man on a tightrope who goes too far to one side, then corrects himself by going too far to the other, the market price oscillates in successive periods above and below equilibrium, tracing out a spider-like cobweb.

What is the final outcome? Figure 8 was drawn with the line of supply SS at E *steeper* than the line of demand DD. So, as can be seen from the diagram, the oscillations finally do dampen and die out: the cobweb winds inward to E. We are then back at equilibrium, were we can stay for a long time, or at least until the next outside disturbance comes to set off still another dying-out oscillation.

Figure 8. Convergent cobweb model.



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Not all equilibrium points are so dynamically stable. Figures 9 and 10 show examples of other possible situations. In Figure 9, line of supply SS has been made *flatter* than line of demand DD. The cobweb diverges outward in an explosive oscillation.

In Figure 10 we get a perfect cobweb: depending how severely the market is disturbed, it will oscillate endlessly around equilibrium, getting neither more nor less violent in its swing. This is like the case of an ideal pendulum, which would repeat its swings forever.

Figure 9. **Divergent cobweb model**.

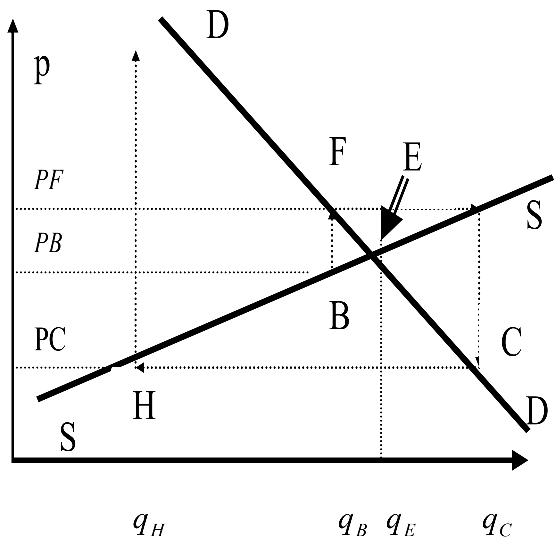

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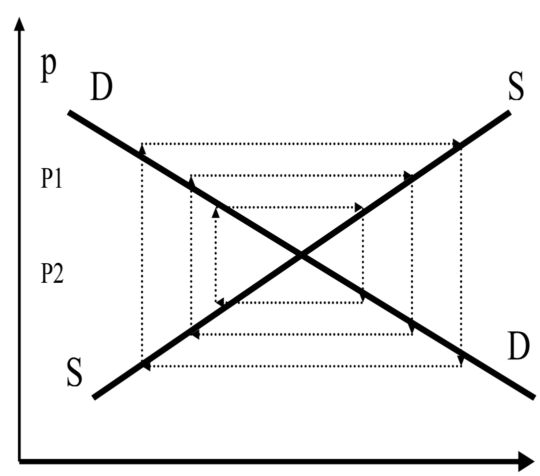
Figure 10. **Oscillating cobweb model**.



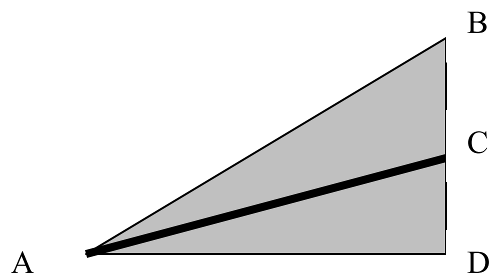
From an economic point of view, Figure 9 means that in each time period, prices will deviate more from the equilibrium value, *PE*, and the amount of pastry demanded or supplied will go farther from the equilibrium value, q_E. In our market (the cafeteria), it could be explained as follows. One day, let the supplier offer fewer boxes of pastry than usual. There could be many reasons for it – auto accident, bad weather, or something else. A shortage of pastry at price *PB* will result in the increase of demand, and, as a consequence, the price will go to the value P1. In the next time period (the next day), suppliers bring more pastry in accordance with the law of supply. Let this value be q_C. But the next day, buyers (students), remembering the prices of the day before, will be reluctant to buy pastry. To get rid of their excess inventories, suppliers need to decrease the price to the value *PC*. Price *PC* is less than the price *PB*. It is evident that in accordance with the law of demand, demand the next day will be greater than the demand q_B from the day before. And, as it follows from the picture, these differences will increase. In real life, such systems cease their existence very soon.

Such a divergent model emerges because the line of supply is flatter than the line of demand. In other words, the rate of supply is less than the rate of demand. It could be concluded that real a market system with a flat line of supply and a steep line of demand is very risky and unstable. Small disturbances lead the market away from the equilibrium state. If we refer to the equations of lines of supply and demand, it is clear that the equilibrium point is the solution of the system of equations for supply and demand.

As for the market portrayed in Figure 10, then, such a situation will not exist in reality for very long. As we know from our own experience, price oscillations are quite unpleasant both for customers and producers. It is very difficult to plan a personal budget or do financial transactions when economic indicators go up and down. For these reasons, state authorities and decision-makers try to find different ways to avoid oscillations in economics. As for geometry, measuring the steepness of the lines of supply and demand helps us to understand in what direction the economy goes and gives us hints as to what to do.

Geometry Focus

The steepness of a line of demand or supply gives important information for decision makers about the business they are involved in. In math, the steepness of the line is called slope. Slope is defined as the ratio of the vertical change of some line to the horizontal change. If we have two lines, AB and AC, then their slope will be the following:



$$slope_{AB} = \frac{BD}{AD}$$

$$slope_{AC} = \frac{CD}{AD}$$

It follows from the picture that BD is greater than CD; consequently, the slope of AB is greater than that of AC.

Slope is a very important indicator of the line. Measurements of the slope for the lines of supply and demand can answer the question of the stability of the market and the speed of its convergence to the equilibrium point. In Lesson Three, there are some activities on this matter.

CONCLUSION

In this unit I tried to systematically apply geometric images for the analysis of certain economic phenomena. At present, geometric ideas and images have penetrated the whole of modern economic theory, and they also have many practical applications. The main aim of my work was to demonstrate the many applications of geometry, both in economics and in other aspects of our everyday life. In this unit, only the simplest geometrical images were considered. I hope that such examples, used in high school, will give students some impetus to be more interested in math. At the same time, when learning the basis of economics, students will have one more opportunity for interdisciplinary perception of the subject.

LESSON PLANS

Lesson One: Measurements of Supply and Demand

Objectives
Students will investigate the law of demand and the law of supply on the basis of real data. They will estimate the areas of rectangles that describe the possible revenue.

Supplies and Materials
CD player
Worksheet with data of prices and sales of gas
Graphing calculator
Graph paper
Ruler

Procedure
The lesson will begin with the discussion of terms, supply and demand. Students will give their own examples of the law of supply and the law of demand. Then, we will consider the equation of the line and discuss how to estimate numerical values of coefficients of the line from the scattered data. I will also ask students to recall the formulas for the area of the rectangle and the triangle as well as how these formulae look for the coordinate plane. Afterwards students will work in groups.

Student Activity: Market Investigations of Fuel Consumption
Students will analyze the data of gas consumption with different prices to estimate the line of demand and the line of supply. After the brief discussion of supply and demand, students will work in two groups. One group will investigate purchases of gas. They will try to arrive at the line of demand. The second group will deal with the supply of gas. The aim of this group is to attain the line of supply. Students will use different worksheets, but at the end, they will combine their results and work together.

Real World Problem for the First Group
"Pilgrims Gas" company needs to know the relationship between the amount of gas it sells (or the amount that customers buy) and the price of gas. One day, the company orders the same type of gas at their gas stations in Houston at different prices. The next day, managers from the headquarters call managers at every gas station and ask them the amount of gas they sold. Afterwards, the economic analyst plots data on the coordinate plane. The coordinate axis means the amount sold at the particular price, and the vertical axis is the axis of price per gallon of gas. If they draw a line through these data, they will see the line. This line will be the line of demand.

Have students sketch a line, taking the horizontal axis for amount of gas sold and the vertical axis for the price per gallon. Estimate the total revenue of each gas station.

Gas station	Price for gas ($ per gallon)	Amount of gas sold/purchased (thou. gallons)	Total revenue
Lubbock	1.50	15	
Braeswood	1.35	17	
Sugar Land	1.30	18	
Spring	1.45	15.5	
Fuqua	1.25	18.5	
Galveston	1.40	16	

Find the equation of the line of demand in the form $p = m \times q + b$. Using a graphing calculator, estimate the slope m and y-intercept b of the line of demand.

Answer the following questions: Does your line look like a demand curve? How can you prove it? What is the slope of the line?

Real World Problem for the Second Group
The same "Pilgrim Gas" tries to get the line of supply for gas. Managers from the supply department contact big companies and ask them the price and the amount the company can sell at this price. The results of such activities are summarized in the table below.

Fuel company	Price for gas ($ per gallon)	Amount of gas offered (1000s of gallons)
Exxon	1.50	19
Shell	1.35	16
BP	1.30	15
Lukoil	1.45	18
Sibneft	1.25	14
Lubec	1.40	17

Students plot these data on the coordinate plane. The horizontal axis is supposed to be an axis of the amount of gas. The vertical axis will represent the price. Draw a line with a ruler through these points. Use a graphing calculator to estimate the equation of the line in the form $p = m \times q + b$.

Students answer the following questions: Does the line you got look like a line of supply? How can you prove it? What is the slope of this line?

During this lesson students use both geometric terms and economic terms. The supply and demand are associated with the increasing and the decreasing line. The amount of possible turnover is represented as an area of the rectangle.

Lesson Two: Analysis of Equilibrium

Objectives
Students will find the equilibrium price on the fuel market as a point of intersection of the line of supply and the line of demand. They estimate the possible surplus and shortage on the gas market as the length of segments between lines of supply and demand.

Supplies
Data from Lesson One
Graph paper
Graphing calculator
Ruler

Procedure
The lesson will begin with a discussion of equilibrium in nature and society. Students will give examples of equilibrium from everyday life (for example, pendulum, swing,

weighting machine, spring). Students will discuss which factors influence the system to lead it to the point of equilibrium. Then, the teacher will ask them to give examples of equilibrium in economics. It will be worthwhile to mention processes of demand and supply in the labor market, in the market of goods or services, and in the securities markets. It is very important to discuss the symptoms of equilibrium and disequilibrium in these markets. Students should discuss which market might be more likely to be stable or unstable. Students will arrive at a hypothesis about the steepness or slope of lines of supply and demand in each case.

The next step will be to analyze the data from the previous lesson. Students will also work in groups. It is important that in each group, there are students from different teams than the previous lesson.

They will plot their lines of supply and demand on the same coordinate plane and geometrically find the point of intersection, $P(q, p)$. The vertical coordinate, p will be the equilibrium price. The teacher should discuss with students of each group the data obtained and compare the equilibrium price from different groups.

For different scenarios of the changes in prices students try to guess the amount of surplus or shortage of gas. They also will try to estimate the amount of revenue firms will have in these scenarios. On the graph paper, they will draw segments of the surplus or the shortage of the gas.

Lesson Three: Achieving the Equilibrium

Objective
Students will analyze the type of equilibrium on the market of gas

Supplies
Data from Lesson One
Graph paper
Graphing calculator
Ruler

Procedure
At the beginning of the lesson, the students and the teacher will try to find examples of periodic systems, events, or phenomena in real world. They should mention examples from science (a pendulum, a swing, changes of day and night, changes of seasons) and economics (the business cycle, the price season cycle, the security exchange cycle). During this discussion, they should also analyze the factors that influence the periodic system to deviate from the equilibrium state. In science such factors could be a friction, a resistance of the air or some fluid or energy dissipation. Students need to consider cases when these factors increase the deviation from the equilibrium as well as cases when these factors eliminate such deviation.

In economics, the best-known periodic phenomena are the business cycle, the seasonal cycle, and the oscillations of the securities markets. Students should find different types of such phenomena. For example, seasonal oscillations could be seasonal price changes, seasonal labor market waves, and demographic waves. Among the factors that influence the economic cycle are credits, bank percent on loans, technological progress, and public relations.

Then, students will analyze the lines of supply and demand in each group. They will compare the absolute values of the slope of these lines. This can be done either geometrically with the graph pare or algebraically, using the well known above formula the slope of the line. Afterwards, they will draw a conclusion about the type of the equilibrium of the gas market. Students will answer the following questions:

- If there is some disturbance, will the deviated prices tend back to the equilibrium or, on the contrary, will the deviation from the equilibrium increase?
- If the market is stable (prices, supply, and demand tend to the equilibrium value) how many cycles will it take to reach the point the equilibrium?
- If the market is unstable (prices, supply, and demand increase with each new cycle), then after how many cycles data become unrealistic, leading to the collapse of the system?

In all these cases, students will draw cobweb models and geometrically try to prove their explanations and speculations regarding the gas market.

APPENDIX A

Texas Essential Knowledge and Skills Objectives Used in This Work

Geometry TEKS

- a2: Students use geometric thinking to understand mathematical concepts and relationships among them
- a4: Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems

Algebra 1 TEKS

- a1: continue to build on mathematical understandings presented in grades K-8
- a3: use functions to represent and model problem situations and to analyze and interpret relationships
- b1B: use data sets to determine functional relationships between quantities
- b1D: represent relationships using concrete models, tables, graphs, diagrams, verbal description, equations, and inequalities
- c1C: translate among and use algebraic, tabular, graphical, or verbal description of linear functions

Math Models TEKS

- c1B: use multiple approaches (algebraic, graphical, and geometric) to solve problems
- c2A: interpret information from various graphs and draw conclusions from the data
- c5A: use rates, linear functions, to solve problems involving personal finance and budgeting
- c3C: determine appropriateness of a model for making predictions
- c3A: formulate meaningful questions, determine data needed, gather data, analyze data, draw conclusions

Introduction to Business TEKS

- c1A: determine the role of wants and needs
- c1B: describe the economic process at it relates to determining wants and needs
- c5A: analyze individual goals and values
- b: Students develop a foundation in the economical aspects of business to become competent consumers, employees, and entrepreneurs. Students incorporate a broad base of knowledge that includes the legal, managerial, marketing, financial, ethical, and international dimensions of business to make appropriate business decisions.
- c1: The student explains the economic process and relates the process to the development of economic system.
- c3: The student researches consumer issues and determines financial implications for individual.

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