

Symmetry, Design, and Patterns

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Introduction

This first part of the unit is meant to introduce the teacher to the subject and vocabulary of the symmetry. The unit will include three activities that the teacher and students can involve themselves in over a two-week period for an hour each day in the classroom culminating in a class field trip. This unit is geared for middle school students, but can be adapted for elementary school students with some modification.

Part of the unit will include hands-on activities, such as using ClarisWorks to design and color forms of symmetry. Students are asked to create shapes in ClarisWorks program, work area of Drawing to test a pattern and design for symmetry.

The approach for this curriculum unit is interdisciplinary: combining math, fine art, computer, and science. Included in the units will be a pre-reading of symmetry from early-recorded history to present day. We will attempt to answer is there symmetry in chaos? How can symmetry be seen where none seems to exist?

“Where is Symmetry in Cultures Found?” – Unit Goals

The goal I have is for the students to develop an awareness of finding symmetry in their environment. Students will also develop an understanding of the different types of symmetry. A transformation is a one-to-one mapping on a set of points. Symmetry is said to exist if an object or figure looks the same under a transformation. Types of symmetries are **rotational symmetry, reflection symmetry, translation symmetry, and glide reflection symmetry**. These four types of symmetries are examples of different types of symmetry on a flat surface called planar symmetry. Rotational symmetry occurs when a pattern that we can turn a pattern by some degree about a point and the pattern looks exactly the same. Reflective symmetry occurs when a pattern can be folded across a mirror so one half lies on top of the other. A reflection flips all the points in the plane over a line, which is called the mirror. Translation symmetry occurs only in patterns that cover a plane. This happens when you can move a copy of a pattern a specific distance in a specific direction so that it lies exactly on top of the original pattern. A translation slides all the points in the plane the same distance in the same direction. This has no effect on the sense of figures in the plane. Glide reflection symmetry is much like translational symmetry. When you move a pattern a given distance in a given direction and then reflect it over a mirror line so that it lies exactly on top of the original pattern, that pattern has glide reflection symmetry. A glide reflection translates the plane and then reflects in across a mirror parallel to the direction of the translations. A glide reflection changes the sense of figures in the plane.

Students will start looking for repeating patterns. Soon they will be able to identify them everywhere. There are the patterns on tile of bathroom floors and kitchens

walls. The simplest of these repeating patterns are those built from a design on a single square whose horizontal and vertical translates fill up the plane. The common feature of all these patterns is that they can be repeated infinitely often. Most flowers possess symmetry and there are many examples of flowers with five petals and fivefold symmetry. Many corporate logos today have chosen symmetry of design for their company symbols. Many of the car manufacturers display ornaments on the hood of cars are examples of symmetry.

What Is a Tessellation? Definition

A dictionary* will tell you that the word "tessellate" means to form or arrange small squares in a checkered or mosaic pattern. The word "tessellate" is derived from the Ionic version of the Greek word "tesseres," which in English means "four." The first tilings were made from square tiles.

A regular polygon has 3, 4, 5 or more sides and all angles, are equal. A regular tessellation means a tessellation made up of congruent regular polygons. [Remember: Regular means that the sides of the polygon are all the same length. Congruent means that the polygons that you put together are all the same size and shape.]

Only three regular polygons tessellate in the Euclidean plane: triangles, squares or hexagons. We can't show the entire plane, but imagine that these are pieces taken from planes that have been tiled. Here are examples of tessellation of



Step-by-step directions to a class.

1. Open Claris Works.
2. Make a new file - select "painting" as the type of file.
3. Go to "save as" - name the file "tess".
4. Select a color from the paint color palette.
5. Select the same color from the pen color palette.
6. Holding the shift key with one hand, draw a square (it will fill it with the color that was selected).
7. Select the lasso.

How to open a new painting file.

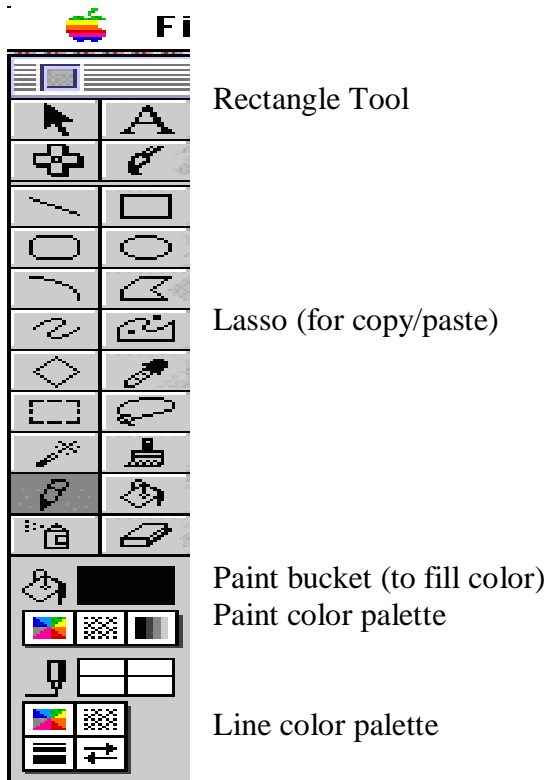
- If you are already in the Claris Works program, go to "file" and select "new".
- A window will appear and you should select "painting" as the type of file.
- It is a good habit, once the file has been opened, to name it.
- Go to "file" and select "save as" -
- A window will appear. Type in "tess" or another appropriate name.
- Press return.

- Open the Claris Works program. The window will appear and you should select "painting" as the type of file and then follow the directions stated in the previous paragraph.

Why is the drawing area so small?

- Painting files in Claris Works require more memory than a word processing file.
- If the area available to paint seems unusually small, quit the program and locate the Claris Works program icon on your desktop.
- Click on it one time (to select it) and go to "file" and select "get info".
- A window will appear where you can increase the amount of memory allotted to the program.

Tools needed to tessellate in Claris Works.



How to draw a square.

- select the rectangle tool
- hold the "shift " key and draw
- you will notice that with the shift key pressed you cannot make a rectangle, only a square.

How to use the lasso.

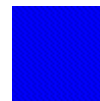
- select the lasso tool.
- move the mouse so that the lasso is positioned at the beginning of what you would like to move
- click and with the mouse pressed down, move through the area of selection.

- once you have completed the loop, release (be careful not to click!)
 - the area selected will be blinking
-

LESSON 1: Making The First Tessellation Pattern

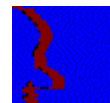
Step One

Select your favorite color from the paint color palette and the line color palette. Use the rectangle tool to draw a square. If the shift key is held down as the "rectangle" is drawn, it will make a square.



Step Two

Using the lasso, start at the top corner, wiggle around as you are going down and exit the lower corner of the square. A section of the square will be selected.



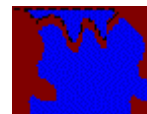
Step Three

Hold the shift key and slide the chunk over to the right. Position the chunk on the right side, leaving no overlap and no gaps. [It would also be possible to start on the right side, select a chunk, and slide it to the left.]



Step Four

Repeat the process but this time go from top to bottom or bottom to top.



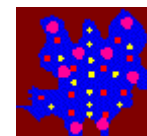
Step Five

Look at the figure. What does it look like to you?



Step Six

Use the pencil or paint brush (with contrasting colors selected) to decorate the figure. Do not add anything to the outer edges of the silhouette.

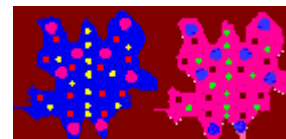


Step Seven

Use the lasso and position the figure up higher on the card. Hold option, slide the copied figure to the side.

- Go to "transform" in the menu and select "invert" and the second figure will change color.

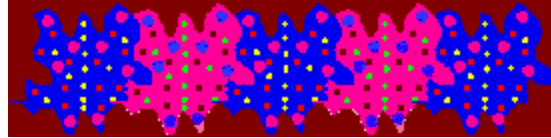
OR



- Select the paint bucket from the tools, also select a contrasting color from the paint color palette and "dump" paint into the various areas until you have a second figure with different colors.

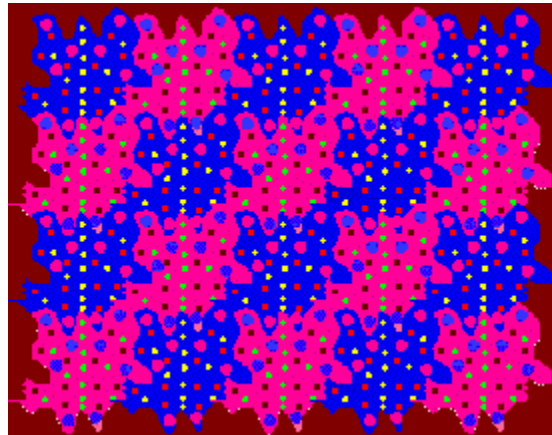
Step Eight

Use the lasso and the option key to copy one figure and position at the beginning of the row. Continue sliding and positioning first one figure and then the second until you have one row completed.



Step Nine

Using the lasso, hold "option" down and "copy" by sliding one figure to the side. Lasso the other figure, hold "option" down, slide it off and fit it into position up against the darker figure. Hint: If it does not fit perfectly, review Steps 1 - 8 and try again. Continue this procedure until you have created a tessellation.



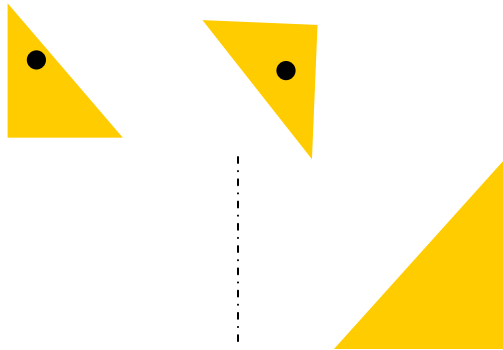
LESSON 2 – Creating a Wallpaper Design

DEFINITION: We will say that a wallpaper pattern is a pattern which covers the entire plane and can be produced by repeatedly applying isometric transformations to a finite motif (and to the images of that motif).

1. Classification of Patterns -
Classification systems are used in many different areas. When you look on store shelves, you have a classification system that sorts products. In a filing cabinet you have a classification system that sorts files, and in a library you have a classification system that sorts books based on their genre. What other examples of classification have you seen?
2. In this lesson you will learn symmetries of repeated patterns. In addition you will learn about a system for sorting these patterns according to their symmetries as mathematicians do. Before we start, here are some terms you need to know.
 - **TRANSFORMATIONS:** A transformation is a one-to-one mapping on a set of points. The most common transformations map the points of the plane onto themselves, in a way which keeps all lengths the same. These transformations are called isometries.

Another common sort of transformation which does not preserve lengths are dilatations. The terms you need to know:

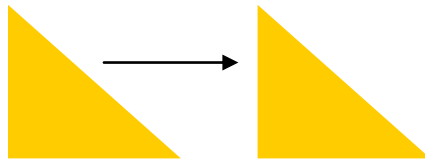
- **ROTATION:** To rotate is to turn about a point. When you make a left-hand turn at a corner, we say you are rotating 90 degrees about the corner.



- **REFLECTION:** When you look in a mirror, your image is reflected back at you. If you imagine that your image actually exists on the other side of the mirror, you get some idea of the mathematical definition of reflection or mirror image. We can reflect figures by folding along the mirror line as you see below.



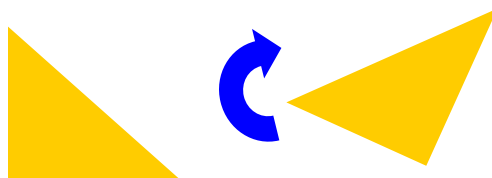
- **TRANSLATION:** Translate means to slide. A translation moves a figure a given distance in a given direction. You can think of a translation as sliding an image across a piece of paper.



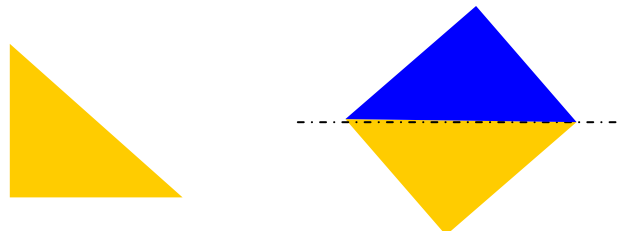
- **GLIDE REFLECTION:** This transformation combines translations and reflections. A glide reflection occurs when you slide an image in one direction and then reflect it over a line.



- **ROTATIONAL SYMMETRY:** We say a pattern has rotational symmetry when we can turn it by some degree about a point and the pattern looks exactly the same.



- REFLECTIVE SYMMETRY: In the same way, we say a pattern has reflective symmetry when we can fold it across a mirror line so that one half lies on top of the other.



- TRANSLATION SYMMETRY: Translation symmetry occurs only in patterns that cover a plane. If you can move a copy of a pattern a specific distance in a specific direction so that it lies exactly on top of the original pattern, then the pattern is said to have translational symmetry.
- GLIDE REFLECTIVE SYMMETRY: Glide Reflective symmetry, like translational symmetry, only occurs in patterns that cover a plane. When you move a pattern a given distance in a given direction and then reflect it over a mirror line so that it lies exactly on top of the original pattern, that pattern has glide reflective symmetry.
- SYMMETRY: Thus to summarize, when you take a copy of an image or a shape in a repeated pattern and move it in some way so that it lies exactly on top of the original, that figure is said to have symmetry. Symmetry means "the same" or "identical." The terms we defined above are examples of different types of symmetry on a flat surface (planar symmetry).

We call patterns that can be repeated to cover entire planes, "**wallpaper patterns.**"

LESSON 3: Graphics Activity - Tessellations Using ClarisWorks

This activity is appropriate for any secondary class as a culmination of a unit on polygons.

Before giving this out the students should know the following:

- rotational symmetry
- measure of angles of regular figures
- what a regular and semi-regular tessellation is
- translations
- how to work Claris works
- sum of angle measure around a point is 360 degrees
- you can only do a translation and have it tessellate if you do it to a figure

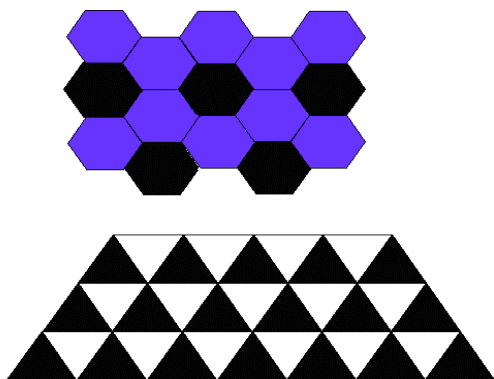
which has at least 2 parallel sides

Give this assignment to students on a worksheet (or possibly on a home page):

Please create the following tessellations:

- 2 regular tessellations
- 2 semi-regular tessellations
- 1 rotational tessellation
- 1 tessellation using a translation
- 1 1 fun tessellation of your choice

(Hint: On the regular and semi-regular tessellation make sure the sum of the interior angles of the figures at a vertex add up to 360 before you try to tessellate them. On the tessellation involving rotations and transformations make sure the figures you start with tessellate before you try adding anything to them.)



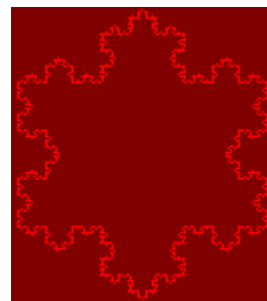
LESSON 4: The Koch Snowflake

You may need triangle grid paper to help you with the drawing.

Directions:

Step One. Start with a large equilateral triangle

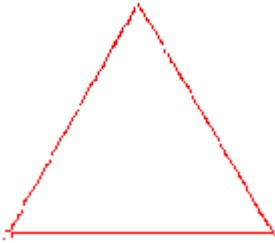
Step Two. Make a star:



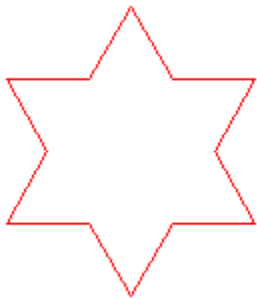
1. Divide one side of the triangle into three parts and remove the middle section.
2. Replace it with two line segments the same length as the section you removed.
3. Do this to all three sides of the triangle.

Want to see what this looks like?

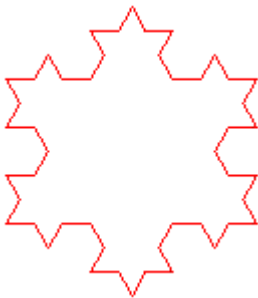
Original Triangle



First Iteration

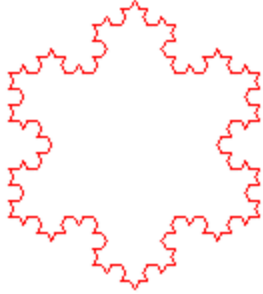


Second Iteration

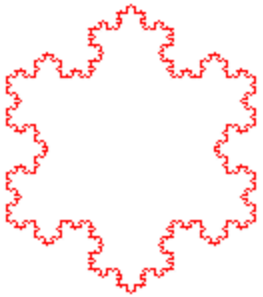


Wow! What did we do to get this figure?
Do you see why mathematicians call it a snowflake?
Did you say do it again?! You said do it again!!

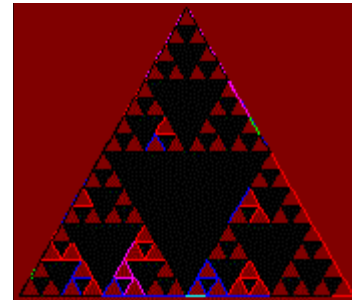
Third Iteration



Fourth Iteration



LESSON 5: The Sierpinski Triangle



Step One

Draw an equilateral triangle with sides of 2 triangle units each.
Connect the midpoints of each side.

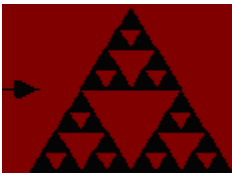
How many equilateral triangles do you now have?
Shade out the triangle in the center.



Step Two

Draw an equilateral triangle with sides of 4 triangle units each. Connect the midpoints of the sides and shade the triangle in the center as before.

Notice the three small triangles that also need to be shaded out in each of the three triangles on each corner.



Step Three

Draw an equilateral triangle with sides of 8 triangle units each. Follow the same procedure as before, making sure to follow the shading pattern. You will have 1 large, 3 medium, and 9 small triangles shaded.



Step Four

How about doing this one on a poster board? Follow the above pattern and complete the Sierpinski Triangle. Use your artistic creativity and shade the triangles in interesting color patterns. Does your figure look like this one? Then you are correct!

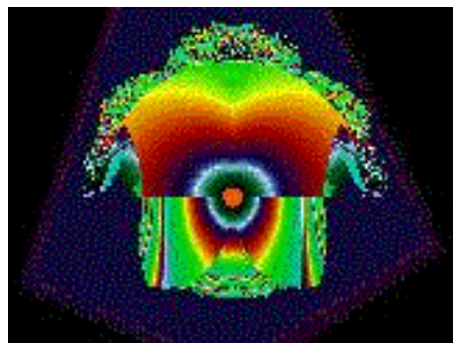
What is a fractal?

"A fractal is a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally self-similar and independent of scale." A rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole.

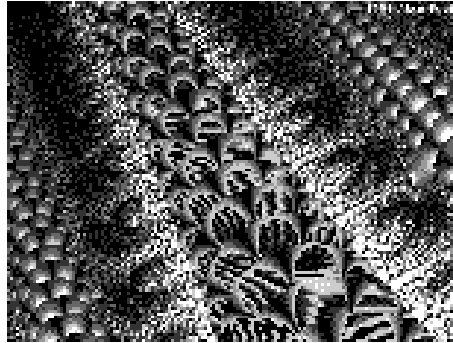
Fractals are geometric figures, just like rectangles, circles and squares, but fractals have special properties that those figures do not have. A fractal is any pattern that reveals greater complexity as it is enlarged. Fractals graphically portray the notion of 'worlds within worlds'.

The word "fractal" was coined less than twenty years ago by one of history's most creative mathematicians, Benoit Mandelbrot, whose seminal work, *The Fractal Geometry of Nature*, first introduced and explained concepts underlying this new vision. Mandelbrot derived the term "fractal" from the Latin verb *frangere*, meaning to break or fragment. Basically, a fractal is any pattern that reveals greater complexity as it is enlarged.

Lamp



Serpentes



Whether generated by computers or natural processes, all fractals are spun from what scientists call a "positive feedback loop." Something--data or matter--goes in one "end," undergoes a given, often very slight, modification and comes out the other. Fractals are produced when the output is fed back into the system as input again and again.

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