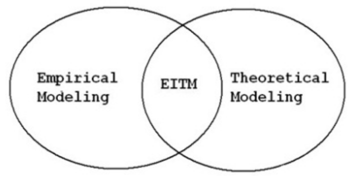
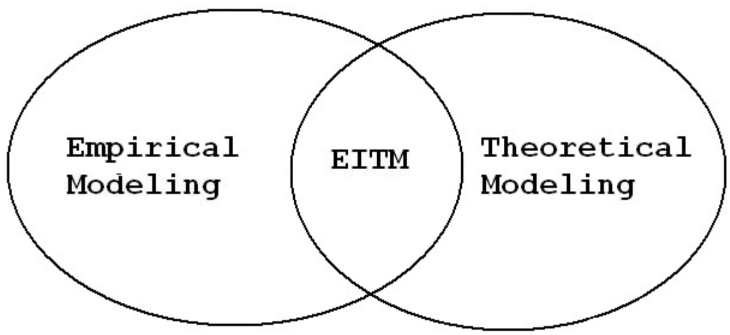
UNIVERSITY of HOUSTON

HOBBY CENTER FOR PUBLIC POLICY

## EITM Summer Training Institute



# The Empirical Implications of Theoretical Models: Unifying Formal and Empirical Analysis in the Political, Social, and Economic Sciences

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To John and Frank

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# Preface

On July 9 and 10, 2001, the Political Science Program of the National Science Foundation (NSF) convened a Workshop seeking ways for improving technical-analytical proficiency in Political Science.[1] This workshop, termed the Empirical Implications of Theoretical Models (hereafter EITM) Workshop, suggested constructive approaches the NSF Political Science Program could employ to foster linkages between formal and empirical modeling.[2]

The workshop acknowledged that a schism had developed between those who engage in formal modeling focusing on quantifying abstract concepts mathematically, and those employing empirical modeling emphasizing applied statistics.[3] As a consequence, a good deal of research in political science is competent in one technical area, but lacking in another. To put this another way, the impaired competency is

[1] The EITM Workshop was recorded and transcribed.
The written transcript is available on the NSF Political Science Program Web Site: www.nsf.gov/sbe/ses/polisci/reports/eitm709.pdf and www.nsf.gov/sbe/ses/polisci/reports/eitm710.pdf .
A report of the EITM initiative is also available at: www.nsf.gov/sbe/ses/polisci/reports/pdf/eitmreport.pdf.

[2] A quantitative model can be characterized as a construct that is represented by a set of logically — and in this case — quantitatively connected variables. Quantitative models include formal analysis (modeling) and applied statistical analysis. Formal analysis (i.e., social choice, linear difference equations, and differential equations) refers to deductive modeling that includes a theorem and proof presentation or computational modeling requiring simulation. Applied statistical analysis involves data analysis (from experimental or secondary sources) using statistical tools. We use the terms *analysis* and *modeling* interchangeably.
In addition, the linkage of formal and applied statistical analysis — the form of methodological unification described in this book — possesses important attributes that aid in falsification, predictive precision and, ultimately, scientific cumulation. Formal models, for example, force clarity about assumptions and concepts; they ensure logical consistency, and they describe the underlying mechanisms, typically behavioral, that lead to outcomes (Powell 1999: 23-39). The other component part of methodological unification — applied statistical models and tests — provide generalizations and rule out alternative explanations through multivariate analysis. Applied statistics assist in distinguishing between causes and effects, allow for reciprocal causation, and also help assess the relative size of the effects.

[3] The participants in the EITM Workshop were senior scholars with research experience in various technical-analytical areas and proven track records in activities that have improved the technical-analytical expertise in various sciences.

reflected by either a formal approach with substandard (or no) empirical tests or an empirical approach without formal clarity. This "siloed" research contributes to a failure to identify the proximate causes explicated in a theory and, in turn, increases the difficulty of achieving a meaningful increase in scientific knowledge.

EITM was also thought to combat a particular set of applied statistical practices that had developed to the point where many statistical procedures were not intended to fix inaccuracies in specification. Rather, they were used to improve the significance levels of estimated parameters — regardless of the structural failure of a model. An emphasis on the t-statistic displaced emphasis on whether a model had systematic (as opposed to random) error.

In this book we extend and address these initial Workshop concerns — and more.

## Purpose

This book provides a framework for demonstrating how to unify formal and empirical analysis not only for political science questions but for questions in the social sciences. By arguing for the scientific benefits of methodological unification, it is shown that the linkage between formal and empirical analysis assists in finding underlying causal mechanisms.[4] We hasten to add that methodological unification in the social and behavioral sciences is not new and can be traced primarily to the accomplishments of the Cowles Commission in the 1930s.[5]

With these potential scientific benefits in mind, EITM is also a response to some current methodological practices (See Achen 2002, 2005 for a review). These practices, for example, borrow applied statistical tools to improve upon older techniques, but this largely comes at the expense of the search for identified and invariant relations. Indeed, with this mindset, the creation of methodologies isolating structural parameters are secondary to the use of applied statistical techniques that end up manipulating standard errors and associated t-statistics. There is no use of formal modeling to aid in this process of identifying causal relations. Moreover, there is

[4] For an extensive and important discussion of this issue and many others relevant to EITM, see Morton (1999).

[5] Created in the 1930s, the Cowles Commission was designed "to foster the development of logical, mathematical, statistical methods of analysis for application in economics and related social sciences." (See http://cowles.econ.yale.edu/about-cf/about.htm). By the mid-1960s the Cowles Commission approach was a standard quantitative framework in economics. However, the approach came under criticism in the early 1970s. Not only were the quantitative models forecasting poorly, but the approach was failing to appropriately address the issues of identification and invariance. Chapter 1 discusses these two issues in greater detail.

little effort to seek unification between formal and empirical approaches.

A few political scientists did see the shortcomings associated with disjointed quantitative work. For example, Aldrich argued in 1980:

> Empirical observation, in the absence of a theoretical base, is at best descriptive. It tells one what happened, but not why it has the pattern one perceives. Theoretical analysis, in the absence of empirical testing, has a framework more noteworthy for its logical or mathematical elegance than for its utility in generating insights into the real world. The first exercise has been described as "data dredging," the second as building "elegant models of irrelevant universes." My purpose is to try to understand what I believe to be a problem of major importance. This understanding cannot be achieved merely by observation, nor can it be attained by the manipulation of abstract symbols. Real insight can be gained only by their combination (page 4).

More than a decade later Bartels and Brady (1993) echoed these sentiments arguing that, in the field of political methodology, "there is still far too much data analysis without formal theory – and far too much formal theory without data analysis" (page 148). In her important treatment on the subject, Morton (1999) discusses these issues in the following terms:

> Political Scientists have become adept at applying – from economics and other disciplines – exciting new statistical methods to analyze data. . . Yet this increase in complexity is not without costs. As the use of methodological techniques in political science has advanced, researchers have found that often their empirical study leads to more questions, questions that need theoretical input. However, because little existing theory is relevant or because the well-developed theory that does exist seems unconnected to the empirical issues, typically the response is to use more sophisticated methods or data gathering to answer the questions without reference to a fully developed theory. But these new methods often lead to still more questions, which in turn result in the use of more sophisticated methods to gather or analyze the data. The connection to theory seems to get lost in the methodological discussion. Rarely do researchers take the empirical results and rework the theoretical framework that began the discussion (page 3).

These concerns emerged in the NSF EITM Workshop held in 2001. The NSF EITM Report concluded the sources for the methodological status quo were deep and would be difficult to overcome. While the issues could extend to various social science disciplines, the report contrasted political science and economics. In the case of political science:

> There are at least two reasons for this state of research competency. One is that rigorous formal and empirical training is a somewhat recent development in political science. Another is that there are significant obstacles in the current political science training environment. The first obstacle is time. Students who desire training in both formal and empirical modeling will take longer to get a Ph.D. and most graduate programs do not have the resources to support students for more than four or five years. Consequently, students take the sequence of formal or empirical modeling classes but seldom both sequences. In addition to classes in formal or empirical modeling, students must take classes in their substantive area. For students in comparative politics there are field work and language requirements. What normally is sacrificed, then, is either the formal or empirical modeling sequence. Taking a single course in formal and empirical modeling is not nearly enough to develop competency to do research. The second obstacle to establishing formal and empirical modeling competency centers on the training itself.

Because of its longer and more extensive formal and empirical modeling traditions — due in part to the influence of the Cowles Commission — the EITM Report found a way to break the status quo in training could be found in economics.

> The economics discipline is illustrative. Economics graduate students are required to take one full year (usually) of mathematics for economists. This mathematical (and quantitative) approach is reinforced in substantive courses which typically are taught as an analytic science in a theorem-proof mode (page 7).

## Ideas

To change the current methodological emphasis — and to build a cumulative social science — an EITM framework is presented in this book. Several ideas provide a foundation for this theme. The first idea is the ultimate focus of a model and test is to support a cumulative scientific process geared toward finding a causal mechanism. The ability of a researcher to parse out specific causal linkages among the many factors is fundamental to the scientific enterprise. But, it also should be noted that no one engaged in the scientific enterprise would call finding causal mechanisms — easy. What makes methodological unification so useful in this process is that specifying a model linking both formal and empirical approaches alerts researchers to outcomes when specific conditions are in place.

To be clear, then, prediction and predictive accuracy are important aspects of the scientific inquiry and scientific cumulation, but they do not exist in a vacuum. Understanding the workings of a system (particulary through formalization), which can sometimes occur long before tests and data are available, is a coequal partner in the process. As Coase (1994) argues:

> The view that the worth of a theory is to be judged solely by the extent and accuracy of its predictions seems to me wrong. Of course, any theory has implications. It tells us that if something happens, something else will follow, and it is true that most of would not value the theory if we did not think these implications corresponded to happenings in the ... system. But, a theory is not like an airline or bus timetable. We are not interested simply in the accuracy of it predictions. A theory also serves as a base for thinking. It helps us to understand what is going on by enabling us to organize our thoughts. Faced with a choice between a theory which predicts well but gives us little insight into how the system works and one which gives us this insight but predicts badly, I would choose the latter...No doubt...that utlimately theory would enable us to make predictions about what would happen in the real world; but since these predictions would emerge at a later date...to assert that the choice between theories depends on their predictive powers becomes completely ambiguous (pages 16-17).

*In sum, the EITM framework is a means — via the dialogue it creates between theory and test — to attain a valid understanding of the workings of a system and assessing a theory's predictive accuracy.*

A second idea is that the methodological isolation of fields and sub-fields in political and social sciences is the status quo. Among the consequences of this isolation is the schism between formal and empirical modeling and the concomitant weaknesses in how social science researchers specify and test their models. A major objective of this book is to select examples from various subfields or disciplines for creating an awareness of EITM-type research and breaking down the barriers to achieving methodological unification.

The third idea follows the second where EITM collaborations in education, knowledge dissemination, and research result in promoting interdisciplinary interactions. The EITM framework presented here originates in political science, but it is based on the original work of the Cowles Commission — a group of quantitatively inclined economists.[6] The contributions of the Cowles Commission rest, in part, on a scientific vision involving the merging of formal and applied statistical analysis. The basis for this linkage is the notion that random samples are governed by a latent and probabilistic law of motion (Haavelmo 1944; Morgan 1990). Further, this view argues that formal models, related to an applied statistical model, could be interpreted as creating a sample drawn from the underlying law of motion. A well-grounded test of a theory could be accomplished then by relating a formal model to an applied statistical model and testing the applied statistical model.[7]

A fourth idea is that EITM extends the Cowles Commission approach.[8] As noted earlier, the Cowles Commission contributed to the rise of quantitative methodology in many ways. The Cowles methodology created new research aimed at determining valid inference by highlighting issues such as identification and invariance. For the first issue, identification, rules (i.e., rank and order conditions) were devised so that an equation of a model could reveal one and only one set of parameters consistent with both the model and the observations (See, for example, Koopmans 1949). A second issue involved the invariance of a (structural) relation. If an underlying mechanism is constant in the past and future, then the path of the relevant variable(s) will be predictable from the past, apart from random disturbances (See, for example, Marschak 1947, 1953). There was no concerted attempt assuring this latter condition obtained and this failure invited both theoretical and empirical criticisms.

These criticisms were fundamental and they figure prominently in social science

[6]Morgan (1990) provides an extensive historical account of the contributions of the Cowles Commission.

[7]Following Haavelmo (1944), this probability approach involved the definition of a precise stochastic model representing the phenomenon under study and the generation of the data. Inference was to be determined within the framework of a complete model, where the model is characterized by as "many relationships as variables to be explained" (Morgan 1990: 114). The application of the probability approach resulted in models (built mostly by economists) appearing as systems of equations with additive random terms. Estimation and testing was done in the context of these complete representations. A typical estimation procedure was ordinary least squares.

[8]Note the EITM framework also shares similarities with Sociologist Guillermina Jasso's (2004) *Tripartite Structure for Social Science Analysis.* See Chapter 12.

progress. Consider that if one were to strictly adhere to the Cowles Commission approach we would, for example, forego the chance of modeling new uncertainty created by shifts in behavioral traits (e.g., public tastes, attitudes, expectations, communication, and learning). The scientific consequence of this omission directly affects the issues of identification and invariance because these unaccounted behavioral shifts of variables would not be linked with the other variables and specified parameters. The EITM framework is devised to deal with these behavioral dynamic concerns. It also takes a more expansive view on the modeling enterprise. Not only does the EITM framework make use of structural equation modeling associated with the Cowles Commission, but it also includes alternative probability approaches, computational methods, and experimental methods (See Poteete, Janssen, and Ostrom 2010).

In sum, the EITM framework builds on the Cowles Commission approach. This framework takes advantage of the mutually reinforcing properties of formal and empirical analysis addressing the challenge(s) above. In addition, this framework focuses on general behavioral concepts integral to many fields of research but seldom are modeled and tested in a direct way. EITM emphasizes discovering ways to model human behavior and action and, thereby, aids in creating realistic representations improving upon simple socio-economic categorization. Numerous social science disciplines focus a good deal of research effort on the interactions between agent behavior and public policies. Yet, current research practices can fail to develop formal models analyzing these interactions. Our approach emphasizes modeling behavior so new uncertainty created by shifts in behavioral traits such as public tastes, attitudes, expectations, learning, and the like are properly studied.[9]

## Features

This book has several distinctive features. The first is a review of some current methodological practices and how they undermine cumulative scientific progress. We then discuss how EITM combats the overemphasis on the t-statistic and the non-falsifiable statistical practices it engenders.

A second feature emphasizes the analytical and technical approach. Formal analysis is merged directly with empirical analysis (*using data or possessing testable implications or both*). The EITM framework builds on the concepts and vision of the

[9]The framework developed also addresses the critique of the Cowles Commission approach leveled by Sims (1980) on devising ways to "credibly" identify a model's parameters (Sims 1980; Freeman, Lin, and Williams 1989). The present framework adds behavioral concepts and analogues, thereby holding the potential to strengthen the credibility for various identification restrictions.

Cowles Commission but it also relies on a significant background literature to serve as the basis for adopting the EITM framework.

A third feature focuses on mechanism operationalization. Operationalizing mechanisms involves creating measurable devices (what we term analogues) on the formal and behavioral side, but also on the empirical side. Behavioral concepts include (but is not limited to) expectations, learning, social interaction, decision making, strategic interaction, and more. Empirical concepts include (but are not limited to) persistence, measurement error, simultaneity, prediction, nominal choice, and more. Traditional operationalization involves finding measures for behavioral concepts. These measures are usually a variable of some sort. Operationalizing mechanisms encompasses this tradition, but also extends it to include the use of operators, frameworks (that can include variables), and tools to represent both behavioral and empirical concepts.[10]

A fourth feature is that the book is self-contained. The examples and tools are blended together so readers can learn how to develop the EITM approach. Each example presented fosters this with appendices added to provide greater detail on the tools used. Where possible the notation is in its original from the original sources. Our view is that demonstrating the differences in notation gives readers an opportunity to see and think of things in ways they otherwise would not have.

## What the Material in this Book Means for Your Research

Who benefits from mastering the material in this book? By its very nature, this book mixes formal analysis and applied statistical analysis. The book is designed first to account for the differences between formal and empirical approaches, and their respective intellectual outlook, skills, and training. In terms of outlook, formal modelers typically emphasize, in minute detail, linkages between concepts, whereas empirical modelers rarely spend their time parsing through minute details that may not add to their understanding. Formal modeling also requires analytical, logical, and mathematical modeling skills, while empirical modeling is inductive and, therefore, places emphasis on descriptive and statistical skills. Empirical modelers devote their energies to data collection, measurement, and statistical matters, but formal modelers focus on mathematical rigor.

[10]See Chapter 3 for more detail on this issue relating to the development of *analogues*.

These differences are eliminated because the tools are merged in accordance with the EITM framework. It is likely that a person using this book will have knowledge with one technical tradition (formal or empirical), but they will also be given the appropriate steps to solve parts less familiar. This is accomplished by not only presenting the various tools and solution procedures but in also featuring examples that may be applied to a variety of social science questions.

The book is presented at a technical level similar to econometric texts. The market it is geared toward are primarily academic and policy. People most comfortable with this material will be graduate students and academics. Policy analysts may strengthen their analyses by applying this approach. This book is also useful as a capstone course for students who have taken both formal and empirical courses.

As a final thought, and if we were to sum up the EITM framework, we would argue it is a guide to unification. But, there is no set formula for what tools to use. What dictates the tools to use is how you characterize your idea. At that point you will need to look in a tool box and use the specific formal and empirical techniques available and appropriate. Our appendices serve this latter purpose. They are certainly not the last word on what the mix is or should be. Competing characterizations are appropriate and should be evaluated on their ability to represent accurately the workings of the system as well as their predictive power.

## The Plan of the Book

The book has two parts. Part I provides the background and framework. Chapter 1 gives a broad overview of how methodology and modeling are integral to the scientific process. We discuss the fundamental scientific ideas — *order, cause, and chance* — and how models can be used to attain these ideas. The discussion also includes various academic institutions that gave rise to the development of formal and applied statistical modeling. These institutions are the precursors to methodological unification — and EITM.

Chapter 2 provides readers an analysis of some contemporary methodological practices. Some of these practices (data mining, overparameterization, and statistical patching ("omega matrices")) contribute to noncumulation. An example is provided on how these practices failure to provide a valid understanding of a system or predictive power.

The EITM framework is presented in Chapter 3. Chapter 3 begins with an example of methodological unification related to the Cowles Commission — the Solow model (1956). An analysis covering the shortcomings of these early forms of methodological unification is presented. The EITM framework then follows.

Part II of the book provides several examples from a variety of research areas in political science, political economy, economics, and sociology. Each chapter is self contained: after the example is presented and related to EITM, there is a discussion of how the example can be extended.[11] As mentioned earlier, each chapter also provides an appendix giving a thorough description of the formal and empirical tools used in the specific example.[12] The tool sections provide the basis for creating an analogue for each concept.

Chapter 4 focuses on economic voting. Using Alesina and Rosenthal's (1995) model it is demonstrated how the model can be extended (See Suzuki and Chappell 1996; Lin 1999). From an EITM perspective this involves the linkage of the behavioral concepts of expectations and uncertainty with the empirical concept of measurement error. The tools in this chapter include the theory of rational expectations (use of linear projections) solution procedures and the technique of signal extraction. The empirical tools include error-in-variables regression and the relation between signal extraction and error-in-variables regression.

Chapter 5 presents an example of macropartisanship and EITM. The EITM linkage cements the relation between the behavioral concept of expectations with the empirical concept of persistence. An example of party identification that provides a linkage between expectations and persistence is Clarke and Granato (2004). The appendix in this chapter includes an extended discussion expectations modeling, including the use of difference equations (various orders), their solution procedures, and stability conditions. Along with these relevant formal modeling tools is a comprehensive discussion of the empirical estimation and properties of autoregressive processes.

Chapter 6 presents a macro political economy example, relating policy to inflation. The macro-policy example used in this chapter is Granato and Wong's (2006) real wage contract model. As with Chapter 5, the EITM linkage offers a relation

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[11] The exception is Chapter 12. That chapter introduces an alternative framework for methodological unification with an example from sociology. The appendix explores an example from the economics discipline.

[12] The appendices can cumulate since there can be some overlap in the tools that are used for each chapter.

between the behavioral concept of expectations and the empirical concept of persistence. The appendix in this chapter provides an extension of the uses of difference equations and adaptive learning procedures.

In Chapter 7, the focus is on social-interaction and learning. The EITM linkage involves relating and unifying the behavioral concepts of expectations, learning, and social interaction with the empirical concept of simultaneity. The example used is Granato, Lo, and Wong (2011). The tools section in this chapter include discussions of techniques in modeling expectations and learning (Evans and Honkapohja 2001). The empirical tools include time series econometrics and multi-equation estimation.

Chapter 8 gives an example of the relation between political parties and political representation. One well researched area in the literature centers on when and why voters choose one party over the others based on the relative political positions of parties. The work of Kedar (2005) is used in this chapter. The EITM linkage in this example is between decision-theoretic models with discrete outcomes. The tools in this chapter include an introduction to discrete choice modeling and decision theory.

Voter turnout is the topic in Chapter 9. Here the EITM linkage is the behavioral concept of learning combined with the empirical concept of discrete choice. The example we use is Achen (2006). The tools in this chapter include the theory of Bayesian learning and discrete choice statistical models.

EITM approaches to international conflict and cooperation is the subject of Chapter 10. The EITM linkage includes the behavioral concept of bargaining and strategic interaction combined with the empirical concept of discrete choice. This linkage is captured in Quantal Response Equilibrium (QRE). QRE — which merges Game Theory and discrete choice models — was developed by McKelvey and Palfrey (1995, 1996, 1998) and was applied by Signorino (1999). In this chapter we use Leblang's (2003) application to currency crises. The tools introduced in this section involve the elements of Game Theory, discrete choice modeling, and how these inform QRE.

In Chapter 11 we show EITM is not simply about closed form solutions or secondary data sources. EITM also includes tools with numerial solution procedures and experiments. Agent based modeling is a case in point. Agent-based modeling (ABM) has been an important element of understanding complex social and economic systems. The EITM linkage is between elements of social interaction (e.g., imitation, invention, communication, and examination) and prediction. We use the work of Arifovic (1994) on learning dynamics and contrast her results with experimental results of Wellford (1989). The tools we introduce in this chapter involve the elements of genetic algorithm simulations and adaptive learning simulations.

An alternative unification framework is presented in Chapter 12. Jasso's (2004) *Tripartite Structure of Social Science Analysis* provides a framework that shares similarities to EITM but also provides a unique perspective. One important emphasis in Jasso's framework is her focus on probability distributions — and the linkage between formalization with known distribution functions. Moreover, Jasso's unification method extends to measurement issues. The example we use in this chapter focuses on a theoretically motivated index measure of justice and relates this index to gender gaps in earnings. The appendix in this chapter differs from others. We deal again with the linkage of behavioral theory and measurement, but this example is from economics. Specifically, we describe how monetary aggregates are based on decision theory, aggregation theory, and index number theory.

Chapter 13 concludes the book. An overview of the obstacles in implementation as well as how training can be reoriented is discussed. The chapter ends with a discussion of how future developments in methodological unification can assist both basic and applied research.

## Acknowledgment

Over the years many individuals have assisted us with this project. We fear in naming some we will unfairly omit others. Yet, two individuals stand out most because their involvement marks the time — from beginning to end. John Aldrich, encouraged a graduate student to think about methodological questions in different ways and also provided support for the added training necessary to do so: his early guidance set the foundation for EITM. And Frank Scioli, who saw the value in EITM, made numerous suggestions pertaining to its intellectual and scientific merit: he was the driving force in seeing its ultimate implementation. This book would not be possible had it not been for them.

# Part I

# EITM: Background and Framework

# Chapter 1

# Insights and Pathbreaking Institutions

EITM is a natural outgrowth of prior modeling and testing approaches — research methods — aimed at fostering social scientific cumulation.[1] The enduring effort — instituting the ideas — to create modeling and testing methods is of vital importance to the social sciences since it "provides a shared language so that even scholars thinking problems with little substantive overlap... can communicate efficiently and productively. It means that we begin with common first principles, and proceed with our research in a way that is commonly understood" (Gerber 2003: 3). Or, as Pearson (1957, 2004) states: "the unity of all science consists alone in its method, not in its material... It is not the facts themselves which make science, but the method by which they are dealt with" (page 12).[2]

With the attributes of shared (and improving) standards, language, and technical-analytical competence, research methods allow us to find ways to implement the fundamental scientific ideas of *order, cause, and chance* (Bronowski 1978):[3]

[1] This chapter borrows from sources such as: Gow (1985), Landreth and Colander (2002), Morgan (1990), Poteete, Janssen and Ostrom (2010: 3-27), Schumpeter (1954, 1994), Sowell (1974, 2006), Worcester (2001), and Zellner (1984).

[2] The American Heritage Dictionary definition of science is "The observation, identification, description, experimental investigation, and theoretical explanation of phenomena."

[3] *Order* is defined as "the selection of one set of appearances rather than another because it gives a better sense of the reality behind the appearances" (Bronowski 1978: 48). Order can require devices which depict relations and predictions. Abstract models and the use of mathematics are natural devices. *Cause* — determining what brings about an effect — was thought by early social scientists (or more accurately political economists) to be a sequential process (See Hoover (2001a: 1-28) for a review of David Hume's influence). For an extensive treatment see Pearl (2000), but see also Kellstedt and Whitten (2009: 45-66) and Zellner (1984: 35-74) for a discussion of causality in applied statistics and econometrics respectively. As for the concept of *chance,* Bronowski (1978) is critical of what he believes is the misuse of the term "cause" and prefers to link it with probabilistic

## 1.1 The Utility of Models and Modeling[4]

Order, cause, and chance can be effectuated by the use of models describing hypothetical worlds whose predictions have testable potential and assist in the systematization of knowledge.[5] With models one may "put all these effects together, and, from what they are separately…collect what would be the effect of all these causes acting at once" (Sowell 1974: 137-138). As Gabaiz and Laibson (2008) argue:

> Models that make quantitatively precise predictions are the norm in other sciences. Models with predictive precision are easy to empirically test, and when such models are approximately empirically accurate, they are likely to be useful (page 299).[6]

If models possess attributes which enhance the scientific process, then how do we go about constructing them? Valid models make use of both deductive and inductive inference. Deductive inference, where "the conclusion is obtained by deducing it from other statements, called premises of the argument. The argument is so constructed that if the premises are true the conclusion must also be true" (Reichenbach 1951: 37).

Inductive inference — because it relies on making inferences from the past to predict the future:

> …enables us to associate probabilities — cause and chance — with propositions and to manipulate them in a consistent, logical way to take account of new information. Deductive statements of proof and disproof are then viewed as limiting cases of inductive logic wherein probabilities approach one or zero, respectively (Zellner 1984: 5).

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statements — "chance" — which "replaces the concept of the inevitable effect by that of the probable trend" (page 87). Modern conceptions on the utility of mathematics also point to how applied statistical analysis aids in the idea of chance (e.g., statistical significance).

[4]There are numerous discussions on the utility of formal analysis and modeling. In political science see Wagner (2001) and Wagner (2007: 1-52) for a review of the ongoing debate over modeling and an application to theories of international relations. See Clarke and Primo (2012), Krugman (1994, 1998), Wolpin (2013), and Jasso (2002) for discussions in the fields of political science, economics, and sociology, respectively.

[5]See Granato (2005) for a discussion of these issues.

[6]The EITM framework can be applied to either observational designs or more controlled settings (See Freedman, Pisani, and Purves (1998: 3-28) and Shively (2010)). For a review of some of the more important developments on research design issues (e.g., counterfactuals) and causality see Morgan and Winship (2007) and Brady (2008). An example of multiple designs and multiple methods can be found in Poteete, Janssen, and Ostrom (2010).

Abstract modeling in the social sciences traces its origins to the early political economists.[7] The initial modeling efforts were deductive in orientation. Mathematics and mathematical models were argued as an attribute for determining order and cause because their logical consistency can be verified using the available operations of mathematics. David Ricardo was one of the first to make use of "abstract models, rigid and artificial definitions, syllogistic reasoning," and applied the conclusions from the highly restrictive models directly to the complexities of the real world (See Sowell 1974: 113 and Landreth and Colander 2002: 113-115).

These early modeling efforts were not without detractors.[8] Richard Jones, for example, argued modeling generalizations were invalid if they ignored things that exist in the world including institutions, history, and statistics. Robert Malthus, Jean Baptiste Say, and J. C. L. Sismonde also criticized attempts at premature generalization (Sowell 1974: 114-116).

These critics gave no consideration that mathematics might be used to contribute to conceptual clarity rather than to derive numerical predictions. Antoine Augustin Cournot pointed out that mathematical analysis was used "not simply to calculate numbers" but to find "relations" (Sowell 1974: 117-118). The criticisms endured. More than a century later Kenneth Arrow (1948) provided the following defense of mathematical modeling:

> It is true that there are certain limitations of mathematical methods in the social sciences. However, it must be insisted that the advantages are equally apparent and may frequently be worth a certain loss of realism. In the first place, clarity of thought is still a pearl of great price. In particular, the multiplicity of values of verbal symbols may be a great disadvantage when it comes to drawing the logical consequences of a proposition (page 131).[9]

In the early 1920s, inductive inference — and linking cause to chance and providing a basis for regression analysis and econometrics — was given important support when the sampling distribution(s) for regression coefficents were established (Fisher

[7]See Sowell (1974) and Landreth and Colander (2002), but for subsequent changes in the use of mathematics see Weintraub (2002).

[8]In the 17th and 18th centuries a break occurred between those who believed political economy should base its method on rigor and precision versus those who emphasized the certainty of the results. The debate focused in part on whether political economic principles should be founded on abstract assumptions or factual premises (Sowell 1974: 117-118).

[9]There are numerous examples where the use of mathematical models uncover logical inconsistencies that would be more difficult to find using verbal argument(s). Viner (1958), for instance, discusses how the aid of mathematics lead to clarification on the uses of average and marginal cost.

1922). This latter contribution was an important precursor to what has been called the "probability approach" to statistical inference (Haavelmo 1944) — and efforts to link deductive and inductive approaches using formal analysis and applied statistical tools.

## 1.2 Institutional Developments

While EITM draws inspiration from the Cowles Commission, it would be a mistake to limit it only to Cowles. The EITM framework builds on a variety of formal institutions and organizations in the social sciences. These institutions — ranging from research organizations to university departments — developed and supported the antecedents of EITM.[10] The entities that supported the creation and development of formal and applied statistical analysis include (but are not limited to):

- The Social Science Research Council (SSRC)
- The Econometric Society and the Cowles Commission
- The Political Science Department at the University of Rochester
- The Political Methodology Society.

### 1.2.1 The SSRC

The 1920s saw movement in the social sciences toward improving quantitative methods of study.[11] One leading figure was Charles Merriam who worked to alter the methods of political study (Merriam 1921, 1923, 1924; Merriam et. al., 1923). At that time he believed the existing methods of analysis failed on a fundamental level — identifying underlying mechanisms:

> The difficulty of isolating political phenomena sufficiently to determine precisely the causal relations between them. We know that events occur, but we find so many alternate causes that we are not always able to indicate a specific cause. For the same reason we are unable to reach an expert agreement upon the proper or scientific policy to pursue and by the same logic we are unable to predict the course of events in future situations (Merriam 1923: 288).

[10] An expanded sample can be found in Mitchell (1930, 1937: 58-71).

[11] Early political science examples can be found in Mayo-Smith (1890), Ogburn and Peterson (1916), and Ogburn and Goltra (1919).

Merriam stressed the need to examine and use multiple methods from numerous social science disciplines (i.e., economics, statistics, history, anthropology, geography, psychology).

> In place of literature Merriam favored a better organized and more consciously scientific and social psychological approach to the study of human behavior. Statistics and other empirical tools would play a critical role in shifting the political and social sciences closer to the "hard sciences" (Worcester 2001: 16-17).

To accomplish these goals, Merriam proposed an interdisciplinary institution to help promote his vision — the SSRC. In the 1920s, the SSRC was considered the first national organization of all the social sciences, and from the outset its goal has been to improve the quality of, and infrastructure for, research in the social sciences.[12] Among the contributions of the SSRC has been its multidisciplinary outlook and emphasis on creating and using data.

### 1.2.2 The Econometric Society and the Cowles Commission

The creation of the SSRC was followed by two other significant institutional developments.

> The Econometric Society was established in 1930...The Society greatly facilitated academic exchanges between European and American scholars not only in the young econometrics profession but also in mathematical statistics. It thus rapidly promoted the growth of econometrics into a separate discipline (Duo 1993: 5).

The Econometric Society sought to use mathematics and statistics to increase the level of rigor in the formulation and testing of economic theory. The society initially featured scholarly meetings and the creation of the journal *Econometrica*.

The Cowles Commission followed in 1932. It was a:

> research institution which contributed uniquely to the formalization of econometrics (See Christ 1952; Hildreth 1986). The Commission had a close connection with the Econometric Society from its beginning (Duo 1993: 5).

[12] Another important research institution, the National Bureau of Economic Research (NBER), was founded in 1920. The NBER had a narrower disciplinary focus (economics), but in many ways shared the same basic vision as the SSRC.

The Cowles Commission advanced the rise and adoption of econometric methodology in two ways.[13] Recall that it developed the *probability approach*. This approach highlighted the issues of *identification and invariance*.[14] Identification was central since a goal of econometrics is to determine the true values of the parameters among all the possible sets of values consistent with the data and with the known or assumed properties of the model.[15]

The second issue was the invariance of a relation. If structure is known to remain in the future what it was in the past, and if the auxiliary variables have constant values through both periods, the path of each variable will be predictable from the past, apart from random disturbances. By addressing the issues of identification and invariance, the probability approach — and the linkage of formal and empirical analysis — provides a connection to falsifiability, predictive precision, and the workings of a system.[16] We would add that models that have these properties also facilitate

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[13]Econometric research associated with the Cowles Commission includes (but is not limited to): Cooper (1948), Haavelmo (1943, 1944), Hood and Koopmans (1953), Klein (1947), Koopmans (1945, 1949, 1950), Koopmans and Reiersol (1950), Marschak (1947, 1953), and Vining (1949). For further background on the Cowles Commission consult the following URL: http://cowles.econ.yale.edu/.

[14]The intuition behind the terms *identify* (i.e., identification) and *invariant* (i.e., invariance) are as follows. For applied statistical models *identification* relates to model parameters (e.g., $\widehat{\beta}$) and whether they indicate the magnitude of the effect for that particular independent variable. Or, in more technical terms, "A parameter is identifiable if different values for the parameter produce different distributions for some observable aspect of the data" (Brady and Collier 2004: 290).

In applied statistical practice, *invariance* refers to the constancy of the parameters of interest. More generally, "the distinctive features of causal models is that each variable is determined by a set of other variables through a relationship (called "mechanism") that remains invariant (constant) when those other variables are subjected to external influences. Only by virtue of its invariance do causal models allow us to predict the effect of changes and interventions..." (Pearl 2000: 63).

[15]An equation of a model is declared to be identifiable in that model if, given a sufficient (possibly infinite) number of observations of the variables, it would be possible to find one and only one set of parameters for it that are consistent with both the model and the observations.

[16]Gabaix and Laibson (2008) argue that falsifiability and predictive precision are among the key properties of useful models (See Gabaix and Laibson 2008). "A model is falsifiable if and only if the model makes nontrivial predictions that can in principle be empirically falsified" (page 295). "Models have *predictive precision* when they make precise — or "strong" — predictions. Strong predictions are desirable because they facilitate model evaluation and model testing. When an incorrect model makes strong predictions, it is easy to empirically falsify the model, even when the researcher has access only to a small amount of data. A model with predictive precision also has greater potential to be practically useful if it survives empirical testing. Models with predictive precision are useful tools for decision makers who are trying to forecast future events or the consequences of new policies" (page 295).

In the language of econometrics, falsification and predictive precision require the mechanisms relating cause and effect be identified. There is a large literature devoted to identification problems (See, for example, Koopmans 1949, Fisher 1966, and Manski 1995), but we use identification in the broadest sense for purposes of attaining some order and underlying cause as well. Since we as social scientists do not have controlled environments to conduct or inquiry, our efforts to achieve order and cause in our models can only come about probabilisitically — by chance.

comparison between rival and competing theories over the same phenomena — and can enhance scientific cumulation (Kuhn 1979).

The Cowles approach also drew criticism. These criticisms for the most part focused on measurement and inferences issues (Keynes 1940) and questions about predictive accuracy (Christ 1951). Despite the criticisms, the Cowles Commission approach was widely adopted and by the mid-1960s was standard in quantitative economics. However, during the 1970s more fundamental criticisms — regarding invariance and identification — arose.

In 1976, Robert Lucas questioned the fragility of invariance when the Cowles approach is used. His formal analysis demonstrated that models based on the Cowles approach were fundamentally flawed in their ability to evaluate the outcomes of alternative economic policies. The reason, he argued, is that in-sample estimation provides little guidance in predicting the effects of policy changes because the parameters of the applied statistical models are unlikely to remain stable under alternative stimuli.[17]

Sims (1980) later challenged the identification procedures inspired by the Cowles Commission. Sims argued against the reliance on "incredible restrictions" to identify structural models. These restrictions had the effect of undermining an understanding of the system. Sims offered a change in emphasis from focusing on individual coefficients, as the structural modeling approach did, to VAR modeling with attention given on the dynamic time series properties of an unrestricted (by theory) system of equations.[18]

Despite these challenges, some of the basic tools and procedures of the probability approach remain. One extension of the structural approach, in part a response to Lucas's criticisms, is "real business cycle modeling" (RBC).[19] Here the focus is on isolating parameters and on making greater explicit use of theory at both the individual and aggregate level of analysis. Where RBC's especially differ from the Cowles Commission is in the use of standard statistical significance testing.[20]

[17] The Lucas *critique* is based on the following intuition: "...given that the structure of an econometric model consists of optimal decision rules ... and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models" (Lucas 1976: 41).

[18] Sims' methodology is grounded in probabilistic inference, imposing only enough economic theory to identify the statistical models and carry out analyses of policy effectiveness. See Freeman, Lin, and Williams (1989) for an application of VAR to political science questions.

[19] See Freeman and Houser (1998) for an application in political economy. For a critique of RBCs see Sims (1996).

[20] The method involves computational experiments. These experiments rely on a sequence of steps including: deriving the equilibrium laws of motion for the model economy from "well-tested

### 1.2.3 The Political Science Department at the University of Rochester

Thanks in part to the SSRC, there was a clear tendency in political science to promote statistical methods.[21] Methodological emphasis was placed on statistical correlation and empirical testing and generally focused on psychological attitudes to derive empirical generalizations. During the late 1950s and early 1960s William Riker and later — the Department of Political Science at the University of Rochester — developed positive political theory.[22]

> The goal of positive political theorists is to make positive statements about political phenomena, or descriptive generalizations that can be subjected to empirical verification. This commitment to scientifically explaining political processes involves the use of formal language, including set theory, mathematical models, statistical analysis, game theory, and decision theory, as well as historical narrative and experiments (Amadae and Bueno de Mesquita 1999: 270).

Riker, while not averse to inductive reasoning, put an emphasis on the deductive approach. In particular,

> The Rochester school has emphasized deriving hypotheses from axioms. Doing so reduces the risk that hypotheses are restatements of already observed patterns in the data. Even when models are constructed specifically to account for known empirical regularities, they are likely to produce new propositions that have not previously been tested. These new

theory," "calibrating" the model using parameter values derived from historical data, generating simulated realizations of the equilibrium processes, determining the sampling distributions of the statistics computed from the simulated data, and comparing these statistics to those computed for data from actual economies. Kydland and Prescott's (1982) "computational experiments" are often referred to as "calibration" because of the use of parameter values derived from simple measures (such as averages) of historical time series to "calibrate" the theoretical models.

[21]See Von Neumann and Morgenstern (1944), Black (1948), Arrow (1951) and Downs (1957) for the start of formal approaches to the study of politics by non-political scientists.

[22]A parallel development, in the early 1960s, was the creation of the Public Choice Society (See http://www.pubchoicesoc.org/about_pc.php). This society's statement of purpose (http://www.pubchoicesoc.org/about_pubchoice.php) is to:

> ...facilitate the exchange of work, and ideas, at the intersection between economics, political science, and sociology. It started when scholars from all three of these groups became interested in the application of essentially economic methods to problems normally dealt with by political theorists. It has retained strong traces of economic methodology, but new and fruitful techniques have been developed that are not clearly identified with any self-contained discipline.

> propositions, of course, create demand tests of the theory. Historical and statistical analyses tend not to hold the relations among variables constant from study to study and so are less likely to test inductively derived hypotheses against independent sources of evidence (Amadae and Bueno de Mesquita 1999: 289).[23]

The advent of positive political theory (and later game theory) provided another social science discipline — political science — the basis for a graduate training and research regimen that continues to grow to this day.

### 1.2.4 The Political Methodology Society[24]

In the early 1970s, political scientists began a process of enhancing the usage of applied statistical procedures. John Sullivan, George Marcus, and Gerald Dorfman created the journal *Political Methodology* (Lewis-Beck 2008). The journal was announced in 1972 and by 1974 the first issue was published. This journal was followed by the creation of the Political Methodology Society in the early 1980s. Like the Econometric Society, the Political Methodology Society developed annual meetings and a journal — *Political Analysis* — which succeeded the earlier *Political Methodology.*

The methodological improvements in political science have accelerated since the early 1970s and 1980s. Brady and Bartels (1993) note political science has now started a series of rigorous literatures in topics ranging from parameter variation and non-random measurement error to dimensional models. A summary of the increasing breadth of this society can be found in Box-Steffensmeier, Brady, and Collier (2008).

## 1.3 Summary

The chapter briefly describes ideas and institutions that provide both the direction and a basis for EITM. These predecessors, while they have some similarities, also have distinct identities ranging from data analysis, multidisciplinarity, applied statistical analysis, formal analysis, and the linkage of the latter two. EITM builds on this foundation and in addressing the scientific ideas — order, cause, and chance — brings "deduction and induction, hypothesis generation and hypothesis testing close together" (Aldrich, Alt, and Lupia 2008: 840).

[23] See Amadae and Bueno de Mesquita (1999) for a list of publications associated, in part, with the Rochester School.

[24] We thank Christopher Achen and Elinor Ostrom for the background information.

# Chapter 2

# Contemporary Methodological Practices

While there has been recent improvement in quantitative social science, there is also cause for concern that social scientists are not absorbing the scientific lessons and emphasis of prior social scientists. The fear is these past contributions have been marginalized and the situation today is one in which so-called technical work is only loosely connected to the fundamental scientific ideas.

In more technical language, the creation of methodologies that isolated structural parameters — to identify these parameters — became secondary to the use of hand-me-down applied statistical techniques that end up manipulating standard errors and their associated t-statistics. The reliance on statistically significant results means virtually nothing when the researcher makes very little attempt to identify the precise origin of the parameters in question. Absent this identification effort, it is not evident where the model is wrong. The dialogue between theory (model) and test is weakened.

We have now reached the point where some contemporary methodological practices contribute to noncumulation. Borrowing and applying statistical tools did seem to improve upon the use of older techniques. But, as this process of replacing old with new techniques took place, the search for causal mechanisms was largely ignored. These practices have not gone unnoticed. Achen (2002) has argued that:

> Dozens of estimators might be used in any of our empirical applications. Too often, applied researchers choose the standard ones because they believe methodologists approve of them, whereas methodologists prefer some new, complicated untested alternative because they know that the

> standard estimators are often ungrounded in substantive theory, and they hope that the new one might stumble onto something better. Few researchers in either group make a convincing case that their estimator is humming rather than clanking on their dataset. Even the creators of estimators usually do not prove that the supporting assumptions would make rational sense or common sense for the political actors being studied. Nor do they carry out the patient data analysis required to show that their estimator, an arbitrary selection from among dozens that might have been proposed, is more than just computable and plausible, but that its assumptions really match up in detail to the data for which it is intended. If the thing might work on some planet, we think our job is done (page 436).

## 2.1 Non-Cumulative Research Practices

Three common practices — *data mining, overparameterization, and the use of statistical weighting and patching (e.g., "Omega Matrices")* — impair scientific cumulation (Granato, Lo, and Wong 2010a; Granato and Scioli 2004). To demonstrate the consequences of separating theory and test consider how these practices affect a widely used test indicator, the $t$-statistic — which is defined as the ratio of an estimated coefficient ($b$) to its standard error ($s.e.(b)$), that is, $\frac{b}{s.e.(b)}$. In the case of a $t$-statistic, this means linking the formal model to the test and focusing on the identification of the parameter $b$.

Below a brief description is presented of these three common practices whether they bear any relation to identifying $b$ and by extension falsifiability, predictive precision, and understanding the inner workings of a system:

**Data Mining.** Data mining involves putting data into a standard statistical package with minimal theory. Regressions (likelihoods) are then estimated until either statistically significant coefficients or coefficients the researcher uses in their "theory" are found. This *step-wise* search is not random and has little relation to identifying causal mechanisms (See Lovell 1983; Denton 1985).

An example of the consequences of date mining is found in Friedman and Schwartz (1991).[1] The case Friedman describes occurred while he was working for the Columbia

[1] This example was originally drawn from Granato and Scioli (2004: 317-318).

University's Statistical Research Group during World War II. Friedman "was to serve as a statistical consultant to a number of projects to develop an improved alloy for use in airplane turbo-chargers and as a lining for jet engines" (page 48). Friedman's task was to determine the amount of time it took for a blade made of an alloy to fracture.

Friedman relied on data from a variety of lab experiments to assist him in addressing this problem. He then used the data to estimate a single equation linear regression. Standard statistical indicators suggested his approach was valid. The analysis predicted that the blade would rupture in "several hundred hours." However, the results of actual laboratory tests indicated that a rupture occurred in "something like 1-4 hours" (page 49). Because of the lab results — and not the linear regression or the data mining — the alloy was discarded. Since Friedman relied primarily on data mining he could not know the various stresses or conditions in which ruptures would occur. He concluded:

> Ever since, I have been extremely skeptical of relying on projections from a multiple regression, however well it performs on the body of data from which it is derived; and the more complex the regression, the more skeptical I am. In the course of decades, that skepticism has been justified time and again. In my view, regression analysis is a good tool for deriving hypotheses. But any hypothesis must be treated with data or nonquantitative evidence other than that used in deriving the regression or available when the regression is derived. Low standard errors of estimate, high $t$ values, and the like are often tributes to the ingenuity and tenacity of the statistician rather than reliable evidence of the ability of the regression to predict data not used in constructing it (page 49).

**Overparameterization.** This practice, related to data mining, involves a researcher including, *absent any systematic specification search*, a plethora of independent variables into a statistical package and obtains significant $t$-statistics. Efforts to identify an underlying causal mechanism are also ineffectual.[2] As Achen (2005) notes:

> ...big, mushy linear regression and probit equations seem to need a great many control variables precisely because they are jamming together all

[2] In the absence of formal models, purely applied statistical procedures relying on systematic rules in a specification search can be cumulative. See Clarke et al. (2004: 79-129) for an example of building *composite* empirical models. Composite models can be crucial in the process of methodological unification.

> sorts of observations that do not belong together. Countries, wars, racial categories, religious preferences, education levels, and other variables that change people's coefficients are "controlled" with dummy variables that are completely inadequate to modeling their effects. The result is a long list of independent variables, a jumbled bag of nearly unrelated observations, and often a hopelessly bad specification with meaningless (but statistically significant with several asterisks!) results (page 227).[3]

**Omega Matrices.** Data mining and overparameterized approaches are virtually guaranteed to break down statistically. The question is what to do when these failures occur (e.g., Friedman and Schwartz 1991). There are elaborate ways of using (error) weighting techniques *to correct* model misspecifications or to use other statistical patches that influence $s.e.(b)$. Many intermediate econometric textbooks contain chapters containing the Greek symbol: Omega ($\Omega$) (e.g., Johnston and DiNardo 1997: 162-164). This symbol is representative of the procedure whereby a researcher weights the arrayed (in matrix form) data so that the statistical errors, ultimately the standard error noted above, is altered and the $t$-statistic is manipulated.

By way of example (using ordinary least squares (OLS)), consider the following model in scalar form (we drop the constant for simplicity):

$$y_t = \beta x_t + \eta_t,$$

and assume there is first-order serial correlation:

$$\eta_t = \rho\eta_{t-1} + \nu_t,$$

where $\nu_t$ is a white noise process. With this estimate of $\rho$ a researcher "removes" the serial correlation:

$$y_t - \rho y_{t-1} = \beta\left(x_t - \rho x_{t-1}\right) + \nu_t.$$

Alternatively, in matrix form, we express this transformation as:

$$\beta^{GLS} = \left(X'\Omega^{-1}X\right) X'\Omega^{-1}Y,$$

[3] Gabaix and Laibson (2008) labels this practice *overfitting* which "occurs when a model works very well in a given situation but fails to make accurate out-of-sample predictions" (page 293). When the "researcher can combine the myriad elements to match almost any given set of facts...[it becomes] easy to explain in-sample data, producing the false impression that the model will have real (out-of-sample) explanatory power" (page 294).

as opposed to the OLS estimator:

$$\beta^{OLS} = (X'X)^{-1} X'Y.$$

The difference is the $\Omega$ matrix which is represented as:

$$\Omega = \begin{bmatrix} 1 & \rho & \rho^2 & ... \\ \rho & 1 & ... & ... \\ \rho^2 & ... & 1 & ... \\ ... & ... & ... & 1 \end{bmatrix}$$

and taking the inverse with the "$\rho$" correction:

$$\Omega^{-1} = \begin{bmatrix} 1 & -\rho & ... & ... \\ -\rho & 1+\rho^2 & ... & ... \\ ... & ... & 1+\rho^2 & ... \\ ... & ... & ... & ... \end{bmatrix} \frac{1}{1-\rho^2}.$$

In principle, there is nothing wrong with knowing the Omega matrix for a particular statistical model. The standard error(s) produced by an Omega matrix can serve as a check on whether inferences have been confounded to such an extent that a Type I or Type II error has been committed. Far too often, however, researchers treat the Omega weights (or alternative statistical patches) as the result of a true model. Given that the parameter of "$\rho$" is based on estimating the error of the regression, researchers that decide to use this weight are taking the mistakes of their model to "fix" their standard errors. It is akin to painting over a crack on a bridge (Hendry 1995). This activity hampers scientific progress because it uses a model's mistakes to obscure flaws.[4]

Similarly, when using a $\rho$-restriction, researchers are imposing a restriction on variables in the estimation system. For time series data, researchers assume all variables in the system *have the same* level of persistence. Matters are worse when data are time series and cross section. The restriction becomes even more severe as researchers assume all the cases *and* all the independent variables have the same level of persistence.

A final scientific problem with $\rho$-restrictions is that they fundamentally alter the original model so there is no longer any relation between the theory and the test.

[4] See Leamer (2010) for a more recent critique.

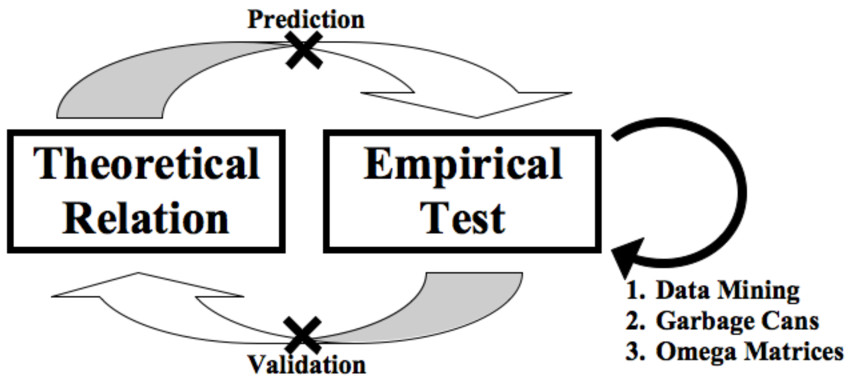


Figure 2.1.1: Noncumulative Practices

A theory assuming, for example, a certain level of persistence (i.e., regularity) in behavior (e.g., party identification) is altered by tools used to "filter out" persistence (Mizon 1995).

### 2.1.1 Assessment

To summarize, these current practices can also be evaluated in relation to how they fail to contribute to a modeling dialogue between theory and test. What we see in Figure 2.1.1 is that the process of theoretical development (understanding the workings of the system), prediction, and validation are never directly applied. Instead, the empirical test(s) remains in a loop or dialogue with itself. An iterative process of data mining, overparameterization, and the use of statistical patches (Omega matrices) *replaces* prediction, validation (falsification), and an understanding of the process.[5] This consequence inevitably follows because current practice does not attempt to identify model parameters ($b$'$s$) with the more general effect to impair scientific progress.[6]

One lesson from this admittedly simplified depiction in Figure 2.1.1 is applied statistical practices, when used in isolation, lack power since they are not linked to

[5]Note when we use the word validation this is not simply a process of a single theory being tested by the accuracy of its predictions but also in comparison to competing theories on the same phenomena. As Kuhn (1979) states:

> Anomalous observations...cannot tempt [a scientist] to abandon his theory until another one is suggested to replace it...In scientific practice the real confirmation questions always involve the comparison of two theories with each other and with the world, not the comparison of a single theory with the world (page 211).

[6]While this example uses linear procedures, non-linear procedures are subject to many of the same weaknesses. Achen (2002), for example, raises questions about the forms of statistical patching in various likelihood procedures and how these practices obscure identification.

a formal model.[7] Of course, formal models are simplifications of what is studied. Nevertheless, they systematically sort rival arguments and confounding factors. If formalized predictions, and the underlying system interactions, *are inconsistent* with empirical tests, then the theory — as represented in the formal model — needs adjustment and the dialogue is maintained.[8]

Even scholars who are sensitive to establishing robustness in their applied statistical results find the tools available are inadequate when used without a formal counterpart. For example, augmenting applied statistical tests with Extreme Bounds Analysis (EBA) (Leamer 1983) provides a check on parameter stability, but the test is ex-post and does not allow for ex-ante prediction.

This should come as no surprise when one considers the effects of previously unspecified covariates in this procedure. Each time an applied statistical model is respecified the entire model is subject to change. All predictions are fragile in that sense, but without a priori use of equilibrium conditions (e.g., stability conditions) in a formal model, the parameter changes in a procedure such as EBA are of unknown origin.[9]

## 2.2 Noncumulative Practices: An Example of What Can Go Wrong

The problem with noncumulative practice is demonstrated in the following macro political economy illustration. We employ a structural model to show a relation between parameters and predictions.[10] This is contrary to current practice which

[7]This criticism extends to progressive applied statistical research strategies (See Hendry 1995). Despite their rigor, specification searches relying on diagnostics, goodness-of-fit metrics, and comparisons to rival models fail to account for ex-ante changes in parameters that a formal model can provide. These applied statistical approaches succeed in improving in-sample accuracy, but lack power out-of-sample particularly where behavioral responses to policy interventions or various shocks occur. Ultimately, the most powerful tests of formal models reside in predictions for other cases and over earlier or future periods.

[8]Experiments serve as empirical tests too. See Ostrom (2010: 71) for a discussion on how empirical results in experiments contributed to the development of an alternative preference function.

[9]Zellner (1984: 9-10) provides other robustness tests, some of which involve the linkage of formal and empirical analysis. These tests include: a) Studying incorrect predictions; b) Studying implications of various equations (alter them); c) Simulating a model's properties; d) Pushing theories to their extreme; e) Observing unusual historical periods; f) Cross level inference; and g) Experiments.

[10]Classic work in macro political economy by political scientists does not generally rely on structural models (e.g., Hibbs 1977), but see Chappell and Keech (1983), Alesina and Rosenthal (1995), and Freeman and Houser (1998) as exceptions. For a more general overview of macro political economy see Drazen (2000) and Persson and Tabellini (2000).

typically ignores structural equation systems (and reduced forms), let alone identification conditions. Then, via simulation, we show how the weaknesses of current practices leads to serious scientific consequences such as incorrect inference and misleading policy advice.

Now consider the relation between a particular macroeconomic policy and outcome: countercyclical monetary policy and inflation. To keep things simple we have left out various political and social influences on the policy rule. While this research area holds great potential for explaining *why* policymakers behave certain ways, it does not affect the methodological point.[11]

The model incorporates a lagged expectations augmented Phillips curve, an IS curve (aggregate demand), and an interest rate policy rule (Taylor 1993). Each of these structural equations is a *behavioral relation* and can be derived from microfoundations.[12] In this example, however, we will also abstract out the microfoundations since they are not central to the demonstration.

The model contains the following structure:

$$y_t = y_t^n + \gamma\left(\pi_t - E_{t-1}\pi_t\right) + u_{1t}, \qquad \gamma > 0, \tag{2.2.1}$$

$$y_t = \lambda_1 + \lambda_2\left(i_t - E_t\pi_{t+1}\right) + \lambda_3 E_t y_{t+1} + u_{2t}, \quad \lambda_1 > 0,\ \lambda_2 < 0, \lambda_3 > 0, \tag{2.2.2}$$

$$i_t = \pi_t + \alpha_y(y_t - y_t^n) + \alpha_\pi\left(\pi_t - \pi_t^*\right) + r_t^*, \tag{2.2.3}$$

where $y_t$ is the log of output, $y_t^n$ is the natural rate of output which follows a linear trend: $\alpha + \beta t$, and $\pi_t$ is price inflation. $E_t$ is an conditional expectations operator such that $E_{t-1}\pi_t$ is expected inflation for period $t$ given the information available up to period $t-1$, $E_t\pi_{t+1}$ is expected inflation one period ahead, and $E_t y_{t+1}$ is expected output one period ahead.[13] The variable $i_t$ is a nominal interest rate that the policymaker can influence, $\pi_t^*$ is the inflation target, $r_t^*$ is the real interest rate, $u_{1t}$ is an iid shock (demand shock), and $u_{2t}$ is an iid shock (supply shock).

At issue is the relation between policy and inflation. The model posits aggregate supply and demand depend on the expectations over the course of policy and this policy follows some stable probability. Furthermore, agents understand the policy rule and augment their behavior to include the expected gains or losses implied by the policy rule and policymaker behavior (Lucas 1976).

[11] See Chappell and Keech (1983) for an application of a structural model that incorporates the length of a presidential term.

[12] See Achen and Shively (1995: 23-25) for an example on the importance of linking individual and aggregate levels of analysis.

[13] See Sargent (1979) for the detailed discussion on the rational expectations operator.

The coefficients $\alpha_y$ and $\alpha_\pi$ represent the aggressiveness policymakers possess in stopping inflationary pressures. Positive values of $\alpha_y$ and $\alpha_\pi$ indicate an aggressive inflation-stabilizing policy tack. These positive parameter values reflect policymaker willingness to raise nominal interest rates in response to excess demand (inflation), whether it is when output is above its natural rate ($y_t > y_t^n$), or when inflation exceeds its prespecified target ($\pi_t > \pi_t^*$). The coefficients typically range between $[0, 2]$ (Clarida, Gali, and Gertler 2000).[14]

With the relation between countercyclical monetary policy and inflation stated, we now solve for inflation using the method of undetermined coefficients. The minimum state variable solution (MSV) is:[15]

$$\pi_t = \left(\frac{J_0}{1 - J_1 - J_2} + \frac{J_2 J_3 \beta}{(1 - J_1 - J_2)^2}\right) + \left(\frac{J_3}{1 - J_1 - J_2}\right) y_t^n + X_t,$$

where:

$$\begin{aligned}
J_0 &= \left(\lambda_1 - \lambda_2 \alpha_\pi \pi_t^* + \lambda_2 r_t^* + \lambda_3 \beta\right) \Theta^{-1}, \\
J_1 &= \left(\gamma - \lambda_2 \alpha_y \gamma\right) \Theta^{-1}, \\
J_2 &= \lambda_2 \Theta^{-1}, \\
J_3 &= \left(\lambda_3 - 1\right) \Theta^{-1}, \\
X_t &= \left[(\lambda_2 \alpha_y - 1) u_{1t} + u_{2t}\right] \Theta^{-1}, \\
\text{and} \quad \Theta &= \gamma \left(1 - \lambda_2 \alpha_y\right) - \lambda_2 (1 + \alpha_\pi).
\end{aligned}$$

In more compact form the solution is:

$$\pi_t = \Xi + \Psi y_t^n + X_{t,} \tag{2.2.4}$$

where:

$$\begin{aligned}
\Xi &= \frac{J_0}{1 - J_1 - J_2} + \frac{J_2 J_3 \beta}{(1 - J_1 - J_2)^2}, \\
\Psi &= \frac{J_3}{1 - J_1 - J_2}.
\end{aligned}$$

[14] Clarida, Gali, and Gertler (2000) refer to (2.2.3) as a "backward looking" rule. They estimate $\alpha_y, \alpha_\pi$ in (2.2.3) for the United States (1960:1-1996:4). They find $\alpha_y$ ranges between 0.0 to 0.39 and $\alpha_\pi$ ranges between 0.86 to 2.55. They conclude the United States monetary authority moved to nearly an exclusive focus on stabilizing inflation.

[15] The MSV solution is the simplest parameterization one can choose using the method of undetermined coefficients (See McCallum 1983). We discuss these tools in Chapters 5 and 6.

Equation (2.2.4) relates parameters $(\alpha_y, \alpha_\pi)$ with $\pi_t$. The problem, however, is — absent added information — it is impossible to explicitly relate policy and treatment *changes* to outcomes.

There are important scientific consequences when we fail to link formal and applied statistical analysis for purposes of deriving structural parameters.[16] First, consider the utility of a reduced form — which is usually not derived in the current methodological environment. A reduced form such as (2.2.4) possesses weaknesses when it comes to making inferences. The reduced form parameters $(J_{i's}, \Xi, \Psi)$ cannot strictly identify the parameters relating cause and effect (i.e., $\alpha_y, \alpha_\pi$). Where reduced forms have power is not in making inferences but in making predictions: reduced form estimates provide some assistance in ex-ante forecasts.

How useful then are reduced forms in making predictions based on specific values of the independent variables — the so-called conditional forecast? With a reduced form a researcher cannot make conditional forecasts. The reason is the system (described in equations (2.2.1) - (2.2.3)) depends on the behavior of the agents and whether responses from agents are invariant. If agents' behavior is not invariant, the parameters (in equations (2.2.1) - (2.2.3)) are not invariant. Without a mechanism detailing how structural parameters (and behavior) remain (in)variant as independent variables change, an applied statistical model fails in assessing alternative independent variable shifts.

While reduced forms lack inferential power necessary to make conditional forecasts, it still has a linkage to theoretical foundations. Things can be far worse, however, if we use a single empirical equation similar to equation (2.2.4), with no formal-theoretical linkage, and rely on current applied statistical practices — data mining, garbage cans, and "omega matrices." These applied statistical practices lack overall robustness as they obscure fundamental specification error. The parameter(s) "identified" by these current practices lack any real meaning or use.

To see the flaws associated with these contemporary practices more straightfor-

[16]Heckman (2000) defines *structural causal effects* as "the direct effects of the variables in the behavioral equations" Furthermore, "When these equations are linear, the coefficients on the causal variables are called *structural parameters* (emphasis added), and they fully characterize the structural effects." (page 59). Heckman also notes there is some disagreement about what constitutes a structural parameter. The disagreement centers on whether one uses a linear model, a non-linear model or, more, recently a fully parameterized model. In the latter case, structural parameters, can also be called "deep" to distinguish between "the derivatives of a behavioral relationship used to define causal effects and the parameters that generate the behavioral relationship" (page 60).

wardly consider equation (2.2.5) as a single equation estimate of $\pi_t$:

$$\pi_t = d_0 + d_1 y_t^n + v_{1t}, \tag{2.2.5}$$

where $v_{1t}$ is the error term. If we contrast the similarly situated parameters in equations (2.2.4) with (2.2.5), note that $d_0$ would serve the purpose of $\Xi$ and $d_1$ serves the purpose of $\Psi$. But, already equation (2.2.5) fails to make any structural statement about the relation between the policy rule in (2.2.3) and inflation in (2.2.5). There is *no explicit* relation between $d_0$ or $d_1$ with $\alpha_y$ or $\alpha_\pi$.

Now, assume the researcher chooses to ignore any process of methodological unification. Instead the analyst tries to estimate a shift in policy regime and reestimates (2.2.5) with new variables. A typical way is to add dummy variables signifying policy shifts due to factors such as partisanship and other factors (e.g., Hibbs 1977). We keep the empirical model small and rewrite (2.2.5) with just one added variable, a policy shift dummy variable ($\text{SHIFT}_t$) signifying a change in the intercept or the level of inflation:

$$\pi_t = \tilde{d}_0 + \tilde{d}_1 y_t^n + \tilde{d}_2 \text{SHIFT}_t + v_{2t}, \tag{2.2.6}$$

where $v_{2t}$ is the error term.

Does this really assist in achieving identification, predictive precision, falsification, and understanding the working of the system? Notice that the parameter $\tilde{d}_2$ in (2.2.6) does not actually reflect actual policy shifts relating policy parameters to inflation (one such as equation (2.2.3)). This shortcoming is severe. It means we do not know what parameters in the system can lead to counterintuitive results and, ultimately, incorrect policy or treatment recommendations.

Consider the following simulation of the system expressed in (2.2.1) , (2.2.2) , and (2.2.3). In this simulation the structural explanation, relating parameters in the system, shows that aggregate demand, as represented by equation (2.2.2), must respond in a certain way to changes in real interest rates and the expected output level. The relation is represented by the parameter $\lambda_3$ in equation (2.2.2). What happens to the relation between inflation-stabilizing policy and inflation if $\lambda_3$ is not invariant and contains alternative responses?

In Figure 2.2.1, Panels A, B, and C report the results in a simulation of the system.[17] Changes in inflation are represented in each panel, given changes to $\lambda_3$.

[17] The simulation parameter values are the following for the AS, IS, and policy rule:
AS: $\alpha = 5.0; \beta = 0.05; y_0^n = 1.0; \gamma = 10; \sigma_{u_1} = 1.0$.
IS: $\lambda_1 = 3.0; \lambda_2 = -1.0; \lambda_3 = (0.5, 1.0, 1.5); \pi^* = 2.0; \sigma_{u_2} = 1.0$.
Policy rule: $\alpha_y = 0.5; \alpha_\pi = 0.5; r^* = 3.0$.

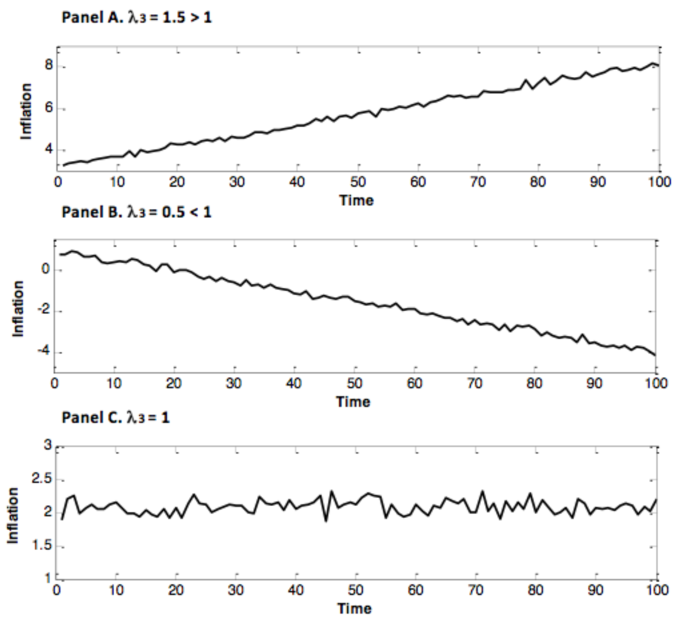


Figure 2.2.1: Simulation Results

In Panel A, when $\lambda_3 > 1$, an aggressive policy $(\alpha_y, \alpha_\pi = 0.5 > 0)$ *does not reduce* inflation. On the other hand, in panel Panel B, when $0 < \lambda_3 < 1$, inflation falls. It is only in the third case, when $\lambda_3 = 1$, we find that an aggressive policy keeps inflation roughly around a 2 percent target (See Panel C).

The implication of these simulations has direct significance for both reduced form estimation and current practice — with the consequences being especially direct for current practice. If a researcher were to estimate the reduced form of inflation as represented in (2.2.4) they would find instability in the forecast and forecast error. However, the researched would have difficulty finding the source of the instability given the lack of information in identifying the relation(s) between the parameters in (2.2.2) and the remaining parameters (variables) in the model.

Yet, this is superior to what occurs using current practices. Specifically, depending on the value of $\lambda_3$, there are alternative outcomes that would not be illuminated using (2.2.6) and the practices associated with that equation. But, now assume we estimate (2.2.6) and find a significant value for $d_2$ indicating that when policy shifts in an aggressive manner, the level of inflation falls. Does this mean that aggressive policy reduces inflation? Using this research practice we do not have an answer. Absent an explicitly posited relation, all we can say is that some parameter, that may or may not relate the independent variable to the dependent variable, is significant.

What is more important is that a formal model would show the substantive findings are not generalizable when alternative and relevant *exogenous* conditions are considered. Indeed, the indeterminacy result shown in Figure 2.2.1 is not robust to the inclusion of nominal variables to the interest rate rule.[18] But, *we would not know what we do not know* under current practices which give us estimates of equations (2.2.5) and (2.2.6).

*The larger lesson from this exercise is to demonstrate how current practice fails to emphasize both the connection between the valid understanding of a system and predictive power.*

## 2.3 Summary

Some contemporary methodology practices have become acts of preemptive scientific surrender. These practices fail to provide the necessary steps in attaining valid inference and prediction. Embedded in *data mining, overparameterization, and omega matrices* is the general view a researcher will:

> Gather the data, run the regression/MLE with the usual linear list of control variables, report the significance tests, and announce that one's pet variable "passed." This dreary hypothesis-testing framework is sometimes seized upon by beginners. Being purely mechanical, it saves a great deal of thinking and anxiety, and cannot help being popular. But obviously, it has to go. Our best empirical generalizations do not derive from that kind of work. (Achen 2002: 442-443).

This scientific weakness is all the more clear when we consider the challenge a social scientist faces. If we simply specify that variable $Y$ is a function of variable $X$, the statistical "tests" estimating a correlation between $X$ and $Y$ cannot determine causation between the two *even when their correlation is statistically significant*. Without unifying formal and empirical analysis we lack a basic analytical attribute suitable for identifying the following possibilities defining the relation between $X$ and $Y$. But, recall that a significant statistical result between $X$ and $Y$ can be due to:

a) $X$ causing $Y$ directly;

b) $X$ causing an unknown variable, $Z$, which causes $Y$; and

c) $X$ and $Y$ are caused by an unknown common factor $W$, but there is no causality between $X$ and $Y$.

[18] Alternatively, McCallum and Nelson (1999) demonstrate that when output is not modeled as a constant, an IS curve of the form (2.2.2) can produce indeterminacies.

Too often researchers, using the methods described above predetermine the significant correlation is result “a.”[19] While EITM does not guarantee a model is correct, it does promote a dialogue between theory and test so that “a” is not the default choice.

[19] See Hoover (2001a,b) for a review of these issues and specific methodologies.

# Chapter 3

# The EITM Framework

The scientific consequences of the current methodological status quo are far reaching. A significant scientific problem with decoupling formal analysis from applied statistical procedures centers on a failure to *identify invariant* parameter estimates. This, in turn, impairs falsification of theories and hypotheses as well as the comparison of rival theories and hypotheses. Predictive precision is also affected since predicting how the behavioral response of an agent influences the success or failure of a policy or treatment is impossible.[1] Developing a basic understanding of how a system operates is a distant hope as well.

We have been arguing that linking mutually reinforcing properties of formal and empirical analysis provides the necessary transparency between theory and test to aid in valid hypothesis testing. This linkage also contributes to the identification of invariant parameter estimates suitable for improving the accuracy of both ex-post and ex-ante predictions and directly addresses the ideas of order, cause, and chance.[2]

## 3.1 Early Methodological Unification: The Solow Model

Methodological approaches to unify formal and empirical analysis are not new. Prior incarnations include research by scholars from organizations such as the Cowles Com-

[1] We will use the word *inference* to refer to a parameter in a regression or likelihood ($b$). We use the word *prediction* to refer to a model's forecast of a dependent variable ($\hat{y}$). For a technical treatment of these two concepts see Engle, Hendry, and Richard (1983).

[2] There is a large literature devoted to identification problems (See, for example, Fisher 1966; Manski 1995). Some researchers treat the issue of simultaneity and identification as one and the same. We consider identification in a broader sense that includes simultaneity, but not limited to simultaneity.

mission presented earlier. Recall the Cowles Commission established conditions in which structural parameters are identified within a model. It explored the differences between structural and reduced-form parameters. Along with the work on structural parameters, Cowles Commission members also gave formal and empirical specificity to issues such as exogeneity and policy invariance (Aldrich 1989; Morgan 1990; Christ 1994; Heckman 2000).

We demonstrate the process of scientific cumulation using the well-known Solow (1956) model of economic development. This example shows how the traditional approach to methodological unification (similar to the Cowles Commission) provides a basis for a modeling dialogue and scientific cumulation. In his seminal paper, Solow (1956) argues capital accumulation and exogenous technological progress are fundamental mechanisms in economic development.

Using a simple Cobb-Douglas production with a dynamic process of capital accumulation, Solow concludes that a country experiences a higher transitory growth rate when the country increases its national saving to stimulate capital accumulation. The formal model has the following structure:

$$Y_t = B_t K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1 \tag{3.1.1}$$

$$S_t = sY_t, \qquad 0 < s < 1 \tag{3.1.2}$$

$$K_{t+1} - K_t = S_t - \delta K_t, \tag{3.1.3}$$

$$L_{t+1} = (1+n) L_t, \tag{3.1.4}$$

$$A_{t+1} = (1+g) A_t, \tag{3.1.5}$$

where aggregate production in year $t$, $Y_t$, is determined by capital, $K_t$, labor, $L_t$, and technological progress, $B_t \equiv A_t^{1-\alpha}$ (Cobb-Douglas specification). $S_t$ is gross domestic savings in year $t$ and the savings rate, $s$, is a constant proportion of total production. Equation (3.1.3) represents the capital accumulation, $K_{t+1} - K_t$, which equals the domestic savings minus capital depreciation, $\delta K_t$ (where $\delta$ is the depreciation rate). Equations (3.1.4) and (3.1.5) show that both labor and technological progress are exogenously increasing at the rates of $n$ and $g$, respectively.

One important prediction from the Solow model is the *conditional convergence hypothesis* which states that countries with lower initial levels of capital and output tend to grow faster when the countries' characteristics are held constant. Using equation (3.1.3), we solve for the transitory growth rate of capital per effective unit of labor $\left(\tilde{k}_t \equiv \frac{K_t}{A_t L_t}\right)$:

$$\frac{\tilde{k}_{t+1}-\tilde{k}_t}{\tilde{k}_t} = \frac{1}{(1+n)(1+g)}\left(s\tilde{k}_t^{\alpha-1} - (n+g+\delta+ng)\right). \tag{3.1.6}$$

In the steady state, the level of capital (per effective unit of labor) is constant over time. It implies that its growth rate is zero. We solve for the steady state level:

$$\tilde{k}^* = \left(\frac{s}{n+g+\delta+ng}\right)^{1/(1-\alpha)}$$

Equation (3.1.6) shows that if $\tilde{k}_t$ is further away from the steady state level $\tilde{k}^*$, then $\tilde{k}_t$ approaches $\tilde{k}^*$ at a faster rate.[3] Therefore, the growth rate of output per effective unit of labor is:

$$\ln\tilde{y}_t - \ln\tilde{y}_{t-1} = \alpha\left(\ln\tilde{k}_t - \ln\tilde{k}_{t-1}\right),$$

where $\ln\tilde{k}_t - \ln\tilde{k}_{t-1}$ is approximately $\frac{\tilde{k}_{t+1}-\tilde{k}_t}{\tilde{k}_t}$ in equation (3.1.6). Using the technique of linear approximation, we can derive the average growth rate of the output level as:

$$\frac{\ln y_T - \ln y_0}{T} \approx g + \frac{1-(1-\lambda)^T}{T}\left(\ln A_0 + \frac{\alpha}{1-\alpha}\left(\ln s - \ln(n+g+\delta+ng)\right) - \ln y_0\right), \tag{3.1.7}$$

where $y_t \equiv \frac{Y_t}{L_t} = A_t\tilde{y}_t$ and $\frac{(\ln y_T - \ln y_0)}{T}$ represents the average (annual) growth rate between period $t=0$ and $t=T$. $A_0$ and $y_0$ are the respective technological level and output levels in the initial period, $t=0$. Equation (3.1.7) shows the level of initial output level $y_0$ is negatively associated with the growth rate of output: a country tends to grow faster if the output level in the country is lower initially.

Extant empirical studies typically regress the average of the annual growth rate (for many countries) on the initial level of real GDP and other related control variables suggested in the model, such as the national savings (or investment) rate and the population growth rate for country $i$:

$$g_{T,0}^i = \beta_0 - \beta_1 \ln y_0^i + \beta_2 Z^i, \tag{3.1.8}$$

where $Z^i \equiv \ln s^i - \ln\left(n^i + g^i + \delta^i + n^i g^i\right)$, and $g_{T,0}^i \equiv \frac{\left(\ln y_T^i - \ln y_0^i\right)}{T}$. From equation

[3] This follows if we define $y_t \equiv \frac{Y_t}{A_t L_t}$ and rewrite equation (3.1.1) as $\tilde{y}_t = \tilde{k}_t^{\alpha}$.

(3.1.7) the parameters $\beta_0$, $\beta_1$ and $\beta_2$ correspond to: $g + \frac{1-(1-\lambda)^T}{T} \ln A_0$, $-\frac{1-(1-\lambda)^T}{T}$, and $\frac{\alpha}{1-\alpha}\left(\frac{1-(1-\lambda)^T}{T}\right)$, respectively.

Using equation (3.1.8), a common empirical finding is that the initial level of GDP possesses a *significant negative correlation* with the economic growth rate in a country. This finding supports the conditional convergence hypothesis that the country with a lower level of real GDP tends to grow faster (Barro and Sala-i-Martin 1992; Mankiw, Romer, and Weil 1992).

### 3.1.1 Leveraging The Solow Model

What role, then, does the Solow model play in cumulation? The typical estimated coefficient of initial real GDP is significantly less than the Solow model prediction. In other words, the Solow model overestimates the speed of convergence. Despite the fact that the Solow model was wrong in this particular case, it had unified formal and empirical analysis and, therefore, provided a foundation upon which to build.

One path taken was to relax the assumption that output production depended on homogeneous labor and capital. Mankiw, Romer and Weil (1992) modified the Solow model by introducing the stock of human capital in the production function. Human capital stock is represented as the sum of education and training that the workers receive. By controlling for the level of human capital (education), the authors find the rate of convergence is approximately two percent per year which is closer to the prediction in the modified Solow model.

The Solow model and the extensions of it provide an example of a cumulative research process. The modifications in the theoretical model were due to the empirical tests, but this is feasible due to methodological unification. The modeling dialogue here gives the researchers a better understanding of the regularities in economic development. If researchers did not consider the Solow model and solely applied an empirical procedure by regressing the average growth rate on the initial real GDP, they could misinterpret the empirical results and fail to understand the mechanism(s) that influence economic development. On the other hand, if the researchers only set up a theoretical model but do not test it empirically, they would not know that the model is actually inconsistent with the empirical observations.

## 3.2 The EITM Framework[4]

This EITM framework builds on the Cowles Commission approach and then places an emphasis on *developing behavioral and applied statistical analogues and linking these analogues*.[5] And while we build on the Cowles Commission approach, there do exist analytical frameworks in other disciplines as well.[6]

EITM includes the following attributes:[7]

- EITM places emphasis on modeling human behavior so new uncertainty created by shifts in behavioral traits such as public tastes, attitudes, expectations, and learning are properly accounted for and studied.

- The Cowles Commission *is* associated with building a system of equations and then following rules (rank and order conditions) for identification that count equations and unknowns. In contrast, our EITM framework is agnostic on the choice to build and relate a system or to partition the system (via assumption) into a smaller set of equations, even a single equation.[8] We place emphasis on the mutually reinforcing properties of formal and empirical analysis.

- A final and related point on model specification relates to the critiques of the structural approach leveled by Sims (1980). It is well known that structural parameters are not identified from reduced form estimates. The practice of finding ways to identify models can lead to "incredible" theoretical specifications (Sims 1980; Freeman, Lin, and Williams 1989). The proposed EITM framework, by adding behavioral concepts and analogues, can address Sims' criticisms in a theoretically meaningful way. Analogues, in particular, have important scientific importance since they hold the promise of *operationalizing mechanisms*.[9]

[4] We thank Douglas Dion for his insights and suggestions.

[5] Analogues are related to operationalizing a concept. An analogue is a device represented by variable — and measurable — quantities. Analogues include variables, operators, or an estimation process that mimic the concept of interest. They serve as analytical devices — not categorical indicators — for behavior and, therefore, provide for changes in behavior as well as a more transparent interpretation of the formal and applied statistical model.

[6] Jasso's (2004) *Tripartite Framework* will be reviewed in Chapter 12.

[7] See Granato, Lo, and Wong (2010a, 2010b, 2011).

[8] This debate about general and partial equilibrium model building can be traced back to at least the 1800s. See Friedman (1953) for descriptions and evaluation of "Walrasian" and "Marshallian" model building practice.

[9] Operationalizing causal mechanisms, as opposed to operationalizing variables, involves the creation of measurable devices (i.e., analogues) on both the formal side and the empirical side. An early example of operationalizing a mechanism can be seen in the work of Converse (1969). He

This EITM framework contains three steps:

**Step 1. Relate and Unify Theoretical Concepts and Applied Statistical Concepts**

The goal of this first step in EITM is to transform the focus from the substantive topic to the underlying behavioral process. We start, however, not with the development of mathematical structures but with the identification of concepts. It is of course standard to suggest that research start with concepts. We have in mind, however, not the substantive concepts central to a discipline, but instead to the general behavioral attributes of the thing being researched.[10]

Concepts of particular concern in this framework reflect many overarching social and behavioral processes. Examples include (but are not limited to):

- decision making
- bargaining
- expectations
- learning
- elements of social interaction (strategic and non-strategic)

It is also important to find an appropriate statistical concept to match with the theoretical concept. Examples of applied statistical concepts include (but are not limited to):

- persistence

advanced the theory that strength of party identification (and voting behavior) is primarily a function of intergenerational transmission plus the number of times one had voted in free elections. To operationalize his proposed mechanism — intergenerational transmission — he made use of the following analogue: the Markov chain. This particular analogue allowed for a particular dynamic prediction he tested with data.

[10] In political science, for example, a student of democracy might focus on *choice*: how do demographic and attitudinal variables drive individual selection over political parties. Another student might focus on *uncertainty and learning*: given the lack of a "track record" among political parties in newly democratizing states, how do individuals come to form expectations regarding those parties, and how do those expectations shift in response to political and economic changes? A third student might concentrate on the idea of *bargaining*: how do the various party leaders face the trade-offs between maximizing their potential influence in the political system and maintaining the promise to democratize? The idea is not to ignore the substantive aspects, but to look at substance from a different perspective, one that not only helps clarify the focus of the research but also suggests common behavioral concerns that make it easier to communicate across subfields and find common elements and approaches. We thank Douglas Dion for this set of examples.

- measurement error
- nominal choice
- simultaneity
- prediction

**Step 2. Develop Behavioral (Formal) and Applied Statistical Analogues**

To link concepts with tests, we need analogues. An analogue is a device representing a concept via a continuous and measurable variable or set of variables. Examples of analogues for the behavioral (formal) concepts such as decision making, expectations, learning, and strategic interaction include (but are not limited to):

- decision theory (e.g., utility maximization)
- conditional expectations (forecasting) procedures
- adaptive and Bayesian learning (information updating) procedures
- game theory

Examples of applied statistical analogues for the applied statistical concepts of persistence[11], measurement error, nominal choice, simultaneity, and prediction include (respectively):

- autoregressive estimation
- error-in-variables regression
- discrete choice modeling
- multi-stage estimation (e.g., two-stage least squares)
- point estimates and distributions

[11] Consider, for example, the concept of persistence. Theoretically, this conjures up a host of potential analogues: coordination equilibria, Markov chain steady-states, locally and globally stable solutions to systems of differential equations, or still lifes in cellular automata models. Similarly, persistence can be tied to a number of statistical models, including autoregressive estimation and asymptotic theory.

**Step 3. Unify and Evaluate the Analogues**

The third step unifies the mutually reinforcing properties of the formal and empirical analogues. By starting with the concept and then moving to the theoretical and applied statistical analogues, we guarantee that there must be something in common between the theory and the empirical analysis. The idea, then, is to locate the parameters of interest in each that reflect the underlying concept, and then use those to build clearer and stronger links between the mechanisms of the theoretical model and the specification of the statistical methods. The specified linkage not only draws theory and empirics closer, but also provides a way for research to build by showing potential sources of inaccuracies and model failure.

## 3.3 Summary

The EITM framework contains three basic steps:

1. Link the theoretical mechanisms and applied statistical concepts.
2. Develop behavioral (formal) and applied statistical analogues. To link concepts with tests, we need analogues. An analogue can be thought of as a device in which a concept is represented by continuously variable — and measurable — quantities. Analogues serve as analytical devices for behavior and, therefore, provide for changes in behavior as well as a more transparent interpretation of the formal and applied statistical model.
3. Link and evaluate the behavioral (formal) and applied statistical analogues.

This EITM framework should not be interpreted as a substitute for pure formal or pure empirical approaches. The criticisms leveled in Chapter 2 stand, but these approaches are valid out of necessity, particularly when theory or data are either underdeveloped, nonexistent, or both. The simple fact is there are numerous examples in many sciences where theory is ahead of data or data are ahead of theory, sometimes for decades (See Rigden 2005). Nor should the quantitative nature of this framework suggest it precludes the use of qualitative procedures (Brady and Collier 2010). Such exclusion would be throwing out information which could otherwise aid in finding underlying mechanisms.

We demonstrate the various linkages in the second part of this book.

# Part II

# EITM in Practice

# Chapter 4

# Economic Voting

Economic voting comprises a substantial literature. A strand starting with Kramer (1983) and extending to work by Alesina and Rosenthal (1995), Suzuki and Chappell (1996), and Lin (1999) contributes to the value of the literature. These studies have refined earlier work and present models of voter sophistication and new applied statistical tests. In the former instance, voters possess the capability to deal with uncertainty in assigning blame or credit to incumbents for good or bad economic conditions. For the latter, applied statistical tests include some of the more advanced tools in time series analysis.

There is another important — EITM related — feature in this work. Some of these authors relate a measurement error problem to the voter capability noted above. This is exactly what EITM and methodological unification accomplish. The theory — the formal model — implies an applied statistical model with measurement error. Consequently, one can examine the joint effects by employing a unified approach.[1]

## 4.1 Step 1: Relating Expectations, Uncertainty, and Measurement Error

Earlier contributors have dealt with this "signal extraction" problem (See the Appendix, Section 4.53). Friedman (1957) and Lucas's (1973) substantive findings would not have been achieved had they treated their research question as a pure measurement error problem requiring only an applied statistical analysis (and "fix" for the

[1] Recall that applied statistical tools lack power in disentangling conceptually distinct effects on a dependent variable. This is noteworthy since the traditional applied statistical view of measurement error is that it creates parameter bias, with the typical remedy requiring the use of various estimation techniques (See the Appendix, Section 4.51) and Johnston and DiNardo (1997:153-159)).

measurement error). Indeed, both Friedman (1957) and Lucas (1973) linked specific empirical coefficients from their respective formal (behavioral) models: among their contributions was to merge "error in variables" regression with formal models of expectations and uncertainty. For Friedman, the expectations and uncertainty involve permanent-temporary confusion, while general-relative confusion is the behavioral mechanism in Lucas's model.

## 4.2 Step 2: Analogues for Expectations, Uncertainty, and Measurement Error

This chapter focuses on Alesina and Rosenthal's (1995) contribution. The formal model representing the behavioral concepts — expectations and uncertainty — is presented. Alesina and Rosenthal (1995) provide the formal model (pages 191-195). Their model of economic growth is based on an expectations augmented aggregate supply curve:

$$\hat{y}_t = \hat{y}^n + \gamma\left(\pi_t - \pi_t^e\right) + \varepsilon_t, \tag{4.2.1}$$

where $\hat{y}_t$ represents the rate of economic growth (GDP growth) in period $t$, $\hat{y}^n$ is the natural economic growth rate, $\pi_t$ is the inflation rate at time $t$, and $\pi_t^e$ is the expected inflation rate at time $t$ formed at time $t-1$.

Having established voter inflation expectations the concept of uncertainty is next. We assume voters want to determine whether to attribute credit or blame for economic growth ($y_t$) outcomes to the incumbent administration. Yet, voters are faced with uncertainty in determining which part of the economic outcomes is due to incumbent "competence" (i.e., policy acumen) or simply good luck.

If the uncertainty is based, in part, from equation (4.2.1), then equation (4.2.2) presents the analogue. It is commonly referred to as a "signal extraction" or measurement error problem (See the Appendix, Section 4.53):

$$\varepsilon_t = \eta_t + \xi_t. \tag{4.2.2}$$

The variable $\varepsilon_t$ represents a "shock" comprised of the two unobservable characteristics noted above — competence or good luck. The first, represented by $\eta_t$, reflects "competence" attributed to the incumbent administration. The second, symbolized as $\xi_t$, are shocks to growth beyond administration control (and competence). Both $\eta_t$ and $\xi_t$ have zero mean with variance(s) $\sigma_\eta^2$ and $\sigma_\xi^2$ respectively. In less technical language Alesina and Rosenthal describe competence as follows:

> The term $\xi_t$ represents economic shocks beyond the governments control, such as oil shocks and technological innovations. The term $\eta_t$ captures the idea of government competence, that is the government's ability to increase the rate of growth without inflationary surprises. In fact, even if $\pi_t = \pi_t^e$, the higher is $\eta_t$ the higher is growth, for a given $\xi_t$. We can think of this competence as the government's ability to avoid large scale inefficiencies, to promote productivity growth, to avoid waste in the budget process, so that lower distortionary taxes are needed to finance a given amount of government spending, etc (page 192).

Note also that competence can persist *and support* reelection. This feature is characterized as an MA(1) process:

$$\eta_t = \mu_t + \rho\mu_{t-1}, \quad 0 < \rho \leq 1 \tag{4.2.3}$$

where $\mu_t$ is $iid\left(0, \sigma_\mu^2\right)$. The parameter $\rho$ represents the strength of the persistence. The lag or lags allow for retrospective voter judgments.

If we reference equation (4.2.1) again, let us assume voters' judgments include a general sense of the average rate of growth ($\hat{y}^n$) and the ability to observe actual growth ($\hat{y}_t$). Voters can evaluate their difference ($\hat{y}_t - \hat{y}^n$). Equation (4.2.1) also suggests that when voters predict inflation with no systematic error (i.e., $\pi_t^e = \pi_t$), the result is non-inflationary growth with no adverse real wage effect.

Next, economic growth performance is tied to voter uncertainty. Alesina and Rosenthal formalize how economic growth rate deviations from the average can be attributed to administration competence or fortuitous events:

$$\hat{y}_t - \hat{y}^n = \varepsilon_t = \eta_t + \xi_t. \tag{4.2.4}$$

Equation (4.2.4) shows when the actual economic growth rate is greater than its average or "natural rate" (i.e., $\hat{y}_t > \hat{y}^n$), then $\varepsilon_t = \eta_t + \xi_t > 0$. Again, the voters are faced with uncertainty in distinguishing the incumbent's competence ($\eta_t$) from the stochastic economic shock ($\xi_t$). However, because competence can persist, voters use this property for making forecasts and giving greater or lesser weight to competence over time.

This behavioral effect is demonstrated by substituting equation (4.2.3) in (4.2.4):

$$\mu_t + \xi_t = \hat{y}_t - \hat{y}^n - \rho\mu_{t-1}. \tag{4.2.5}$$

Equation (4.2.5) suggests that voters can observe the composite shock $\mu_t + \xi_t$ based on the observable variables, $\hat{y}_t$, $\hat{y}^n$, and $\mu_{t-1}$ which are available at time $t$ and $t-1$. Determining the optimal estimate of competence, $\eta_{t+1}$, when the voters observe $\hat{y}_t$. Alesina and Rosenthal demonstrate this result making a one-period forecast of equation (4.2.3) and solving for its expected value (conditional expectation) at time $t$ (See the Appendix, Section 4.52):

$$E_t(\eta_{t+1}) = E_t(\mu_{t+1}) + \rho E(\mu_t|\hat{y}_t) = \rho E(\mu_t|\hat{y}_t), \tag{4.2.6}$$

where $E_t(\mu_{t+1}) = 0$. Alesina and Rosenthal (1995) argue further that rational voters would not use $\hat{y}_t$ as the only variable to forecast $\eta_{t+1}$. Instead, they use all available information, including $\hat{y}^n$ and $\mu_{t-1}$. As a result, a revised equation (4.2.6) is:

$$\begin{aligned} E_t(\eta_{t+1}) &= E_t(\mu_{t+1}) + \rho E(\mu_t|\hat{y}_t - \hat{y}^n - \rho\mu_{t-1}) && (4.2.7) \\ &= \rho E(\mu_t|\mu_t + \xi_t). && (4.2.8) \end{aligned}$$

Using this analogue for expectations in equation 4.2.7, competence, $\eta_{t+1}$, can be forecasted by predicting $\mu_{t+1}$and $\mu_t$. Since there is no information available for forecasting $\mu_{t+1}$, rational voters can only forecast $\mu_t$ based on observable $\hat{y}_t - \hat{y}^n - \rho\mu_{t-1}$ (at time $t$ and $t-1$) from equations 4.2.7 and 4.2.8.

## 4.3 Step 3: Unifying and Evaluating the Analogues

The method of recursive projection and equation (4.2.5) illustrates how the behavioral analogue for expectations is linked to the empirical analogue for measurement error (an error-in-variables "equation"):

$$E_t(\eta_{t+1}) = \rho E(\mu_t|\hat{y}_t) = \rho\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2}(\hat{y}_t - \hat{y}^n - \rho\mu_{t-1}), \tag{4.3.1}$$

where $0 < \rho\frac{\sigma_\mu^2}{\sigma_\mu^2+\sigma_\xi^2} < 1$. Equation (4.3.1) shows voters can forecast competence using the difference between $\hat{y}_t - \hat{y}^n$, but also the "weighted" lag of $\mu_t$ (i.e., $\rho\mu_{t-1}$).

In equation (4.3.1), the expected value of competence is *positively* correlated with economic growth rate deviations. Voter assessment is filtered by the coefficient, $\frac{\sigma_\mu^2}{\sigma_\mu^2+\sigma_\xi^2}$, representing a proportion of competence voters are able to interpret and observe.

The behavioral implications are straightforward. If voters interpret that the variability of economic shocks come solely from the incumbent's competence (i.e., $\sigma_\xi^2 \to 0$), then $\frac{\sigma_\mu^2}{\sigma_\mu^2+\sigma_\xi^2} \to 1$. On the other hand, the increase in the variability of uncontrolled shocks, $\sigma_\xi^2$, confounds the observability of incumbent competence since the signal-noise coefficient $\frac{\sigma_\mu^2}{\sigma_\mu^2+\sigma_\xi^2}$ decreases. Voters assign less weight to economic performance in assessing the incumbent's competence.

Alesina and Rosenthal test the empirical implications of their theoretical model with U.S. data on economic outcomes and political parties for the period 1915 to 1988. They first use the growth equation (4.2.1) to collect the estimated exogenous shocks ($\varepsilon_t$) in the economy. With these estimated exogenous shocks, they then construct their variance-covariance structure.

Since competence ($\eta_t$) in equation (4.2.3) follows an MA(1) process, they hypothesize that a test for incumbent competence, as it pertains to economic growth, can be performed using the covariances between the current and preceding year. The specific test centers on whether the changes in covariances with the presidential party in office are statistically larger than the covariances associated with a change in presidential parties. They report null findings (e.g., equal covariances) and conclude that there is little evidence to support that voters are retrospective and use incumbent competence as a basis for support.

## 4.4 Leveraging EITM and Extending the Model

Alesina and Rosenthal provide an EITM connection between equations (4.2.1) , (4.2.3) and their empirical tests. They link the behavioral concepts — expectations and uncertainty — with their respective analogues (conditional expectations and measurement error) and devise a signal extraction problem. While the empirical model resembles an error-in-variables specification, testable by dynamic methods such as rolling regression (Lin 1999), they instead estimate the variance-covariance structure of the residuals.

Their model is testable in other ways. We can, for example, leverage equation (4.3.1) and account for other forms of uncertainty. Suzuki and Chappell (1996) (and numerous others) provide such tests without any formalization. The formalization of Alesina and Rosenthal can be used and linked to Suzuki and Chappell's test.

Recall that the competence analogue ($\eta_t$) in their model is set up to be part of the aggregate supply (AS) shock ($\varepsilon_t = \eta_t + \xi_t$). Accordingly, competence ($\eta_t$) is defined as the incumbent's ability to promote economic growth via policies along the AS curve. Let us assume voters are sophisticated enough to not reward incumbent politicians for unusual economic growth resulting from an aggregate demand (AD) policy or shock. Rather, voters think the AS policy is the source of long-lasting (permanent) economic growth since it adds to productive capacity.[2] On the other hand, AD policy can at best produce temporary output gains and eventually leaves the economy with higher inflation.[3]

By leveraging the EITM framework, these studies lead to a direct relation between the parameters of the formal and empirical models. In particular, the competence equation (4.3.1) can be evaluated with the empirical tests and measures Suzuki and Chappell use for permanent and temporary changes in economic growth.

## 4.5 Appendix

The tools in this chapter are used to establish a transparent and testable relation between expectations (uncertainty) and forecast measurement error. The applied statistical tools provide a basic understanding of:

- Measurement error in a linear regression context — error-in-variables regression.

The formal tools include a presentation of:

- A linkage to linear regression.
- Linear projections.
- Recursive projections.

These tools, when unified, produce the following EITM relations consistent with research questions termed *signal extraction*. The last section of this appendix demon-

[2] AS policies provide positive technology shocks. These policies range from government protection of property rights to the provision of public infrastructure.

[3] Achen (2012) adds yet another wrinkle to how competence is characterized. A key feature of his extension is to alter the MA(1) characterization by adding a constant term. This term signifies average competence and provides memory on incumbent administration competence. Achen's modification has important implications on how mypopic voters are and what circumstances can affect retrospection. Achen's work also opens the possibility for using an AR(1) process and he discusses this alternative.

strates signal extraction problems which are directly related to Alesina and Rosenthal's model and test.

### 4.5.1 Empirical Analogues

#### Measurement Error and Error in Variables Regression

In a regression model it is well known that endogeneity problems (e.g., a relation between the error term and a regressor) can be due to measurement error in the data. A regression model with mis-measured right-hand side variables gives least squares estimates with bias. The extent of the bias depends on the ratio of the variance of the signal (true variable) to the sum of the variance of the signal and the variance of the noise (measurement error). The bias increases when the variance of the noise becomes larger in relation to the variance of the signal. Hausman (2001: 58) refers to the estimation problem with measurement error as the "Iron Law of Econometrics" because the magnitude of the estimate is usually smaller than expected.

To demonstrate the downward bias consider the classical linear regression model with one independent variable:

$$Y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \quad t = 1, ..., n \tag{4.5.1}$$

where $\varepsilon_t$ are independent $N(0, \sigma_\varepsilon^2)$ random variables. The unbiased least squares estimator for regression model (4.5.1) is:

$$\hat{\beta}_1 = \left[\sum_{t=1}^{n}(x_t - \bar{x})^2\right]^{-1} \sum_{t=1}^{n}(x_t - \bar{x})(Y_t - \bar{Y}). \tag{4.5.2}$$

Now instead of observing $x_t$ directly, observe its value with an error:

$$X_t = x_t + e_t, \tag{4.5.3}$$

where $e_t$ is an $iid(0, \sigma_e^2)$ random variable. The simple linear error-in-variables model can be written as:

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 x_t + \varepsilon_t, \quad t = 1, ..., n \\ X_t &= x_t + e_t. \end{aligned} \tag{4.5.4}$$

In model (4.5.4), an estimate of a regression of $Y_t$ on $X_t$, with an error term mixing the effects of the true error $\varepsilon_t$ and the measurement error $e_t$ is presented.[4] It follows that the vector $(Y_t, X_t)$ is distributed as a bi-variate normal vector with mean vector and covariance matrix defined as (4.5.5) and (4.5.6), respectively:

$$E\left\{(Y, X)\right\} = (\mu_Y, \mu_X) = (\beta_0 + \beta_1\mu_x, \mu_x) \tag{4.5.5}$$

$$\begin{bmatrix} \sigma_Y^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_X^2 \end{bmatrix} = \begin{bmatrix} \beta_1^2\sigma_x^2 + \sigma_\varepsilon^2 & \beta_1\sigma_x^2 \\ \beta_1\sigma_x^2 & \sigma_x^2 + \sigma_e^2 \end{bmatrix} \tag{4.5.6}$$

The estimator for the slope coefficient when $Y_t$ is regressed on $X_t$ is:

$$\begin{aligned} E(\hat{\beta}_1) &= E\left\{\left[\sum_{t=1}^{n}(X_t - \bar{X})^2\right]^{-1}\sum_{t=1}^{n}(X_t - \bar{X})(Y_t - \bar{Y})\right\} \qquad (4.5.7) \\ &= (\sigma_X^2)^{-1}\sigma_{XY} \\ &= \beta_1\left(\frac{\sigma_x^2}{\sigma_x^2 + \sigma_e^2}\right). \end{aligned}$$

The resulting estimate is smaller in magnitude than the true value of $\beta_1$. The ratio of $\lambda = \frac{\sigma_x^2}{\sigma_X^2} = \frac{\sigma_x^2}{\sigma_x^2+\sigma_e^2}$ defines the degree of attenuation. In applied statistics, this ratio, $\lambda$, is termed the *reliability ratio.* A traditional applied statistical remedy is to use a "known" reliability ratio and weight the statistical model accordingly.[5] As presented above (4.5.7) the expected value of the least squares estimator of $\beta_1$ is the true $\beta_1$ multiplied by the reliability ratio, so it is possible to construct an unbiased estimator of $\beta_1$ if the ratio of $\lambda$ is known.

### 4.5.2 Formal Analogues[6]

#### Least Squares Regression

Normally we think of least squares regression as an empirical tool, but in this case it serves as a bridge between the formal and empirical analogues ultimately creating

[4] To demonstrate this results, we derive $Y_t = \beta_0 + \beta_1 X_t + (\varepsilon_t - \beta_1 e_t)$ from (4.5.4)). Assuming the $x_t's$ are random variables with $\sigma_x^2 > 0$ and $(x_t, \varepsilon_t, e_t)'$ are *iid* $N[(e_x, 0, 0)', diag(\sigma_x^2, \sigma_\varepsilon^2, \sigma_e^2)]$ where $diag(\sigma_x^2, \sigma_\varepsilon^2, \sigma_e^2)$ is a diagonal matrix with the given elements on the diagonal.

[5] See Fuller (1987) for other remedies based on the assumption some of the parameters of the model are known or can be estimated (from outside sources). Alternatively, there are remedies which do not assume any prior knowledge for some of the parameters in the model (See Pal 1980).

[6] The following sections are based on Whittle (1963, 1983), Sargent (1987), and Woolridge (2008).

a behavioral rationale for the ratio in equations (4.2.6) and (4.3.1). This section is a review following Sargent (1987: 223-229).

Assume there is a set of random variables, $y$, $x_1, x_{2,}, \ldots, x_n$. Consider that we estimate the random variable $y$ which is expressed as a linear function of $x_i$:

$$\hat{y} = b_0 + b_1 x_1 + \cdots + b_n x_n, \tag{4.5.8}$$

where $b_0$ is the intercept of the linear function, and $b_i$ presents the partial slope parameters on $x_i$, for $i = 1, 2, \ldots, n$. As a result, by choosing the $b_i$, $\hat{y}$ is the "best" linear estimate which minimizes the "distance" between $y$ and $\hat{y}$:

$$\begin{aligned} &\min\nolimits_{a_i} E\left(y - \hat{y}\right)^2 \\ \Rightarrow \quad & E\left[y - \left(b_0 + b_1 x_1 + \cdots + b_n x_n\right)\right]^2, \end{aligned} \tag{4.5.9}$$

for all $i$. To minimize equation (4.5.9), a necessary and sufficient condition is (in the normal equation(s)):

$$E\left\{\left[y - \left(b_0 + b_1 x_1 + \cdots + b_n x_n\right)\right] x_i\right\} \quad = \quad 0 \tag{4.5.10}$$

$$E\left[\left(y - \hat{y}\right) x_i\right] \quad = \quad 0, \tag{4.5.11}$$

where $x_0 = 1$.

The condition expressed in equation (4.5.11) is called the *orthogonality principle*. It implies that the difference between observed $y$ and the estimated $y$ according to the linear function, $\hat{y}$, is not linearly dependent with $x_i$ for $i = 1, 2, \ldots, n$.

**Linear Projections**

A least squares projection begins with:

$$y = \sum_{i=0}^{n} b_i x_i + \varepsilon, \tag{4.5.12}$$

where $\varepsilon$ is the forecast error, $E\left(\varepsilon \sum b_i x_i\right) = 0$ and $E\left(\varepsilon x_i\right) = 0$, for $i = 0, 1, \cdots, n$. Note also that the random variable $\hat{y} = \sum_{i=0}^{n} b_i x_i$, is based on $b_i^{\prime} s$ chosen to satisfy the least squares orthogonality condition. This is called the projection of $y$ on $x_0, x_1, \ldots, x_n$.

Mathematically, it is written:

$$\sum b_i x_i \equiv P\left(y\,|1, x_1, x_2, \cdots, x_n\right), \tag{4.5.13}$$

where $x_0 = 1$. Assuming orthogonality, the equation (4.5.10) can be rewritten as a set of normal equations:

$$\begin{bmatrix} Ey \\ Eyx_1 \\ Eyx_2 \\ \vdots \\ Eyx_n \end{bmatrix} = \begin{bmatrix} 1 & Ex_1 & Ex_2 & \cdots & Ex_n \\ Ex_1 & Ex_1^2 & Ex_1x_2 & \cdots & \\ Ex_2 & Ex_1x_2 & \ddots & & \\ \vdots & \vdots & & \ddots & \\ Ex_n & & & & Ex_n^2 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}. \tag{4.5.14}$$

Given that the matrix of $Ex_ix_j$ in equation (4.5.14) is invertible for $i, j \in \{1, 2, \ldots, n\}$, and solving for each coefficient $(b_i)$:

$$\begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_n \end{bmatrix} = \left[Ex_ix_j\right]^{-1}\left[Eyx_k\right]. \tag{4.5.15}$$

Applying the above technique to a simple example:

$$y = b_0 + b_1x_1 + \varepsilon,$$

and:

$$\begin{bmatrix} Ey \\ Eyx_1 \end{bmatrix} = \begin{bmatrix} 1 & Ex_1 \\ Ex_1 & Ex_1^2 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}. \tag{4.5.16}$$

Using normal equation(s), the following estimates are derived for the intercept and slope:

$$b_0 = Ey - b_1Ex_1,$$

and:

$$\begin{aligned} b_1 &= \frac{E\left(y - Ey\right)\left(x_1 - Ex_1\right)}{E\left(x_1 - Ex_1\right)^2} \\ &= \frac{\sigma_{x_1y}}{\sigma_{x_1}^2}, \end{aligned}$$

where $\sigma_{x_1y}$ is the covariance between $x_i$ and $y$, and $\sigma^2_{x_1}$is the variance of $x_1$.[7]

### Recursive Projections

The linear least squares identities can be used in formulating how agents update their forecasts (*expectations*). Recursive projections are a key element of deriving the optimal forecasts, such as the one shown in equation (4.3.1). These forecasts are updated consistent with the linear least squares rule described above. The simple univariate projection can be used (recursively) to assemble projections on many variables, such as $P\left(y\,|1, x_1, x_2, \cdots, x_n\right)$.

For example, when there are two independent variables, equation (4.5.13) can be rewritten for $n = 2$ as:

$$y = P\left(y\,|1, x_1, x_2\right) + \varepsilon, \tag{4.5.17}$$

implying:

$$y = b_0 + b_1 x_1 + b_2 x_2 + \varepsilon, \tag{4.5.18}$$

where $E\varepsilon = 0$. Assume that equations (4.5.17) and (4.5.18) satisfy the orthogonality conditions: $E\varepsilon x_1 = 0$ and $E\varepsilon x_2 = 0$. If we omit the information from $x_2$ to project

[7]From equation (4.5.16), we derive a similar equation expressed in equation (4.5.15):

$$\begin{aligned}
\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} &= \begin{bmatrix} 1 & Ex_1 \\ Ex_1 & Ex_1^2 \end{bmatrix}^{-1} \begin{bmatrix} Ey \\ Eyx_1 \end{bmatrix} \\
&= \begin{bmatrix} Ex_1^2 & -Ex_1\left(Ex_1^2 - (Ex_1)^2\right)^{-1} \\ -Ex_1\left(Ex_1^2 - (Ex_1)^2\right)^{-1} & \left(Ex_1^2 - (Ex_1)^2\right)^{-1} \end{bmatrix} \begin{bmatrix} Ey \\ Eyx_1 \end{bmatrix}.
\end{aligned}$$

$b_1$ can be expressed as:

$$\begin{aligned}
b_1 &= -\frac{Ex_1}{Ex_1^2 - (Ex_1)^2} Ey + \frac{Eyx_1}{Ex_1^2 - (Ex_1)^2} \\
&= \frac{-Ex_1 Ey + Eyx_1}{Ex_1^2 - (Ex_1)^2}.
\end{aligned}$$

For simplicity, we assume $Ex_1 = 0$ and $Ey = 0$. Consequently:

$$\begin{aligned}
b_1 &= \frac{-Ex_1 Ey + Eyx_1}{Ex_1^2 - (Ex_1)^2} \\
&= \frac{Eyx_1}{Ex_1^2} \\
&= \frac{\sigma_{x_1y}}{\sigma^2_{x_1}}.
\end{aligned}$$

$y$, then the projection of $y$ can only be formed based on the random variable $x_1$:

$$P(y|1,x_1) = b_0 + b_1x_1 + b_2P(x_2|1,x_1). \tag{4.5.19}$$

In equation (4.5.19), $P(x_2|1,x_1)$ is a component where $x_2$ is projected using 1 and $x_1$ to forecast $y$. Formally, equation (4.5.19) can be separated into three projections:

$$P(y|1,x_1) = P(b_0|1,x_1) + b_1P(x_1|1,x_1) + b_2P(x_2|1,x_1). \tag{4.5.20}$$

Equation (4.5.20) demonstrates that the projection of $y$ given $(1,x_1)$ is a linear function of the three projections:[8]

$$\begin{aligned} P(b_0|1,x_1) &= b_0, \\ P(x_1|1,x_1) &= x_1, \text{ and} \\ P(\varepsilon|1,x_1) &= 0. \end{aligned}$$

An alternative expression is to rewrite the forecast error of $y$ given $x_1$ as simply the "forecast" error of $x_2$ given $x_1$ and a stochastic error term $\varepsilon$. Mathematically, equation (4.5.18) is subtracted from equation (4.5.19):

$$y - P(y|1,x_1) = b_2[x_2 - P(x_2|1,x_1)] + \varepsilon, \tag{4.5.21}$$

and simplified to:

$$z = b_2w + \varepsilon,$$

where $z = y - P(y|1,x_1)$, and $w = [x_2 - P(x_2|1,x_1)]$. Note that $x_2 - P(x_2|1,x_1)$ is also orthogonal to $\varepsilon$, such that, $E\{\varepsilon[x_2 - P(x_2|1,x_1)]\} = 0$ or $E(\varepsilon w) = 0$.

[8]The first two conditions can be interpreted as follows. First, when predicting a constant $b_0$ using 1 and $x_1$, we are still predicting a constant $b_0$. As a result, $P(b_0|1,x_1) = b_0$. Second, when predicting $x_1$ using 1 and $x_1$, we can also predict $x_1$, which is $P(x_1|1,x_1) = x_1$.

To show the results mathematically, rewrite the projection as the following linear function: $P(b_0|1,x_1) = t_0 + t_1x_1$, where $t_0$ and $t_1$ are parameters. Using normal equations, we can derive $t_0$ and $t_1$: $t_0 = Eb_0 - t_1Ex_1$, and $t_1 = \frac{E(b_0-Eb_0)(x_1-Ex_1)}{E(x_1-Ex_1)^2}$. Since $Eb_0 = b_0$, then: $t_1 = \frac{E(b_0-Eb_0)(x_1-Ex_1)}{E(x_1-Ex_1)^2} = 0$, and $t_0 = Eb_0 = b_0$. Therefore, $P(b_0|1,x_1) = t_0 + t_1x_1 = b_0$.

For $P(x_1|1,x_1) = x_1$, we perform the same operations: $P(x_1|1,x_1) = t_0 + t_1x_1$. Now $t_0 = Ex_1 - t_1Ex_1$, and $t_1 = \frac{E(x_1-Ex_1)(x_1-Ex_1)}{E(x_1-Ex_1)^2} = \frac{E(x_1-Ex_1)^2}{E(x_1-Ex_1)^2} = 1$. Therefore $t_0 = Ex_1 - Ex_1 = 0$, and $P(x_1|1,x_1) = t_0 + t_1x_1 = 0 + x_1 = x_1$. As a result, $P(x_1|1,x_1) = x_1$.

We rely on the orthogonality condition for the last expression: $E(\varepsilon) = E(\varepsilon x_1) = 0$. This gives us $P(\varepsilon|1,x_1) = t_0 + t_1x_1$. Now $t_0 = E\varepsilon - t_1Ex_1$ and:$t_1 = \frac{E(\varepsilon-E\varepsilon)(x_1-Ex_1)}{E(x_1-Ex_1)^2} = \frac{E(\varepsilon x_1 - \varepsilon Ex_1 - E\varepsilon x_1 + E\varepsilon x_1)}{E(x_1-Ex_1)^2} = 0$. Since $t_1 = 0$, we find $t_0 = E\varepsilon - t_1Ex_1 = E\varepsilon = 0$. Therefore, $P(\varepsilon|1,x_1) = 0$.

Now writing the following expression as a projection of the forecast error of $y$ that depends on the forecast error of $x_2$ given $x_1$:

$$P\left[y - P\left(y\,|1, x_1\right)|x_2 - P\left(x_2\,|1, x_1\right)\right] = b_2\left[x_2 - P\left(x_2\,|1, x_1\right)\right], \qquad (4.5.22)$$

or in simplified form:

$$P\left(z|w\right) = b_2 w.$$

By combining equations (4.5.21) and (4.5.22), the result is:

$$y = P\left(y\,|1, x_1\right) + P\left[y - P\left(y\,|1, x_1\right)|x_2 - P\left(x_2\,|1, x_1\right)\right] + \varepsilon. \qquad (4.5.23)$$

Consequently, equation (4.5.23) can also be written as:

$$P\left(y\,|1, x_1, x_2\right) = P\left(y\,|1, x_1\right) + P\left[y - p\left(y\,|1, x_1\right)|x_2 - P\left(x_2\,|1, x_1\right)\right], \qquad (4.5.24)$$

where $P\left(y\,|1, x_1, x_2\right)$ is called a bivariate projection. The univariate projections are given by:

$P\left(x_2\,|1, x_1\right)$, $P\left(y\,|1, x_1\right)$, and $P\left[y - P\left(y\,|1, x_1\right)|x_2 - P\left(x_2\,|1, x_1\right)\right]$.

In this case, the bivariate projection equals three univariate projections. More importantly, equation (4.5.24) is useful for purposes of describing optimal updating (learning) by the least squares rule:

$$y = P\left(y\,|1, x_1\right) + P\left[y - P\left(y\,|1, x_1\right)|x_2 - P\left(x_2\,|1, x_1\right)\right] + \varepsilon,$$

where $y - P\left(y\,|1, x_1\right)$ is interpreted as the prediction error of $y$ given $x_1$, and $x_2 - P\left(x_2\,|1, x_1\right)$ is interpreted as the prediction error of $x_2$ given $x_1$.

If initially we have data only on a random variable $x_1$, the linear least squares estimates of $y$ and $x_2$ are $P\left(y\,|1, x_1\right)$ and $P\left(x_2\,|1, x_1\right)$ respectively:

$$P\left(y\,|1, x_1\right) = b_0 + b_1 x_1 + b_2 P\left(x_2\,|1, x_1\right). \qquad (4.5.25)$$

Intuitively, we forecast $y$ based on two components: (i) $b_1 x_1$ alone, and (ii) $P\left(x_2\,|1, x_1\right)$, that is, the forecast of $x_2$ given $x_1$. When an observation $x_2$ becomes available, according to equation (4.5.24), the estimate of $y$ can be improved by adding to $P\left(y\,|1, x_1\right)$, and the projection of unobserved "forecast error" $y - P\left(y\,|1, x_1\right)$ on the observed forecast error $x_2 - P\left(x_2\,|1, x_1\right)$.

In equation (4.5.24), $P\left(y\,|1, x_1\right)$ is interpreted as the original forecast, $y - P\left(y\,|1, x_1\right)$

is the forecast error of $y$, given $x_1$, and $x_2 - P(x_2 |1, x_1)$ is the forecast error of $x_2$ to forecast the forecast error of $y$ given $x_1$. The above concept can be summarized in a general expression:

$$P(y|\Omega, x) = P(y|\Omega) + P\{y - P(y|\Omega)|x - P(x|\Omega)\},$$

where $\Omega$ is the original information, $x$ is the new information, and $P(y|\Omega)$ is the prediction of $y$ using the original information. The projection, $P\{y - P(y|\Omega)|x - P(x|\Omega)\}$, indicates new information has become available to update the forecast. It is no longer necessary to use the original information to make predictions. In other words, one can obtain $x - P(x|\Omega)$, the difference between the new information and the "forecasted" new information, to predict the error of $y$: $y - P(y|\Omega)$.

### 4.5.3 Signal-Extraction Problems

Based on these tools it can now be demonstrated how conditional expectations with recursive projections has a mutually reinforcing relation with measurement error and error-in-variables regression. There are many examples of this "EITM-like" linkage and they generally fall under the umbrella of signal extraction problems. Consider the following examples.[9]

#### Application 1: Measurement Error

Suppose a random variable $x^*$ is an indepenent variable. However, measurement error, $e$, exists so that the variable $x$ is only observable:

$$x = x^* + e, \tag{4.5.26}$$

where $x^*$ and $e$ have zero mean, finite variance, and $Ex^*e = 0$. Therefore, the projection of $x^*$ given an observable $x$ is:

$$P(x^* |1, x) = b_0 + b_1 x.$$

Based on the least squares and the orthogonality conditions, we have:

$$b_1 = \frac{E(xx^*)}{Ex^2} = \frac{E[(x^* + e)x^*]}{E(x^* + e)^2} = \frac{E(x^*)^2}{E(x^*)^2 + Ee^2}, \tag{4.5.27}$$

[9] The first example can be found in Sargent (1987: 229).

and

$$b_0 = 0. \tag{4.5.28}$$

The projection of $x^*$ given $x$ can be written as:

$$P\left(x^* \left| 1, x\right.\right) = \frac{E\left(x^*\right)^2}{E\left(x^*\right)^2 + Ee^2} x, \tag{4.5.29}$$

where $b_1 = \frac{E(x^*)^2}{E(x^*)^2+Ee^2}$ is between zero and one.

The "measurement error" attenuation is now transparent. As $\frac{E(x^*)^2}{Ee^2}$ increases, $b_1 \rightarrow 1$: the greater $\frac{E(x^*)^2}{Ee^2}$ is, the larger the fraction of variance in $x$ is due to variations in the actual value (i.e., $E\left(x^*\right)^2$).

### Application 2: The Lucas (1973) Model (Relative-General Uncertainty)

An additional application is the case where there is general-relative confusion. Here, using Lucas's (1973) supply curve, producers observe the prices of their own goods ($p_i$) but not the aggregate price level ($p$).

The relative price of good $i$ is $r_i$ is defined as:

$$r_i = p_i - p. \tag{4.5.30}$$

The observable price $p_i$ is a sum of the aggregate price level and its relative price:

$$p_i = p + (p_i - p) = p + r_i. \tag{4.5.31}$$

Assume each producer wants to estimate the real relative price $r_i$ to determine their output level. However, they do not observe the general price level. As a result, the producer forms the following projection of $r_i$ given $p_i$:

$$P\left(r_i \left| p_i\right.\right) = b_0 + b_1 p_i. \tag{4.5.32}$$

According to (4.5.32), the values of $b_0$ and $b_1$ are:

$$b_0 = E\left(r_i\right) - b_1 E\left(p_i\right) = E\left(p_i - p\right) - b_1 E\left(p_i\right) = -b_1 E\left(p_i\right), \tag{4.5.33}$$

and:

$$\begin{aligned} b_1 &= \frac{E\left[r_i - E\left(r_i\right)\right]\left[p_i - E\left(p_i\right)\right]}{E\left[p_i - E\left(p_i\right)\right]^2} \\ &= \frac{E\left[r_i - E\left(r_i\right)\right]\left[\left(p + r_i\right) - E\left(p + r_i\right)\right]}{E\left[\left(p + r_i\right) - E\left(p + r_i\right)\right]^2} \\ &= \frac{Er_i^2}{Er_i^2 + Ep^2} \qquad (4.5.34) \\ &= \frac{v_r}{v_r + v_p}, \qquad (4.5.35) \end{aligned}$$

where $v_r = Er_i^2$ is the variance of the real relative price, and $v_p = Ep^2$ is the variance of the general price level. Inserting the values of $b_0 = -b_1 E\left(p\right)$ and $b_1$ into the projection (4.5.32), we have:

$$P\left(r_i \left| p_i \right.\right) = b_1\left[p_i - E\left(p\right)\right] = \frac{v_r}{v_r + v_p}\left[p_i - E\left(p\right)\right]. \qquad (4.5.36)$$

Next factoring in an output component — the labor supply — and showing it is increasing with the projected relative price we have:

$$l_i = \beta E\left(r_i \left| p_i \right.\right), \qquad (4.5.37)$$

and:

$$l_i = \frac{\beta v_r}{v_r + v_p}\left[p_i - E\left(p\right)\right]. \qquad (4.5.38)$$

If aggregated over all producers and workers, the average aggregate production is:

$$y = b\left[p - E\left(p\right)\right], \qquad (4.5.39)$$

where $b = \frac{\beta v_r}{v_r + v_p}$.

Lucas's (1973) empirical tests are directed at output-inflation trade-offs in a variety of countries.[10] Equation (4.5.39) represents the mechanism of the general-relative price confusion:

$$y = \beta\frac{v_r}{v_r + v_p}\left[p - E\left(p\right)\right], \qquad (4.5.40)$$

where $v_p$ is the variance of the nominal demand shock, and $p - E\left(p\right)$ is the nominal demand shock.

[10] The empirical tests are described in Romer (1996: 253-254).

**Application 3: The Derivation of the Optimal Forecast of Political Incumbent Competence**

This application uses the techniques of recursive projections and signal extraction to derive the optimal forecast of political incumbent competence in equation (4.3.1). In Section 4.2, the public's conditional expectations of an incumbent's competence at time $t+1$ (as expressed in equations (4.2.7) and(4.2.8)) is:

$$\begin{aligned} E_t(\eta_{t+1}) &= E_t(\mu_{t+1}) + \rho E(\mu_t|\hat{y}_t - \hat{y}^n - \rho\mu_{t-1}) \\ E_t(\eta_{t+1}) &= \rho E(\mu_t|\mu_t + \xi_t), \end{aligned} \tag{4.5.41}$$

where $E_t(\mu_{t+1}) = 0$.

Using recursive projections, voters forecast $\mu_t$ using $\mu_t + \xi_t$ and obtain the forecasting coefficients $a_0$ and $a_1$:

$$P(\mu_t|\mu_t + \xi_t) = a_0 + a_1(\mu_t + \xi_t), \tag{4.5.42}$$

with:

$$\begin{aligned} a_1 &= \frac{cov(\mu_t, \mu_t + \xi_t)}{var(\mu_t + \xi_t)} \\ &= \frac{E(\mu_t(\mu_t + \xi_t))}{E[(\mu_t + \xi_t)(\mu_t + \xi_t)]} \\ &= \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2}, \end{aligned}$$

and:

$$a_0 = E(\mu_t) - a_1 E(\mu_t + \xi_t) = 0,$$

where $E(\mu_t) = E(\mu_t + \xi_t) = 0$. The projection for $\mu_t$ is written as:

$$\begin{aligned} E_t(\mu_t|\mu_t + \xi_t) = P(\mu_t|\mu_t + \xi_t) &= a_0 + a_1(\mu_t + \xi_t) \\ &= \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2}(\mu_t + \xi_t). \end{aligned} \tag{4.5.43}$$

Placing equation (4.2.5) into equation (4.5.43):

$$E_t(\mu_t|\mu_t + \xi_t) = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2}(\hat{y}_t - \hat{y}^n - \rho\mu_{t-1}). \tag{4.5.44}$$

The final step is inserting equation (4.5.44) in equation (4.5.41) and obtaining the optimal forecast of competence at $t+1$:

$$\begin{aligned} E_t\left(\eta_{t+1}\right) &= \rho E\left(\mu_t | \mu_t + \xi_t\right) \\ &= \rho \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2}\left(\hat{y}_t - \hat{y}^n - \rho\mu_{t-1}\right). \end{aligned}$$

This is the expression in equation (4.3.1).

# Chapter 5

# Strategists and Party Identification

An important debate in political science centers on the persistence of party identification, termed *macropartisanship* (See Erikson, MacKuen, and Stimson (2002: 109-151)). Using Clarke and Granato's (2004) example of an EITM formulation it is assumed political campaign advertisements influence the public's party identification. In particular, party identification persistence can be influenced by a rival political party strategist's use of campaign advertisements. Consequently, shocks to macropartisanship can either be amplified or die out quickly depending on the rival political strategist's actions.

The EITM linkage is the relation between the behavioral concept of expectations and the empirical concept of persistence. Empirical tools for this example require a basic understanding of autoregressive processes. Formal tools include an extended discussion of conditional expectations modeling, difference equations (various orders), their solution procedures, and relevant stability conditions.

## 5.1 Step 1: Relating Behavioral and Applied Statistical Concepts: Expectations and Persistence

Clarke and Granato (2004) relate agent expectations to the persistence of agent behavior. It is demonstrated how a rival political strategist can use campaign advertisements to influence aggregate persistence in party identification.

The model is based on three equations. Each citizen ($i$) is subject to an event ($j$) at time ($t$). Clarke and Granato then aggregate across individuals and events so the notation will only have the subscript $t$.

$$M_t = a_1 M_{t-1} + a_2 E_{t-1} M_t + a_3 F_t + u_{1t} \tag{5.1.1}$$

$$F_t = b_1 F_{t-1} + b_2 A_t + u_{2t} \tag{5.1.2}$$

$$A_t = c_1 A_{t-1} + c_2 \left(M_t - M^*\right) + c_3 F_{t-1}. \tag{5.1.3}$$

The first equation (5.1.1) specifies what influences aggregate party identification ($M_t$). The variable $M_{t-1}$ accounts for the empirical concept of persistence. The behavioral concept in the model is citizen expectations. It is assumed citizens have an expectation of what portion of the population identifier with a particular political party ($E_{t-1}M_t$). In forming their expectations, citizens use all available and relevant information (up to time $t-1$) as specified in this model (i.e., rational expectations).[1] Further party identification depends on how favorably a citizen views the national party ($F_t$). Finally, party identification can be subject to unanticipated stochastic shocks (realignments) ($u_{1t}$) where $u_{1t} \sim N\left(0, \sigma^2_{u_{1t}}\right)$. These relations are assumed to be positive — $a_1, a_2, a_3 \geq 0$.

Equation (5.1.2) represents citizens' impression and sense of favorability about a political party ($F_t$). In this equation, favorability is a linear function of the lag of favorability ($F_{t-1}$) and an advertising resource variable ($A_t$). $u_{2t}$ is a stochastic shock representing unanticipated events (uncertainty), where $u_{2t} \sim N\left(0, \sigma^2_{u_{2t}}\right)$. The parameter $b_1 \geq 0$, while $b_2 \gtreqless 0$ depending on the tone and content of the advertisement.

Equation (5.1.3) presents the contingency plan or rule that (rival) political strategists use. Clarke and Granato posit that political strategists track their previous period's advertising resource expenditures ($A_{t-1}$) and react to that period's favorability rating for the (rival) national party ($F_{t-1}$). The strategists also base their current expenditure of advertisement resources on the degree to which macropartisanship ($M_t$) approximates a prespecified and desired target ($M^*$).

Ideally, political strategists want $(M_t - M^*) = 0$. The parameters $c_1$ and $c_3$ are positive. The parameter $c_2$ is countercyclical ($-1 \leq c_2 < 0$): it reflects a willingness to increase or conserve their advertising resources depending on whether macropar-

[1] Rational expectations is only one type of expectation modeling. It has particular implications for how fast citizen's adjust to new information, which in this case is political adverstisements. See the Appendix, Section 5.5.2 for a discussion on the speed of adjustment.

tisanship is above or below the target.

## 5.2 Step 2: Analogues for Expectations and Persistence[2]

The reduced form for macropartisanship is determined by substituting (5.1.3) into (5.1.2). Note that there is an autoregressive component ($\Theta_1 M_{t-1}$) in the reduced form for macropartisanship:

$$M_t = \Theta_0 + \Theta_1 M_{t-1} + \Theta_2 E_{t-1} M_t + \Theta_3 A_{t-1} + \Theta_4 F_{t-1} + \varepsilon_t^*, \tag{5.2.1}$$

where:

$$\Theta_0 = \frac{b_2 c_1 Y^*}{(1 - a_2 b_2 c_2)},$$

$$\Theta_1 = \frac{a_1}{(1 - a_2 b_2 c_2)},$$

$$\Theta_2 = \frac{a_2}{(1 - a_2 b_2 c_2)},$$

$$\Theta_3 = \frac{b_2 c_1}{(1 - a_2 b_2 c_2)},$$

$$\Theta_4 = \frac{(b_1 + b_2 c_3)}{(1 - a_2 b_2 c_2)},$$

$$\varepsilon_t^* = \frac{u_{2t} + u_{1t}}{(1 - a_2 b_2 c_2)}.$$

The system is simplified to a model of macropartisanship that depends on lagged macropartisanship and also a conditional expectation at time $t-1$ of current macropartisanship. This lagged dependent variable is the analogue for *persistence* (See the Appendix, Section 5.5.1). Note that the prior values of advertising and favorability may also have an effect.

Because (5.2.1) possesses a conditional expectations operator we must make it a function of other variables (not operators) (See the Appendix, Section 5.5.2). In this example, "closing the model" and finding the rational expectations equilibrium (REE) involves taking the conditional expectation at time $t-1$ of equation (5.2.1)

[2] In this example, the applied statistical analogue is an autoregressive process and the formal analogue is conditional expectations. The description of theses analogues and the tools to develop these analogues can be found in the Appendix, Sections 5.5.1, and 5.5.2.

and then substituting this result back into equation (5.2.1):

$$M_t = \Pi_1 + \Pi_2 M_{t-1} + \Pi_3 A_{t-2} + \Pi_4 F_{t-2} + \xi_t'. \tag{5.2.2}$$

Equation (6.3.1) is the minimum state variable (MSV) solution (McCallum, 1983) for macropartisanship.[3] Macropartisanship ($M_t$) depends also on its past history, the autoregressive component, ($M_{t-1}$).

## 5.3 Step 3: Unifying and Evaluating the Analogues

The persistence of macropartisanship ($\Pi_2$) is now shown as dependent on the persistence and willingness of rival political strategists to maintain a rival macropartisanship target ($c_2$). In other words, the EITM linkage is the MSV with the AR(1) component in (6.3.1).

The linkage is this case is the reduced form AR(1) coefficient expression, $\Pi_2$:

$$\Pi_2 = \frac{a_1 + b_2 c_2 \left(c_1 + b_1 + b_2 c_3\right)}{1 - b_2 c_2 - a_2}. \tag{5.3.1}$$

Taking the derivative of (5.3.1) with respect to ($c_2$) and finding the following relation we have:

$$\frac{\partial \Pi_2}{\partial c_2} = \frac{b_2 \left(a_1 + (1 - a_2) A\right)}{\left(1 - a_2 - b_2 c_2\right)^2}, \tag{5.3.2}$$

where $A = (b_1 + c_1 + b_2 c_3)$. Given the assumptions about the signs of the coefficients in the model, the numerator is positive when $a_2 < 1$. Therefore, under these conditions, the relation is positive $\left(\frac{\partial \Pi_2}{\partial c_2} > 0\right)$.[4]

The relation between $c_2$ and $\Pi_2$ is demonstrated in Figure 5.3.1. Using the following values: $a_1 = a_2 = b_1 = b_2 = c_1 = c_3 = 0.5$. The parameter $c_2$ ranges from 0.0 to -1.0. The value of $c_2$ is varied between 0.0 and $-1.0$ and we find that the persistence (autocorrelation) in macropartisanship ($\Pi_2$ ) — all things equal — is zero when $c_2 = -0.8$.

On the other hand, macropartisanship persistence increases ($\Pi_2 \to 1.0$) when rival political strategists fail to react ($c_2 \to 0.0$) to deviations from their prespecified target. *A conclusion derived from this model is that negative advertisements from*

[3]Note: $\Pi_1 = \left(\frac{\Theta_0}{1-\Theta_2} - \left[\frac{\Theta_3}{1-\Theta_2} - \frac{\Theta_4}{1-\Theta_2} b_2\right] c_2 Y^*\right)$, $\Pi_2 = \left(\frac{\Theta_1}{1-\Theta_2} + \left[\frac{\Theta_3}{1-\Theta_2} + \frac{\Theta_4}{1-\Theta_2} b_2\right] c_2\right)$, $\Pi_3 = \left(\left[\frac{\Theta_3}{1-\Theta_2} + \frac{\Theta_4}{1-\Theta_2} b_2\right] c_1\right)$, $\Pi_4 = \left(\frac{\Theta_3}{1-\Theta_2} c_3 + \frac{\Theta_4}{1-\Theta_2}\left[b_1 + b_2 c_3\right]\right)$,and $\xi_t' = \left(\frac{\Theta_4}{1-\Theta_2} u_{2t} + \varepsilon_t^*\right)$.

[4]Note, for $\frac{\partial \Pi_2}{\partial c_2} > 0 : a_2 > \frac{-A}{1-A}$ is the *necessary condition* and $a_2 < 1$is the *sufficient condition.*

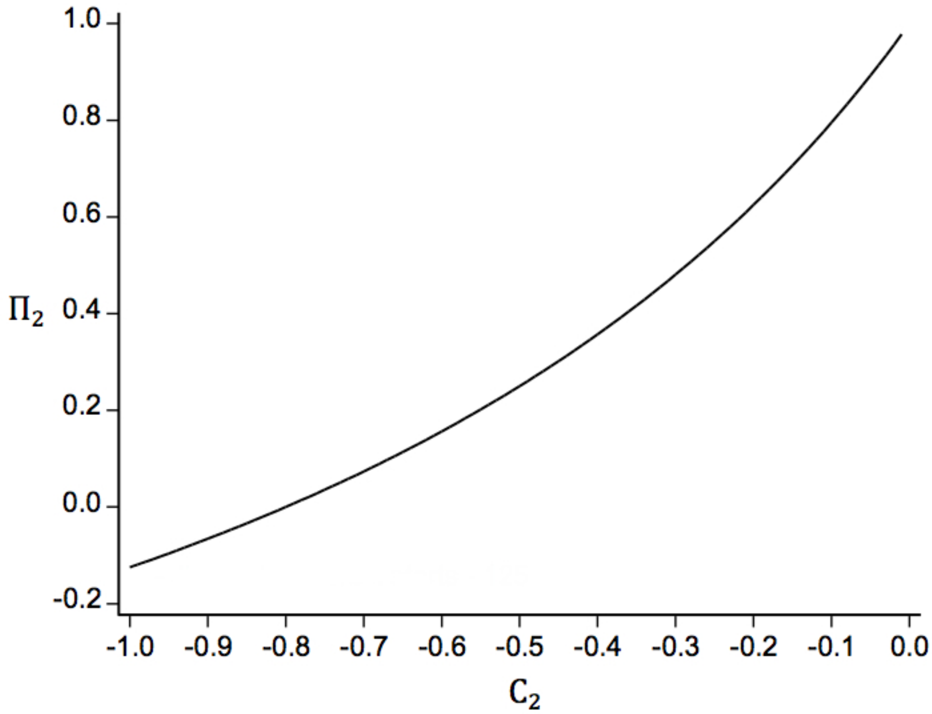


Figure 5.3.1: Simulation Results

*rival political parties can influence the persistence of their opponents national party identification.*

## 5.4 Leveraging EITM and Extending the Model

Among the ways to extend the model is to use an alternative way to model citizen expectations. In this model the use of RE can limit the complexities of expectation formation. Alternatives could include the use of expectations formation where the public updates at a far slower pace and using information sets that are far more limited. There is also the question of data. Currently, it is difficult to measure and link specific advertisements to response in real time. One way to deal with this particular design concern is to use experiments and ascertain the treatment and response effects with lags of far shorter duration.

## 5.5 Appendix

To assess the degree of persistence in a variable, autoregressive estimation is the most frequently used technique in empirical research. However, in EITM, persistence is behaviorally based. Persistence can be due to many things including brand loyalty and party loyalty. Persistence might also arise because of habit formation, meaning choosing an option in the present period directly increases the probability of choosing it again in future periods.[5]

While the example in this chapter is about party identification — and an appropriate province of political science — the foundations for developing the tools to model expectations is drawn from economics. Since Muth (1961), a great deal of theoretical research uses RE. RE is a particular equilibrium concept representing the optimal choice of the decision rule used by agents depending on the choices of others. "An RE equilibrium (REE) imposes a consistency condition that each agents' choice is a best response to the choices of others" (Evans and Honkapohja 2001: 11) . Muth (1961) defined expectations to be rational if they were formed according to the model describing the behavior.

It is also possible to relate RE to autoregressive processes. Let there be a time series $(z_t)$ generated by a first-order autoregression:

$$z_t = \lambda_0 + \lambda_1 z_{t-1} + \nu_t, \quad for\ z_{t-1} \in I_{t-1} \tag{5.5.1}$$

where $\nu_t$ are independent $N(0,\, \sigma_\nu^2)$ random variables, $|\lambda_1| < 1$, and $I_{t-1}$ represents all possible information at period $t-1$. If the agent acts rationally, the equation (5.5.1) is treated as the data generating process (DGP). The mathematical expression of RE corresponding to equation (5.5.1) is:

$$E[z_t|z_{t-1}] = z_t^e = \lambda_0 + \lambda_1 z_{t-1}. \tag{5.5.2}$$

With this simple linkage in mind, the tools in this chapter are used to establish a transparent and testable relation between expectations and persistence. The applied statistical tools provide a basic understanding of:

- Autoregressive processes.

The formal tools include a presentation of:

- Conditional expectations (naive, adaptive, and rational).

[5] See Shachar (1992) for the role of habit formation in voting decisions.

- Difference equations.
- Method of undetermined coefficients (minimum state variable procedure).

These tools are used in various applications for models where RE is assumed just as in this chapter.

### 5.5.1 Empirical Analogues

#### Autoregressive Processes

An autoregressive process for the time series of $Y_t$ is one in which the current value of $Y_t$ depends upon past values of $Y_t$ and a stochastic disturbance term.[6] A convenient notation for an autoregressive process is $AR(p)$, where $p$ denotes the maximum lag of $Y_t$ , upon which $Y_t$ depends. Note that in an $AR(p)$ process, lags are assumed to be present from 1 through to $p$.

For simplicity, we now use $AR(1)$ for illustration. An $AR(1)$ process represents a first-order autoregressive process where $Y_t$ depends upon $Y_{t-1}$ and a disturbance term, $\varepsilon_t$:

$$Y_t = \phi_1 Y_{t-1} + \varepsilon_t, \tag{5.5.3}$$

where $\varepsilon_t$ is a white noise that has zero mean, constant variance, and zero autocorrelation. The autoregressive parameter $\phi_1$ in equation (5.5.3) can take on values with distinct empirical implications bearing on the persistence of a process. In particular, if $\phi_1 > 1$, the process is explosive meaning $Y_t$ will grow without limit.

[6] Time series data are discretely ordered by some period. They differ from cross-sectional data in that unlike their cross-sectional cousin, time series are a sequence of data points of the same entity over a period of time. For political science, examples include presidential approval and macropartisanship, while in economics, many macroeconomic data, such as gross domestic product and unemployment rates, are time series. A key property of time series data is stationarity. The consequences of having stationary processes is not trivial. In fact, it is a crucial requirement in that, among other things, most probability moments — the mean, the variance — and all the constituent statistics that derive from these moments are based on the assumption of a stationary time series. No valid inference is achievable absent some assurance that the data and model are stationary. The reason is that non-stationary data affects the moments (mean, variance, for example) of the series and these moments are used in all sorts of inferential statistics such as the t- and F-test. With this in mind, an intuitive definition for stationarity is:

> A data series (or model) is stationary if there is no systematic change in the mean (e.g., no trend), no systematic stochastic variation, and if strict periodic variations (seasonal) are stable. Time plays no role in the sample moments.

**Random Walk Processes and the Persistence of Random Shocks**

A special case arises when $\phi_1 = 1$. In this case, equation (5.5.3) can be written as:

$$Y_t = Y_{t-1} + \varepsilon_t. \tag{5.5.4}$$

Equation (5.5.4) is termed a "pure random walk" process. A pure random walk process is a best guess of $Y_{t+1}$, given information at period $t$, is $Y_t$. The relation between a random walk process and the "persistence" of random shocks is also of importance.

To see this relation, consider an $AR(1)$ process with a unit root, $\phi_1 = 1$ (i.e., equation (5.5.4)). If the pure random walk process of equation (5.5.4) starts at $t = 1$, the process then is $Y_1 = Y_0 + \varepsilon_1$. In the next period, $t = 2$, the process is $Y_2 = Y_1 + \varepsilon_2 = (Y_0 + \varepsilon_1) + \varepsilon_2$. Generalizing:

$$Y_t = Y_0 + \sum_{i=1}^{t} \varepsilon_t. \tag{5.5.5}$$

Equation (5.5.5) indicates the impact of a particular shock persists and will never die out. Also, it can be demonstrated from equation (5.5.5) that the mean value of $Y_t$ wanders over time.

Now, using equation (5.5.3), trace how persistence evolves if we have a stationary process, $0 < \phi_1 < 1$. The result is the shock does die out over time. The process is also mean reverting. To illustrate this, consider an $AR(1)$ process with $\phi_1 = 0.5$:

$$Y_t = 0.5Y_{t-1} + \varepsilon_t. \tag{5.5.6}$$

If we start at $t = 1$, the process is: $Y_1 = 0.5Y_0 + \varepsilon_1$. In successive periods we have:

$$\begin{aligned} Y_2 &= 0.5Y_1 + \varepsilon_2 \\ &= 0.5\,(0.5Y_0 + \varepsilon_1) + \varepsilon_2 \\ &= 0.5^2 Y_0 + 0.5\varepsilon_1 + \varepsilon_2, \end{aligned}$$

and

$$\begin{aligned} Y_3 &= 0.5Y_2 + \varepsilon_3 \\ &= 0.5\left[0.5^2 Y_0 + 0.5\varepsilon_1 + \varepsilon_2\right] + \varepsilon_3 \\ &= 0.5^3 Y_0 + 0.5^2\varepsilon_1 + 0.5\varepsilon_2 + \varepsilon_3. \end{aligned}$$

In general:

$$Y_t = 0.5^t Y_0 + \sum_{i=1}^{t} 0.5^{t-i} \varepsilon_i. \tag{5.5.7}$$

Equation (5.5.7) indicates the effect of a particular shock, say at period 1, on all the subsequent periods does die out when $t \to \infty$ (i.e, the shock is not persistent).[7]

### 5.5.2 Formal Analogues

### Conditional Expectations

The use of expectations in economic models has a long history. While RE is featured in this particular chapter, there are many ways to model expectations and each method has distinct behavioral implications. Here background is provided on three approaches:

1. Naive or static expectations.

2. Adaptive expectations.

3. Rational expectations.

The solution procedures for RE are then presented. Because the development of expectations modeling was largely the creation of economics, the variables and examples are economic in nature. We stay true to those original examples and the variables used. However, as this chapter demonstrates, the application of these tools can be used for any social science question where expectations are a behavioral and theoretical component.

### Static Expectations: The Cobweb Model[8]

Static expectations (also called naive expectations) assume agents form their expectations of a variable based on their previous period ($t - 1$) observation of the variable. An example illustrating the use of static expectations is the traditional cobweb model which was used to determine the dynamic process of prices in agricultural markets.

[7]Autoregressive processes can be estimated using ordinary least squares (OLS). See Box and Jenkins (1970, 1976), for an extensive discussion on the estimation of autoregressive processes. A comprehensive discussion of time series methods can be found in Hamilton (1994). Also, Johnston and DiNardo (1996) provide a basic framework in time series methods, within the broader context of econometric methods.

[8]See Enders (2009) for background material for the following sections.

The cobweb model consists of demand and supply curves, respectively:

$$q_t^d = \alpha - \beta p_t + \epsilon_t^d, \tag{5.5.8}$$

and:

$$q_t^s = \gamma + \lambda p_t^e + \epsilon_t^s, \tag{5.5.9}$$

where $\beta > 0$, $\lambda > 0$, $\alpha > \gamma > 0$. $\epsilon_t^d \sim iid\left(0, \sigma_{\epsilon^d}^2\right)$ and $\epsilon_t^s \sim iid\left(0, \sigma_{\epsilon^s}^2\right)$ are stochastic demand and supply shocks with zero mean and constant variance, respectively.

Equation (5.5.8) is a demand schedule where consumers decide the level of quantity demanded $\left(q_t^d\right)$ given the current price level in the market $(p_t)$ and other stochastic factors $\left(\epsilon_t^d\right)$ at time $t$. From equation (5.5.9), we assume producers make decisions on the production level $(q_t^s)$ based on *the expected price level* at time $t$, $p_t^e$. Since the actual market price $p_t$ is not revealed to producers until goods have been produced in the market, producers make a decision on the level of production by forecasting the market price.

The market equilibrium, where $q_t^d = q_t^s$, gives us the dynamic process of the price level:

$$p_t = \left[\frac{\alpha - \gamma}{\beta}\right] - \left(\frac{\lambda}{\beta}\right) p_t^e + \left[\frac{\epsilon_t^d - \epsilon_t^s}{\beta}\right]. \tag{5.5.10}$$

Equation (5.5.10) is called the *cobweb model*: the current price level $(p_t)$ depends on the expected price level $(p_t^e)$ and a composition of stochastic shocks. Producers form static expectations where they choose the level of production $q_t^s$ at time $t$ by observing the previous price level at time $t-1$ (i.e., $p_t^e = p_{t-1}$). Substituting $p_t^e = p_{t-1}$ into equation (5.5.10):

$$p_t = \left[\frac{\alpha - \gamma}{\beta}\right] - \left(\frac{\lambda}{\beta}\right) p_{t-1} + \left[\frac{\epsilon_t^d - \epsilon_t^s}{\beta}\right]. \tag{5.5.11}$$

Equation (5.5.11) shows the current price level is determined by the past price level and stochastic shocks. Since the initial price level $p_t$ is not in a stationary equilibrium, the price approaches the equilibrium $p^*$ in the long run when certain conditions exist. In this model, $\left|\frac{\lambda}{\beta}\right| < 1$: $\lim_{t\to\infty} p_t = p^*$. The converging process is shown in Figure (5.5.1).

## The Use of Difference Equations

The result in Figure 5.5.1 can be demonstrated using stochastic difference equations. Equation (5.5.11) is also called a stochastic first-order difference equation with a

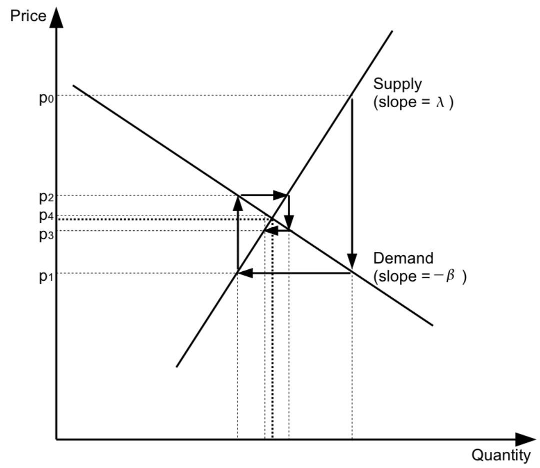


Figure 5.5.1: Cobweb Model with Static Expectation Formation

constant. The equation (5.5.2) can be presented in a simpler form:

$$p_t = a + bp_{t-1} + e_t, \tag{5.5.12}$$

where $a = \frac{\alpha-\gamma}{\beta}$, $b = -\frac{\lambda}{\beta}$, and $e_t = \frac{\epsilon_t^d - \epsilon_t^s}{\beta}$. To see the sequence of the price level we solve by iteration. Assuming the initial price level is $p_{t=0} = p_0$, the price level at time $t = 1$ is:

$$p_1 = a + bp_0 + e_1.$$

Using the above equation, we solve for $p_2$:

$$\begin{aligned} p_2 &= a + bp_1 + e_2 \\ &= a + b\left(a + bp_0 + e_1\right) + e_2 \\ &= a + ab + b^2p_0 + be_1 + e_2. \end{aligned}$$

With a similar substitution, $p_3$ is:

$$\begin{aligned} p_3 &= a + bp_2 + e_3 \\ &= a + b\left(a + ab + b^2p_0 + be_1 + e_2\right) + e_3 \\ &= a + ab + ab^2 + b^3p_0 + b^2e_1 + be_2 + e_3. \end{aligned} \tag{5.5.13}$$

If we iterate the equation $n$ times, we have (for $n \geq 1$):

$$p_n = a\sum_{i=0}^{n-1} b^i + b^n p_0 + \sum_{i=0}^{n-1} b^i e_{n-i}, \tag{5.5.14}$$

and by extension we can show if $t = n = 3$, then:

$$\begin{aligned}
p_3 &= a\sum_{i=0}^{3-1} b^i + b^3 p_0 + \sum_{i=0}^{3-1} b^i e_{3-i} \\
&= a\sum_{i=0}^{2} b^i + b^3 p_0 + \sum_{i=0}^{2} b^i e_{3-i} \\
&= a\left(b^0 + b^1 + b^2\right) + b^3 p_0 + \left(b^0 e_{3-0} + b^1 e_{3-1} + b^2 e_{3-2}\right) \\
&= a + ab + ab^2 + b^3 p_0 + e_3 + be_2 + b^2 e_{1.} \qquad (5.5.15)
\end{aligned}$$

Note equations (5.5.13) and (5.5.15) are identical.

Using equation (5.5.14), the current price level $p_t$ depends on the initial level $p_0$ and the sequence of stochastic shocks $\{e_i\}_{i=1}^{t}$. Assuming $|b| < 1$, then $\lim_{n\to\infty} b^n = 0$, and $\lim_{n\to\infty}\left(b^0 + b^1 + \cdots + b^n\right) = \lim_{n\to\infty}\sum_{i=0}^{n} b^i = \frac{1}{1-b}$. Therefore, in the long run, the price level equals:

$$p_{n\to\infty} = \frac{a}{1-b} + \sum_{i=0}^{\infty} b^i e_{n-i}. \qquad (5.5.16)$$

Equations (5.5.14) and (5.5.16) show transitory and stationary levels of price, respectively. Using the previous parameter values in the cobweb model: $a = \frac{\alpha-\gamma}{\beta} = 10$, and $b = -\frac{\lambda}{\beta} = -0.8$, we replicate Figure (5.5.2). Based on equations (5.5.14) and (5.5.16) by assuming $p_0 = p^* = 5.56$, $e_1 = 4.44$, and $e_i = 0$ for $i > 1$ we have:

$$\begin{aligned}
p_n &= 10\sum_{i=0}^{n-1}(-0.8)^i + (-0.8)^n p_0 + \sum_{i=0}^{n-1}(-0.8)^i e_{n-i} \\
&= 10\sum_{i=0}^{n-1}(-0.8)^i + (-0.8)^n (5.56) + (-0.8)^{n-1}(4.44).
\end{aligned}$$

As $t = n = \infty$, we have:

$$p_{n\to\infty} = p^* = 10\sum_{i=0}^{\infty}(-0.8)^i = \frac{10}{1-(-0.8)} = 5.56,$$

where:

$$\lim_{n\to\infty}(-0.8)^n = 0$$

and:

$$\lim_{n\to\infty}\left[(-0.8)^0 + (-0.8)^1 + \cdots + (-0.8)^n\right] = \lim_{n\to\infty}\sum_{i=0}^{n}(-0.8)^i = \frac{1}{[1-(-0.8)]}.$$

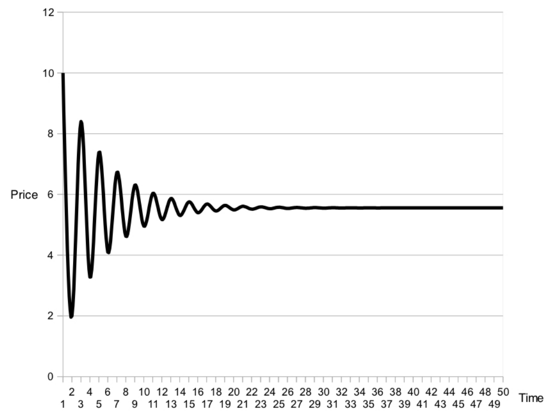


Figure 5.5.2: Price Movements in the Cobweb Model

**Expectational Errors: Speed of Adjustment**

An important issue in expectations modeling is the speed of adjustment. Naive or static expectation models contain agents who are relatively slow to adjust and update their forecasts. The movement of price over time is assumed by the constant terms and slope coefficients for demand and supply: $\alpha = 20$, $\gamma = 10$, $\beta = 1$, $\lambda = 0.8$, and $|\lambda/\beta| = 0.8 < 1$. Recall the stationary equilibrium $p^*$ is 5.56. At time $t = 1$ assume there is a stochastic shock — to either demand or supply (or both). This moves the price level from $p^* = 5.56$ to $p_1 = 10$. In Figure (5.5.2), we see that the price level fluctuates and approaches the equilibrium $p^* = 5.56$ in the long run.

Intuitively, if there is a one-time shock that shifts the demand or supply curve (or both) producers are assumed to *passively* determine the current level of production by observing the previous price level. A surplus or shortage, would exist while the market price deviated from the equilibrium until $t \to \infty$.

The behavioral implication when agents "naively" form expectations based on the past period's observation are as follows: agents systematically forecast above or below the actual value for an *extensive time period.* McCallum (1989) terms this sluggishness in error correction: *systematic expectational errors.*

## Adaptive Expectations

Of course, agents can actively revise their expectations when they realize their forecasting mistakes. This alternative formation of expectations is called adaptive expectations. The revision of current expectations is a function of the difference between the actual observation and the past expectation:

Expectational Revision = Function (Actual Observation - Past Expectation).

If agents make forecast errors in the previous period, then they revise their current expectations. Mathematically, adaptive expectations can be written as:

$$p_t^e - p_{t-1}^e = (1-\theta)\left(p_{t-1} - p_{t-1}^e\right), \tag{5.5.17}$$

where $0 < \theta < 1$ represents the degree (or speed) of expectational revision. When $\theta = 0$, $p_{t-1}^e = p_{t-1}$, agents do not revise their expectations: they have static expectations. On the other hand, if $\theta = 1$, $p_t^e = p_{t-1}^e$, then agents form their current expectations $(p_t^e)$ based on the past expectations $\left(p_{t-1}^e\right)$ only. By arranging equation (5.5.17), we have:

$$p_t^e = \theta p_{t-1}^e + (1-\theta)\, p_{t-1}. \tag{5.5.18}$$

Equation (5.5.18) also shows that the expectation of the current price $(p_t^e)$ is the weighted average of past expectation $\left(p_{t-1}^e\right)$ and the past observation $(p_{t-1})$. To recover the expected price level at time $t$, the method of iterations is applied. The expectations at time $t-1$ and $t-2$ is:

$$p_{t-1}^e = \theta p_{t-2}^e + (1-\theta)\, p_{t-2}, \tag{5.5.19}$$

and:

$$p_{t-2}^e = \theta p_{t-3}^e + (1-\theta)\, p_{t-3}, \tag{5.5.20}$$

respectively. Substituting (5.5.20) into (5.5.19) and then substituting it back to equation (5.5.18), we have:

$$\begin{aligned} p_t^e &= \theta\left\{\theta\left[\theta p_{t-3}^e + (1-\theta)\, p_{t-3}\right] + (1-\theta)\, p_{t-2}\right\} + (1-\theta)\, p_{t-1} \\ &= \theta^3 p_{t-3}^e + \theta^2 (1-\theta)\, p_{t-3} + \theta (1-\theta)\, p_{t-2} + (1-\theta)\, p_{t-1}. \end{aligned} \tag{5.5.21}$$

Iterating equation (5.5.18) $n$ times:

$$p_t^e = \theta^n p_{t-n}^e + (1-\theta) \sum_{i=1}^{n-1} \theta^{i-1} p_{t-i}.$$

If $n \to \infty$, then:

$$p_t^e = (1-\theta)\sum_{i=1}^{\infty}\theta^{i-1}p_{t-i}, \tag{5.5.22}$$

for $|\theta| < 1$.

Equation (5.5.18) shows the current expectation is the weighted average of the last period expectation and observations. An alternative interpretation based on equation (5.5.22) is that the expected price level for the current period is a weighted average of *all price levels* observed in the past (with geometrically declining weights).

Under adaptive expectations, agents make their forecast of a variable by weighting its past behavior (Cagan 1956; Friedman 1957; Nerlove 1958). However, just as with the assumption of static expectations, *systematic expectational errors* can still be generated. Unexpected stochastic shocks have *permanent* effects on future expectations formation.

This result is inconsistent with central tenets in microeconomic theory. If agents know that such errors are systematically generated, they have incentives to avoid them. For example, agents have the incentive to collect other (or even *all* available) information for improving the forecast of the observed variable. Theoretically, one way to avoid the problem of having agents make systematic errors is to assume they have rational expectations (RE) (Muth 1961; Lucas 1972, 1973).

## Rational Expectations

Under RE, agents are assumed to take conditional (mathematical) expectations of *all* relevant variables. Agents form their expectations according to all of the information available at time $t$. The behavioral implications are very different from static or adaptive expectations when it comes to the speed of correcting forecast errors. RE also has very different implications for persistence.

Mathematically, RE can be written as the projection:

$$p_{t+j}^e = E\left(p_{t+j}\,|I_t\right), \tag{5.5.23}$$

where $p_{t+j}^e$ is the subjective expectations of $p_{t+j}$ formed in time $t$, and $E\left(p_{t+j}\,|I_t\right)$ is a mathematical expectations of $p_{t+j}$ given the information $I_t$ available at time $t$. Statistically, $E\left(p_{t+j}\,|I_t\right)$ is interpreted as the mean of the conditional probability distribution of $p_{t+j}$ based on available information $I_t$ at time $t$. Equation (5.5.23) implies agents use *all* information available at time $t$ to forecast the variable of interest for time $t+j$.

More importantly, agents' ability to form a conditional probability distribution of $p_{t+j}$ also implies that agents "know" the structure of the model. For example, agents are able to form a conditional distribution of $p_t$ given the parameters of $\alpha$, $\beta$, $\gamma$, and $\lambda$ as known in equation 5.5.10. It is difficult to imagine agents can know the "true" model in the first place and then construct a probability distribution based on the model.[9]

As mentioned earlier, systematic expectational errors are generated when agents form their adaptive expectations given the related information available. These systematic expectational errors can be eliminated under RE. Defining the expectational error as the difference between actual observation at time $t+1$ and the expectation for time $t+1$:

$$p_{t+1} - p_{t+1}^{e}.$$

If agents systematically *over*-predict or *under*-predict the variable of interest, then the "average" of the expectational errors is either larger than or less than zero. Under RE there is no systematic forecast error.

To demonstrate this result under RE we calculate the expected value of the expectational errors as:

$$\begin{aligned} E\left(p_{t+1} - p_{t+1}^{e}\right) &= E\left[p_{t+1} - E\left(p_{t+1} \,|I_t\,\right)\right] \\ &= E\left(p_{t+1}\right) - E\left[E\left(p_{t+1} \,|I_t\,\right)\right] \\ &= E\left(p_{t+1}\right) - E\left(p_{t+1}\right) = 0, \end{aligned} \tag{5.5.24}$$

where $E\left[E\left(p_{t+1} \,|I_t\,\right)\right] = E\left(p_{t+1}\right)$ is the unconditional expectation of the conditional expectations of $p_{t+1}$, given the information set $I_t$. This is simply the unconditional expectation of $p_{t+1}$.[10] This result can also be explained by a statistical property

[9] An alternative assumption is agents "learn" the structure of the model over time by least squares to form optimal conditional expectations (Bray 1982; Bray and Savin 1986; Evans 1985; Marcet and Sargent 1986, 1987; Evans and Honkapohja 2001). This is called the *adaptive learning* approach and is discussed in chapters 6 and 7.

[10] To show $E\left[E\left(p_{t+1} \,|I_t\,\right)\right] = E\left(p_{t+1}\right)$, it is necessary to review some important statistical properties. Let us generalize the statements below and use a variable $X$. Assume $X$ is a random variable where its numerical values are randomly determined. For the discrete case, the variable $X$ has a set of $J$ random numeral values, $x_1, x_2, \ldots, x_J$. The probability of any numerical value, $x_j$, for $j = 1, 2, \ldots, J$, can be represented by a probability density function $f\left(x_j\right) = Prob\left\{X = x_j\right\} \geq 0$. Note that the sum of the probability for all possible numerical values is $\sum_{j=1}^{J} f\left(x_j\right) = 1$, and $f\left(x_k\right) = 0$, for any $x_k \notin \left\{x_1, \ldots x_J\right\}$. Based on the density function, we calculate the (unconditional) expected value of the random variable $X$:

$$E\left(X\right) = \sum_{j=1}^{J} x_j f\left(x_j\right). \tag{5.5.25}$$

called *the law of iterated expectations* [11]

The law of iterated expectations suggests that, given an information set $\Omega$ and a subset of information $\omega \subset \Omega$, for a variable of interest $x$, the conditional expectations of the conditional expectations of $x$, given a larger information set is just the

If $g(x_j)$ is defined as a function for any random value $x_j$ for all $j = 1, 2, \ldots, J$, then the expected value of $g(X)$ is:

$$E[g(X)] = \sum_{j=1}^{J} g(x_j) f(x_j). \tag{5.5.26}$$

Assuming there is another random variable $Y$ which has a set of $M$ random values, $y_1, y_2, \ldots, y_M$. Assuming further that $X$ and $Y$ are jointly distributed random variables such that the joint probability density function is $f(x_j, y_m) = Prob\{X = x_j \text{ and } Y = y_m\} \geq 0$, for $j = 1, 2, \ldots J$, and $m = 1, 2, \ldots, M$. Again, note $\sum_{m=1}^{M} \sum_{j=1}^{J} f(x_j, y_m) = 1$, and $f(x_k, y_h) = 0$, for any $x_k \notin \{x_1, \ldots x_J\}$ or $y_h \notin \{y_1, \ldots y_M\}$. Based on the joint density function $f(x_j, y_m)$, the single density function can be calculated for the random variable $X$ by summing up all joint probability of $f(x_j, y_m)$ for any given $x_j$:

$$f(x_j) = \sum_{m=1}^{M} f(x_j, y_m). \tag{5.5.27}$$

Similarly, the single density function can be derived for the random variable $Y$:

$$f(y_m) = \sum_{j=1}^{J} f(x_j, y_m). \tag{5.5.28}$$

In addition, if there is a multivariate function $g(x_j, y_m)$, then the expected value of $g(X, Y)$ is:

$$E[g(X,Y)] = \sum_{m=1}^{M} \sum_{j=1}^{J} g(x_j, y_m) f(x_j, y_m). \tag{5.5.29}$$

The last statistical property introduced is the conditional probability density function. This is defined as the conditional probability density function of $y$ given $x$ (subscripts are dropped for convenience) as:

$$f(y|x) = \frac{f(x,y)}{\sum_y f(x,y)} = \frac{f(x,y)}{f(x)}, \tag{5.5.30}$$

for $f(x) > 0$. As before, the conditional probability density function is the same form but now it is of $x$ given $y$:

$$f(x|y) = \frac{f(x,y)}{\sum_x f(x,y)} = \frac{f(x,y)}{f(y)}, \tag{5.5.31}$$

for $f(y) > 0$. Equation (5.5.30) shows the probability of any numerical value $y_m$ given a specific value of a random variable $X$. Therefore, we define the conditional expected value of $Y$ given $X$ as:

$$E(Y|X) = \sum_{y} y f(y|x). \tag{5.5.32}$$

Based on the above statistical properties, we are able to show that $E[E(Y|X)] = E(Y)$ by using the fact that $E[g(X,Y)] = \sum_y \sum_x g(x,y) f(x,y)$ in equation (5.5.29) and assuming that $E(Y|X) = \sum_y y f(y|x) = g(x,y)$ in equation (5.5.32). All we need to show is that $E[E(Y|X)] = E[g(X|Y)] = E(Y)$. This result validates $E[E(p_{t+1}|I_t)] = E(p_{t+1})$ in condition (5.5.24).

[11] Note that the expectation operator, $E(\cdot)$, is in linear form. The ideas of recursive expectations and the law of iterated expectations are demonstrated in the discussin of recursive projections in

conditional expectations of $x$ given a subset of information.[12] Mathematically:

$$E\left[E\left(x\left|\Omega\right.\right)\left|\omega\right.\right]=E\left(x\left|\omega\right.\right). \tag{5.5.33}$$

If the conditional expectation of $x$ is formed *over time* according to the available information, then equation (5.5.33) is rewritten as:

$$E\left[E\left(x_{t+1}\left|I_t\right.\right)\left|I_{t-1}\right.\right]=E\left(x_{t+1}\left|I_{t-1}\right.\right),$$

where $I_{t-1}\subset I_t$ for all $t$.

The second implication of RE is that the expectational errors are uncorrelated with any information available at time $t$: any information available to agents to form expectations at time $t$ *does not systematically* generate forecast errors. To demonstrate this result under RE, consider any information, $w_t$, where $w_t\in I_t$ :

$$\begin{aligned}
E\left[\left(p_{t+1}-p^e_{t+1}\right)w_t\right] &= E\left[\left(p_{t+1}-E\left(p_{t+1}\left|I_t\right.\right)\right)w_t\right]\\
&= E\left(p_{t+1}w_t\right)-E\left[E\left(p_{t+1}\left|I_t\right.\right)w_t\right]\\
&= E\left(p_{t+1}w_t\right)-E\left(p_{t+1}w_t\right)=0,
\end{aligned}$$

where $E\left[E\left(p_{t+1}\left|I_t\right.\right)w_t\right]=E\left[E\left(w_tp_{t+1}\left|I_t\right.\right)\right]=E\left(p_{t+1}w_t\right)$ and can be shown using the law of iterated expectations.

## Solving Rational Expectations Models

The solution procedures for RE models require a different approach.[13] RE models do not rely merely on a mathematical expectation, which is a summary measure (expected value). Rather, RE models are based on conditional expectations, which is a mathematical expectation with a modified probability distribution ("information set"). Solution procedures involve "closing the model" where unknown variables (i.e., expectations) are expressed in terms of other "known" variables. The method of undetermined coefficients is a particular solution process that closes a model, and the minimum state variable (MSV) solution is the simplest solution when using the method of undetermined coefficients.

Chapter 4, Appendix, Section 4.5.2.

[12] See Wooldridge (2008), Appendix B for an introductory discussion of conditional expectations.

[13] See Enders (2009), Chapter 1 for an introduction to these tools.

**Application 1: A Simple Cobweb Model**

The solution(s) for RE models include a REE. The REE imposes a consistency condition that an agent's choice is a best response to the choices made by others (Evans and Honkapohja 2001: 11). A simple way to demonstrate an REE is to use the cobweb model presented in equation (5.5.10). This particular equation shows that the movements of the price level at time $t$, (i.e., $p_t$) depend on the RE of the price level form at $t-1$, $\left(p_t^e = E\left(p_t|I_{t-1}\right)\right)$, and a composite stochastic error term, $e_t$:

$$p_t = a + bE\left(p_t|I_{t-1}\right) + e_t, \tag{5.5.34}$$

where $a = \frac{\alpha-\gamma}{\beta}$, $b = -\frac{\lambda}{\beta} < 0$, and $e_t = \frac{\epsilon_t^d - \epsilon_t^s}{\beta}$. Agents "know" the model when they form their conditional expectations so their expectations can be written as:

$$\begin{aligned}
E\left(p_t|I_{t-1}\right) &= E\left\{\left[a + bE\left(p_t|I_{t-1}\right) + e_t\right]|I_{t-1}\right\} \\
&= E\left(a|I_{t-1}\right) + E\left\{\left[bE\left(p_t|I_{t-1}\right)\right]|I_{t-1}\right\} + E\left(e_t|I_{t-1}\right) \\
&= a + bE\left[E\left(p_t|I_{t-1}\right)|I_{t-1}\right] \\
&= a + bE\left(p_t|I_{t-1}\right),
\end{aligned} \tag{5.5.35}$$

where $E\left(a|I_{t-1}\right) = a$, $E\left[E\left(p_t|I_{t-1}\right)|I_{t-1}\right] = E\left(p_t|I_{t-1}\right)$, and $E\left(e_t|I_{t-1}\right) = 0$.[14]

Following equation (5.5.35), the right-hand-side expression of $E\left(p_t|I_{t-1}\right)$ is moved to the left hand side of the equation:

$$(1-b)\,E\left(p_t|I_{t-1}\right) = a,$$

and $E\left(p_t|I_{t-1}\right)$ is equal to:

$$E\left(p_t|I_{t-1}\right) = \frac{a}{1-b}. \tag{5.5.36}$$

Equation (5.5.36) shows that agents form their conditional expectations of $p_t$ using the structural parameters $a$ and $b$. Inserting equation (5.5.36) into equation (5.5.34)

[14] These identities are based on the following. Since agents know the structure of the model, that is, the parameters of $a$ and $b$, the existing information set would not affect the parameter values. Therefore, $E\left(a|I_{t-1}\right) = a$. We can show, using the law of iterated expectations, that $E\left[E\left(p_t|I_{t-1}\right)|I_{t-1}\right] = E\left(p_t|I_{t-1}\right)$. Intuitively, if an agent forms an expectation of a conditional expectation (based on the same information set), the conditional expectation does not change since there is no added information. Lastly, the conditional expectational of a stochastic error term is zero, $\left(E\left(e_t|I_{t-1}\right) = 0\right)$, since an agent is unable to "forecast" white noise, $e_t$, given past information $I_{t-1}$.

yields:

$$\begin{aligned} p_t &= a + bE\left(p_t|I_{t-1}\right) + e_t \\ &= a + b\left[\frac{a}{1-b}\right] + e_t \\ &= \frac{a\left(1-b\right)+ab}{1-b} + e_t \\ p_t^{RE} &= \frac{a}{1-b} + e_t. \end{aligned} \tag{5.5.37}$$

Equation (5.5.37) is the REE and shows the movements of the price level over time given the RE in equation (5.5.36).

Furthermore, equations (5.5.36) and (5.5.37) also suggest the agents have made an optimal forecast in the model since the expectational error is simply stochastic noise:

$$p_t - E\left(p_t|I_{t-1}\right) = \left(\frac{a}{1-b} + e_t\right) - \frac{a}{1-b} = e_t. \tag{5.5.38}$$

The average "expected value" of the expectational effects is zero:

$$E\left[p_t - E\left(p_t|I_{t-1}\right)\right] = E\left(e_t\right) = 0.$$

**Application 2: A Cobweb Model with Observable Variables**

In the previous section, it was demonstrated that the variable of interest — the price level at time $t$ — depends on its conditional expectations and a composite stochastic error term in equation (5.5.34). Assuming there are other observable variable(s), $w_{t-1}$, influencing the quantity supplied in equation (5.5.9):

$$q_t^s = \gamma + \lambda p_t^e + \delta w_{t-1} + \epsilon_t^s. \tag{5.5.39}$$

For convenience $E_{t-1}$ is used as an expectation operator to represent the conditional expectations given information available at time $t-1$. The conditional expectations of price level at time $t$ given the information available at time $t-1$ is written as:

$$E_{t-1}p_t = E\left(p_t|I_{t-1}\right). \tag{5.5.40}$$

In general, the conditional expectations of $p_t$ given the information available at $t-j$ can be written as:

$$E_{t-j}p_t = E\left(p_t|I_{t-j}\right),$$

for all $j$.

To solve for the reduced form of the price level, both equations (5.5.39) and (5.5.8) are set equal to each other:

$$p_t = \left[\frac{\alpha - \gamma}{\beta}\right] - \left(\frac{\lambda}{\beta}\right) p_t^e - \left(\frac{\delta}{\beta}\right) w_{t-1} + \left[\frac{\epsilon_t^d - \epsilon_t^s}{\beta},\right]$$

or:

$$p_t = a + bp_t^e + dw_{t-1} + e_t',$$

where $a = \frac{\alpha-\gamma}{\beta}$, $b = -\frac{\lambda}{\beta}$, $d = -\frac{\delta}{\beta}$, and $e_t' = \frac{\epsilon_t^d - \epsilon_t^s}{\beta}$. The RE price at time $t$ is written as:

$$p_t^e = E\left(p_t | I_{t-1}\right) = E_{t-1} p_t.$$

Therefore, the revised model of the price level is:

$$p_t = a + bE_{t-1}p_t + dw_{t-1} + e_t'. \tag{5.5.41}$$

Equation (5.5.41) is very similar to equation (5.5.34). But, equation (5.5.41) shows the price level, $p_t$, now depends on an extra observable variable, $w_{t-1}$. To solve for the REE, conditional expectations of both sides in equation (5.5.41) are taken:

$$\begin{aligned} E_{t-1}p_t &= E_{t-1}\left(a + bE_{t-1}p_t + dw_{t-1} + e_t'\right) \\ &= E_{t-1}a + E_{t-1}\left(bE_{t-1}p_t\right) + E_{t-1}\left(dw_{t-1}\right) + E_{t-1}e_t' \\ &= E_{t-1}a + bE_{t-1}\left(E_{t-1}p_t\right) + dE_{t-1}w_{t-1} + E_{t-1}e_t'. \end{aligned} \tag{5.5.42}$$

Note $E_{t-1}a = a$, $E_{t-1}\left(E_{t-1}p_t\right) = E_{t-1}p_t$, $E_{t-1}w_{t-1} = w_{t-1}$, and $E_{t-1}e_t' = 0$, thus we have:

$$\begin{aligned} E_{t-1}p_t &= a + bE_{t-1}p_t + dw_{t-1} \\ E_{t-1}p_t &= \frac{a}{1-b} + \frac{d}{1-b}w_{t-1}. \end{aligned} \tag{5.5.43}$$

Now substituting equation (5.5.43) into equation (5.5.41) and solving for the REE:

$$\begin{aligned} p_t &= a + b\left(\frac{a}{1-b} + \frac{d}{1-b}w_{t-1}\right) + dw_{t-1} + e_t' \\ p_t^{RE} &= \frac{a}{1-b} + \frac{d}{1-b}w_{t-1} + e_t'. \end{aligned} \tag{5.5.44}$$

Equation (5.5.44) is the REE where the price level depends on a constant term, an observable variable, $w_{t-1}$, and a composite stochastic error term, $e'_t$.

### Application 3: The Cagan Hyperinflation Model

A well-known model, with implications for RE, is the Cagan Hyperinflation model (Cagan 1956) that describes the fundamental relation between the aggregate price level and the money supply. To begin, assume the quantity of real money demanded $(m_t - p_t)^d$ depends on the expected change in the price level:

$$(m_t - p_t)^d = \alpha - \beta \left( E_t p_{t+1} - p_t \right) + \epsilon_t,$$

where $\alpha, \beta > 0$. Assume also the quantity of real money supplied $(m_t - p_t)^s$ is determined by policymakers:

$$(m_t - p_t)^s = m_t - p_t,$$

where $m_t$ and $p_t$ are the log levels of money stock and price, respectively, $E_t p_{t+1}$ is the conditional expectations of $p_{t+1}$ formed at $t$, and $\epsilon_t$ is a stochastic money demand shock. The quantity of money demanded is set with the quantity of money supplied to determine price level dynamics. The reduced form is:

$$p_t = a + bE_t p_{t+1} + dm_t + e_t, \tag{5.5.45}$$

where $a = -\frac{\alpha}{1+\beta}$, $b = \frac{\beta}{1+\beta}$, $d = \frac{1}{1+\beta}$, and $e_t = -\frac{\epsilon_t}{1+\beta}$.

### The Cagan Model with a Constant Policy or Treatment

To make the model as simple as possible the "treatment" is assumed to be constant. In the Cagan model, the "treatment" or "policy" are monetary policy rules. For example, assume the treatment or policy, in this case assume the money stock $m_t$, does not change over time (i.e., $m_t = \bar{m}$). This implies policymakers decide to fix the money stock level in the economy. Equation (5.5.45) is rewritten as:

$$p_t = a' + bE_t p_{t+1} + e_t, \tag{5.5.46}$$

where $a' = a + d\bar{m}$, and $e_t = -\frac{\epsilon_t}{1+\beta}$.

*The method of undetermined coefficients* is used to solve for the model (5.5.46). From equation (5.5.46), the price level depends only on a constant term, its expectations, and a stochastic error term. We conjecture the RE solution is in the following form:

$$p_t = \Pi + e_t, \tag{5.5.47}$$

where $\Pi$ is an unknown coefficient. Equation (5.5.47) is extended *one period forward* and conditional expectations for time $t$ are taken:

$$\begin{aligned} E_t p_{t+1} &= E_t\left(\Pi + e_{t+1}\right) \\ &= E_t\Pi + E_t e_{t+1} \\ &= \Pi, \end{aligned} \tag{5.5.48}$$

where $E_t e'_{t+1} = 0$. Substituting equation (5.5.48) into equation (5.5.46):

$$\begin{aligned} p_t &= a' + bE_t p_{t+1} + e_t \\ &= a' + b\Pi + e_t. \end{aligned} \tag{5.5.49}$$

In equation (5.5.49), we see the *actual law of motion* (ALM) of $p_t$ depends only on a constant term, $a' + b\Pi$, and a stochastic term, $e_t$, when RE is formed. By comparing equations (5.5.47) and (5.5.49), the result is:

$$\text{[From Equation (5.5.47)] } \Pi = a' + b\Pi \text{ [From Equation (5.5.49)].} \tag{5.5.50}$$

It is straight-forward to solve the unknown parameter $\Pi$ from (5.5.50):

$$\Pi = \frac{a'}{1-b}. \tag{5.5.51}$$

Equation (5.5.51) is put back in equation (5.5.48):

$$E_t p_{t+1} = \frac{a'}{1-b}. \tag{5.5.52}$$

Equation (5.5.52) is the RE agents form. Inserting (5.5.52) into (5.5.46), to get the dynamics of the price level:

$$\begin{aligned} p_t &= a' + bE_t p_{t+1} + e_t \\ &= a' + b\left[\frac{a'}{1-b}\right] + e_t \\ p_t^{RE} &= \frac{a'}{1-b} + e_t. \end{aligned} \tag{5.5.53}$$

### The Cagan Model with an Autoregressive Policy or Treatment

We can also have alternative treatment regimes. Assume the movement of the money supply follows a first-order autoregressive (AR(1)) policy rule:

$$m_t = \lambda + \gamma m_{t-1} + \xi_t, \tag{5.5.54}$$

where $\xi_t$ is a stochastic factor. Substituting equation (5.5.54) into equation (5.5.45) to get the reduced form for the price level:

$$p_t = a'' + bE_t p_{t+1} + hm_{t-1} + u_t + e_t, \tag{5.5.55}$$

and: $a'' = \frac{\lambda-\alpha}{1+\beta}$, $b = \frac{\beta}{1+\beta}$, $h = \frac{\gamma}{1+\beta}$, $u_t = \frac{\xi_t}{1+\beta}$, and $e_t = -\frac{\epsilon_t}{1+\beta}$.

Applying the method of undetermined coefficient coefficients, based on equation (5.5.55), our conjecture for the RE solution is:

$$p_t = \Pi_0 + \Pi_1 m_{t-1} + \Pi_2 u_t + \Pi_3 e_t. \tag{5.5.56}$$

Using equation (5.5.56), the equation is moved one period forward and then expectations for $t$ are taken:

$$\begin{aligned}
E_t p_{t+1} &= E_t \left(\Pi_0 + \Pi_1 m_t + \Pi_2 u_{t+1} + \Pi_3 e_{t+1}\right) \\
&= E_t \Pi_0 + \Pi_1 E_t m_t + \Pi_2 E_t u_{t+1} + \Pi_3 E_t e_{t+1} \\
&= \Pi_0 + \Pi_1 E_t m_t \\
&= \Pi_0 + \Pi_1 m_t, \qquad (5.5.57)
\end{aligned}$$

where $E_t m_t = m_t$, and $E_t u_{t+1} = E_t e_{t+1} = 0$. Substituting equation (5.5.54) into equation (5.5.57):

$$\begin{aligned}
E_t p_{t+1} &= \Pi_0 + \Pi_1 \left(\lambda + \gamma m_{t-1} + \xi_t\right) \\
&= \Pi_0 + \Pi_1 \lambda + \Pi_1 \gamma m_{t-1} + \Pi_1 \xi_t. \qquad (5.5.58)
\end{aligned}$$

Inserting equation (5.5.58) into equation (5.5.55):

$$\begin{aligned} p_t &= a'' + b\left(\Pi_0 + \Pi_1\lambda + \Pi_1\gamma m_{t-1} + \Pi_1\xi_t\right) + h m_{t-1} + u_t + e_t \\ &= \left(a'' + b\Pi_0 + b\lambda\Pi_1\right) + \left(b\gamma\Pi_1 + h\right) m_{t-1} + b\Pi_1\xi_t + u_t + e_t \\ &= \left(a'' + b\Pi_0 + b\lambda\Pi_1\right) + \left(b\gamma\Pi_1 + h\right) m_{t-1} + b\Pi_1\left[\left(1+\beta\right) u_t\right] + u_t + e_t \\ &= \left(a'' + b\Pi_0 + b\lambda\Pi_1\right) + \left(b\gamma\Pi_1 + h\right) m_{t-1} + \left[b\left(1+\beta\right)\Pi_1 + 1\right] u_t + e_t, \end{aligned} \tag{5.5.59}$$

where $u_t = \frac{\xi_t}{1+\beta}$, and now $\xi_t = (1+\beta)\, u_t$. According to equations (5.5.56) and (5.5.59), these two equations are identical when:

$$\Pi_0 = a'' + b\Pi_0 + b\lambda\Pi_1, \tag{5.5.60}$$

$$\Pi_1 = b\gamma\Pi_1 + h, \tag{5.5.61}$$

$$\Pi_2 = b\left(1+\beta\right)\Pi_1 + 1, \tag{5.5.62}$$

$$\Pi_3 = 1. \tag{5.5.63}$$

From conditions (5.5.61)-(5.5.63), the unknown coefficients can be solved:

$$\Pi_0 = \frac{a'' + b\lambda\Pi_1}{1-b} = \frac{a''\left(1-b\gamma\right) + bh\lambda}{\left(1-b\gamma\right)\left(1-b\right)}, \tag{5.5.64}$$

$$\Pi_1 = \frac{h}{1-b\gamma}, \tag{5.5.65}$$

$$\Pi_2 = b\left(1+\beta\right)\Pi_1 + 1 = \frac{bh\left(1+\beta\right)}{1-b\gamma} + 1, \tag{5.5.66}$$

$$\Pi_3 = 1. \tag{5.5.67}$$

Substituting solutions (5.5.64)-(5.5.67) into equation (5.5.56), the RE solution is obtained:

$$p_t^{RE} = \frac{a''\left(1-b\gamma\right) + bh\lambda}{\left(1-b\gamma\right)\left(1-b\right)} + \frac{h}{1-b\gamma} m_{t-1} + \left[\frac{bh\left(1+\beta\right)}{1-b\gamma} + 1\right] u_t + e_t. \tag{5.5.68}$$

**Application 4: Models with Multiple Expectations**

In this application a RE model is introduced with two rational expectations formulations. An example is Sargent and Wallace's (1975) "ad hoc" model consisting of an aggregate supply equation, an IS equation and an LM equation. A general reduced-form model is:

$$y_t = a + bE_{t-1}y_t + dE_{t-1}y_{t+1} + e_t. \tag{5.5.69}$$

Equation (5.5.69) implies agents' expectations of $y_t$ and $y_{t+1}$ are formed at time $t-1$.

Using the simplest REE:

$$y_t = \Pi_0 + \Pi_1 e_t. \tag{5.5.70}$$

The expression of equation (5.5.70) one period forward is:

$$y_{t+1} = \Pi_0 + \Pi_1 e_{t+1}. \tag{5.5.71}$$

Taking expectations of equations (5.5.70) and (5.5.71) at time $t-1$, respectively:

$$E_{t-1} y_t = \Pi_0, \tag{5.5.72}$$

and:

$$E_{t-1} y_{t+1} = \Pi_0. \tag{5.5.73}$$

Substituting equations (5.5.72) and (5.5.73) into equation (5.5.69):

$$y_t = a + b\Pi_0 + d\Pi_0 + e_t. \tag{5.5.74}$$

Solving for $\Pi_0$:

$$\Pi_0 = \frac{a}{1-b-d}.$$

From equation (5.5.74), we see that:

$$\Pi_1 = 1.$$

Therefore, the REE is:

$$y_t^{RE} = \frac{a}{1-b-d} + e_t. \tag{5.5.75}$$

Equation (5.5.75) is also called the *minimum state variable* (MSV) solution or "fundamental" solution (McCallum 1983). This is a linear solution that depends on a minimal set of variables. In this example, the REE of $y_t$ depends only on an intercept, $\frac{a}{1-b-d}$, and a stochastic error term $(e_t)$. Note, a variation of this procedure is applied in this chapter.

Another possible solution for model (5.5.69) is an AR(1) solution. We conjecture the AR(1) solution:

$$y_t = \Pi_0 + \Pi_1 y_{t-1} + \Pi_2 e_t. \tag{5.5.76}$$

The expectations of $y_t$ and $y_{t+1}$ formed at $t-1$ are, respectively:

$$E_{t-1}y_t = \Pi_0 + \Pi_1 y_{t-1}, \tag{5.5.77}$$

and:

$$\begin{aligned} E_{t-1}y_{t+1} &= \Pi_0 + \Pi_1 E_{t-1}y_t \\ &= \Pi_0 + \Pi_1 \left(\Pi_0 + \Pi_1 y_{t-1}\right) \\ &= \Pi_0 + \Pi_0\Pi_1 + \Pi_1^2 y_{t-1}. \end{aligned} \tag{5.5.78}$$

Substituting equations (5.5.77) and (5.5.78) into equation (5.5.69):

$$\begin{aligned} y_t &= a + b\left(\Pi_0 + \Pi_1 y_{t-1}\right) + d\left(\Pi_0 + \Pi_0\Pi_1 + \Pi_1^2 y_{t-1}\right) + e_t \\ &= \left(a + b\Pi_0 + d\Pi_0 + d\Pi_0\Pi_1\right) + \left(b\Pi_1 + d\Pi_1^2\right) y_{t-1} + e_t. \end{aligned} \tag{5.5.79}$$

Using equations (5.5.76) and (5.5.79), $\Pi_0$, $\Pi_1$, and $\Pi_2$ can be solved:

$$\Pi_0 = -\frac{a}{d},$$

$$\Pi_1 = \frac{1-b}{d},$$

and:

$$\Pi_2 = 1.$$

Therefore, the AR(1) REE is:

$$y_t^{RE} = -\frac{a}{d} + \frac{1-b}{d} y_{t-1} + e_t. \tag{5.5.80}$$

McCallum (1983) also terms this AR(1) REE a *"bubble"* solution since it involves the concept of a "self-fulfilling prophecy." The reason is equation (5.5.75) can fundamentally determine the dynamic behavior of $y_t$, but if agents "believe" and use $y_{t-1}$ to form expectations, then the RE solution becomes equation (5.5.80) and a self-fulfilling prophecy. McCallum (1983) argues the MSV solution — not necessarily the AR(1) REE — should be the solution of interest unless an alternative assumption is made to focus on the bubble solution in the model.[15]

[15] A more general REE for this model can be derived. The general solution is:

$$y_t^{RE} = -\frac{a}{d} + \frac{1-b}{d} y_{t-1} + e_t + he_{t-1} + ku_{t-1}, \tag{5.5.81}$$

where $h, k$ are arbitrary values of coefficients, and $u_t$ is an extra stochastic term (i.e., a *sunspot* variable) where $E_{t-1}u_{t-1} = 0$. This general solution in equation (5.5.81) is also called the ARMA(1,1) sunspot solution for model (5.5.69).

# Chapter 6

# Macro Policy

Post World War II economies have experienced various regime shifts in macroeconomic policy. In addition, this has resulted in a research emphasis on overall monetary policy effectiveness (See Bernanke et. al., 1999 and Taylor 1999). One particular line of research has focused on the use of interest rate rules in new Keynesian models (See Clarida, Gali, and Gertler 2000). Recently, Granato and Wong (2006) used a model with new Keynesian properties for determining the relation between inflation stabilizing policy, associated with inflation targeting, inflation persistence and volatility, and business cycle fluctuations.

The EITM linkage is the relation between the behavioral concepts — expectations and learning — *and* the applied statistical concept — persistence. Substantively speaking, the model and test show implementation (and shifts) in the aggressiveness of maintaining an inflation target affects inflation persistence. Consequently, the empirical tools for this example require a basic understanding of autoregressive processes. These tools have been presented earlier (Chapter 5). The formal tools include an extended presentation of tools from Chapter 4 and Chapter 5 (i.e., RE and linear of difference equations (variations on the minimum state variable solution procedure)) in addition to the components of adaptive learning. These adaptive learning components include RE modeling, an understanding of recursive stochastic algorithms, and relevant stability conditions.

## 6.1 Step 1: Relating Expectations, Learning, and Persistence

The model's intuition is as follows: monetary policy influences inflation expectations by encouraging the public to substitute an inflation target for past inflation. The testable prediction is a negative relation between periods of aggressive inflation-stabilizing policy and inflation persistence.[1] The model is a small structural model of macroeconomic outcomes and policy in the Cowles Commission tradition, but it also includes behavioral analogues for expectations and learning (with a *unique and stable* REE (Evans and Honkapohja 2001)). Under the REE, aggressive implementation of an inflation target guides agents to the stable equilibrium and thereby reduces inflation persistence.[2]

However, in this chapter, Granato and Wong do not impose RE. Rather they leave the possibility open that an REE can be reached via adaptive learning. Under adaptive learning expectations are formed by extrapolating from the historical data. One of the key differences between the assumption of RE and that of adaptive learning concerns whether the agent uses full information for forecasting.

Unlike RE — which assumes agents exhaust all possible information for forecasting — adaptive learning assumes agents choose only to use cost-effective information in a presumably known econometric model for forecasting.[3] Over time, by updating the data and running the same econometric model repeatedly, the agent is expected to learn and obtain the REE. The ability to reach the REE is formalized via *stability conditions*. These conditions are important because they have direct implications for how, and if, agents learn from policymakers. Adaptive learning models make use of what are called *E-stability* conditions (See the Appendix, Section 6.5.1).

[1]Aggressive inflation-stabilizing policy is defined as one that includes a willingness to respond forcefully to deviations from a prespecified implicit or explicit inflation target.

[2]Adaptive learning has gained popularity recently in inflation persistence research (See Milani (2007) as a representative study).

[3]Or, in more technical terms, adaptive learning is used so that agents update parameters of a forecasting rule (a perceived law of motion (PLM)) — associated with the stochastic process of the variable in question — to learn an REE. This process requires a condition establishing convergence to the REE — the E-stability condition. The E-stability condition determines the stability of the equilibrium in which the perceived law of motion (PLM) parameters adjust to the implied actual law of motion (ALM) parameters. See Evans and Honkapohja (2001) for details.

## 6.2 Step 2: Analogues for Expectations, Learning, and Persistence

The model is based on a two-period contract. For simplicity, prices reflect a unitary markup over wages. The price at time $t$, $p_t$, is expressed as the average of the current ($x_t$) and the lagged ($x_{t-1}$) contract wage:[4]

$$p_t = \frac{1}{2}\left(x_t + x_{t-1}\right), \tag{6.2.1}$$

where $p_t$ is the logarithm of the price level, and $x_t$ is the logarithm of the wage level at period $t$.

Additionaly, agents are concerned with their real wages over the lifetime of the contract:

$$x_t - p_t = \frac{1}{2}\left[x_{t-1} - p_{t-1} + E_t\left(x_{t+1} - p_{t+1}\right)\right] + \theta z_t, \tag{6.2.2}$$

where $x_t - p_t$ represents the real wage rate at time $t$, $E_t\left(x_{t+1} - p_{t+1}\right)$ is the expectation of the future real wage level at time $t+1$ formed at time $t$, and $z_t = y_t - y_t^n$ is the excess demand for labor at time $t$.

Next, the inflation rate ($\pi_t$) is defined as the difference between the current and lagged price level ($p_t - p_{t-1}$). With this definition, substituting equation (6.2.2) into equation (6.2.1) obtains:

$$\pi_t = \frac{1}{2}\left(\pi_{t-1} + E_t\pi_{t+1}\right) + \theta z_t + u_{1t}, \tag{6.2.3}$$

where $E_t\pi_{t+1}$ is the expected inflation rate over the next period and $u_{1t}$ is $iid\left(0, \sigma_{u_1}^2\right)$. Equation (6.2.3) captures the main characteristic of inflation persistence. Since agents make plans about their real wages over both past and future periods, the lagged price level ($p_{t-1}$) is taken into consideration as they adjust (negotiate) their real wage at time $t$. This model feature allows the inflation rate to depend on both the expected inflation rate as well as past inflation.

Letting equation (6.2.4) represent a standard IS curve: the quantity demanded on output relative to natural output ($z_t$) is negatively associated with the changes in real interest rates:

$$z_t = -\varphi\left(i_t - E_t\pi_{t+1} - r^*\right) + u_{2t}, \tag{6.2.4}$$

where $i_t$ is nominal interest rate, $r^*$ is the target real interest rate, $u_{2t}$ is $iid\left(0, \sigma_{u_2}^2\right)$,

[4] See Wang and Wong (2005) for the details of the general theoretical framework.

and $\varphi > 0$.

Assume policymakers use an interest rate rule in linking policy and outcomes — the Taylor rule (Taylor 1993) — when conducting monetary policy:

$$i_t = \pi_t + \alpha_y z_t + \alpha_\pi (\pi_t - \pi^*) + r^*. \tag{6.2.5}$$

Positive values of $\alpha_\pi$ and $\alpha_y$ indicate a willingness to raise (lower) nominal interest rates in response to the positive (negative) deviations from either the target inflation rate $(\pi_t - \pi^*)$, the output gap $(z_t)$, or both. An aggressive inflation-stabilizing policy is consistent with $\alpha_\pi > 0$.

The equilibrium inflation rate can be found by solving for the reduced form of the system. Substitute equation (6.2.5) into equation (6.2.4) to solve for $z_t$ and then put that result into equation (6.2.3). If we solve this expression for $\pi_t$ the result is:

$$\pi_t = \Gamma_0 + \Gamma_1 \pi_{t-1} + \Gamma_2 E_t \pi_{t+1} + \xi_t, \tag{6.2.6}$$

where:

$$\begin{aligned}
\Gamma_0 &= (\theta\varphi\alpha_\pi\pi^*)\,\Phi^{-1}, \\
\Gamma_1 &= (1 + \varphi\alpha_y)\,(2\Phi)^{-1}, \\
\Gamma_2 &= (1 + \varphi\alpha_y + 2\theta\varphi)\,(2\Phi)^{-1}, \\
\xi_t &= [\theta u_{2t} + (1 + \varphi\alpha_y)\,u_{1t}]\,\Phi^{-1}, \\
\Phi &= 1 + \varphi\alpha_y + \theta\varphi\,(1 + \alpha_\pi).
\end{aligned}$$

Equation (6.2.6) shows current inflation depends on the first-order lag of inflation and also expected inflation. When (6.2.6) is "closed," the MSV solution can be expressed as an AR(1) process. Thus, the AR(1) process is the empirical analogue for persistence.

## 6.3 Step 3: Unifying and Evaluating the Analogues

Solving for the REE ensures methodological unification since this solution involves merging the behavioral analogue of expectations with the empirical analogue for persistence. By taking the conditional expectations at time $t + 1$ of equation (6.2.6) and substituting this result into equation (6.3.1) :

$$\pi_t = A + B\pi_{t-1} + \tilde{\xi}_t, \tag{6.3.1}$$

where:

$$A = \Gamma_0 \left(1 - \Gamma_2 B - \Gamma_2\right)^{-1},$$
$$B = \left(1 \pm \sqrt{1 - 4\Gamma_1\Gamma_2}\right)\left(2\Gamma_2\right)^{-1},$$
$$\tilde{\xi}_t \equiv \xi_t \left(1 - \Gamma_2 B\right)^{-1}.$$

Equation (6.3.1) is the MSV solution of inflation — which depends solely on the lagged inflation rate.[5]

This solution also highlights an important formal modeling and analogue attribute. Using an adaptive learning analogue, one potential confounding factor that we are alerted to, with important empirical implications, is the nature of the coefficient of lagged inflation, ($B$). This parameter is a quadratic where the two values are defined as:

$$B^+ = \frac{1 + \sqrt{1 - 4\Gamma_1\Gamma_2}}{2\Gamma_2},$$
$$B^- = \frac{1 - \sqrt{1 - 4\Gamma_1\Gamma_2}}{2\Gamma_2}.$$

Behaviorally, when policymakers adopt an aggressive inflation-stabilizing policy, a stationary AR(1) solution can be obtained (i.e., $B^-$) while an explosive AR(1) solution (i.e., $B^+$) would also be possible. Here adaptive learning serves as an important selection criteria (i.e., determining stable solutions) where only the stationary solution (i.e., $B^-$) is attainable and the explosive solution (i.e., $B^+$) is not possible (See the Appendix, Section 6.5.1 and McCallum 2003).

In other words, if agents learn the equilibrium in an adaptive manner and they form expectations as new data becomes available over time, $B^-$ is the only learnable (E-stable) equilibrium when policymakers aggressively stabilize inflation (i.e., $\alpha_\pi > 0$). The intuition with this selection critierion is that the model is internall consistent: people can learn the inflation target and begin to rely less on the past history of inflation in making their forecasts.[6]

[5] The MSV is solved for in Section 6.5.1.

[6] The stability condition(s) show that this hypothesis is possible in this model: $B^-$ is a unique stationary solution when $\alpha_\pi \geq 0$. The empirical implications of the model — and ex-ante prediction — as represented in equation (6.3.1) is that an increase in $\alpha_\pi$ reduces persistence under $B^-$.

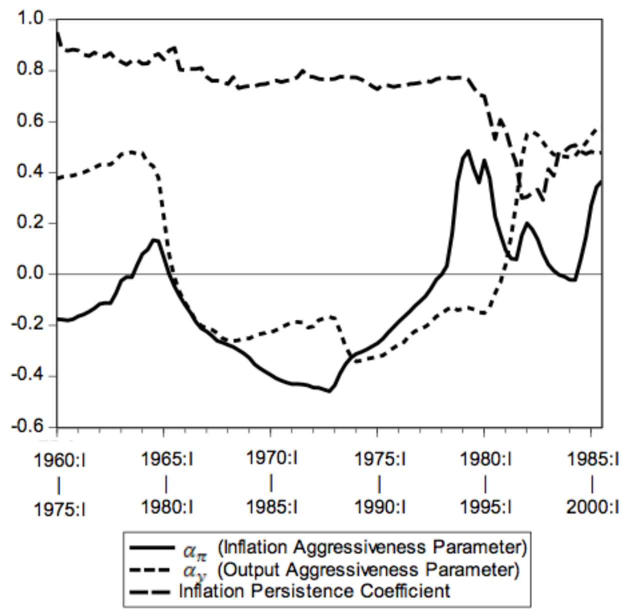


Figure 6.3.1: Policy and Persistence

To test the relation between the policy parameter(s) and inflation persistence quarterly, U.S. data are used (for the period 1960:I to 2000:III). According to the model, inflation persistence should fall significantly under an aggressive inflation-stabilizing policy (i.e., $\alpha_\pi > 0$.). From equation (6.3.1) Granato and Wong estimate a first-order autoregressive process (i.e., AR(1)) of the U.S. inflation rate. As a consequence of the more aggressive inflation-stabilizing policy stance during the Volcker-Greenspan period (August, 1979 through August, 2000), the expectation is that the inflation-persistence, parameter ($B_t$) in the Volcker-Greenspan period to be smaller (statistically) relative to the pre-Volcker period.

Granato and Wong estimate equation (6.2.5) to contrast the parameter movements in $\alpha_\pi$ and $\alpha_y$.[7] Figure 6.1 provides point estimates of inflation persistence ($B_t$) and policy rule parameters, $\alpha_\pi$ and $\alpha_y$, for a 15-year rolling sample starting in the first quarter of 1960 (1960:I). The results show that after 1980, inflation persistence starts falling. Figure 6.1 also indicates both $\alpha_\pi$ and $\alpha_y$ de-emphasize inflation and output stability in approximately 1968. Prior to 1968, policy emphasized output stability ($\alpha_y > 0$). Aggressive inflation stabilizing policy occurs only after 1980, when $\alpha_\pi > 0$.

[7] See also Granato and Wong (2006: 198-211).

## 6.4 Leveraging EITM and Extending the Model

In this example, there is little in the way of microfoundations or the strategic interaction between policymakers and the public. Moreover, the policy rule (6.2.5) is devoid of any political and social factors. Both the inflation target variable ($\pi^*$) and the response parameters ($\alpha_\pi, \alpha_y$) could be made endogenous to political and social factors, including (but not limited to) partisanship, elections, and social interaction where public information levels are heterogeneous.[8]

## 6.5 Appendix

Using the tools in this chapter a transparent and testable relation is established between expectations, learning, and persistence. The applied statistical analogue for persistence is located in Section 5.5.1 and will not be repeated. The formal tools include a presentation of adaptive learning building on tools used in Chapter 4 and 5. The formal tools include a presentation of:

- Extended discussion of difference equations (variations on the minimum state variable solution procedure)
- Adaptive learning (recursive stochastic algorithms and relevant stability conditions).

These tools are subsequently used in various applications for models where RE is assumed. The discussion ends by showing how adaptive learning, under certain conditions, translates into showing how an REE can be attained.

### 6.5.1 Formal Analogues

Equation (6.2.6) can be solved using an MSV solution and a special case of the method of undetermined coefficients. Refer to Section 5.5.2. The technique is extended to RE models with lagged variables.

Equation (6.2.6) is of a particular form. A general RE model with persistence is presented as follows:

$$y_t = a + bE_t y_{t+1} + dy_{t-1} + e_t, \tag{6.5.1}$$

[8] See Granato, Guse, and Wong (2008) for a model with heterogeneous information levels and adaptive learning.

where $y_t$ is a variable of interest (e.g., the inflation rate at time $t$), $E_t y_{t+1}$ is the rational expectation of $y_{t+1}$ formed at time $t$, $y_{t-1}$ is the lagged dependent variable, and $e_t$ is a stochastic term.

Solving for the RE in model (6.5.1), conjecture the solution is:

$$y_t = \Pi_0 + \Pi_1 y_{t-1} + \Pi_2 e_t \tag{6.5.2}$$

Moving equation (6.5.2) one period forward and then taking expectations at time $t$:

$$\begin{aligned} E_t y_{t+1} &= E_t\left(\Pi_0 + \Pi_1 y_t + \Pi_2 e_{t+1}\right) \\ &= \Pi_0 + \Pi_1 y_t. \end{aligned} \tag{6.5.3}$$

Equation (6.5.3) indicates agents form their expectations of $y_{t+1}$ based on the current information $y_t$. Substituting equation (6.5.3) into equation (6.5.1):

$$y_t = a + b\left(\Pi_0 + \Pi_1 y_t\right) + d y_{t-1} + e_t.$$

After algebraic manipulations:

$$y_t = \frac{a + b\Pi_0}{1 - b\Pi_1} + \frac{d}{1 - b\Pi_1} y_{t-1} + \frac{1}{1 - b\Pi_1} e_t. \tag{6.5.4}$$

Comparing equations (6.5.2) and (6.5.4):

$$\Pi_0 = \frac{a + b\Pi_0}{1 - b\Pi_1} \tag{6.5.5}$$

$$\Pi_1 = \frac{d}{1 - b\Pi_1} \tag{6.5.6}$$

$$\Pi_2 = \frac{1}{1 - b\Pi_1}. \tag{6.5.7}$$

Next, use (6.5.6):

$$\Pi_1 = \frac{d}{1 - b\Pi_1}$$

to find:

$$b\Pi_1^2 - \Pi_1 + d = 0. \tag{6.5.8}$$

Equation (6.5.8) demonstrates there are two possible solutions for the model:

$$\Pi_1 = \frac{1 \pm \sqrt{1 - 4bd}}{2b}. \tag{6.5.9}$$

From equation (6.5.9), we define $\Pi_1^+ = \frac{1+\sqrt{1-4bd}}{2b}$, and $\Pi_1^- = \frac{1-\sqrt{1-4bd}}{2b}$.

Although there are two possible solutions in the model, it is straight-forward to "eliminate" one of them. Assuming the lagged dependent variable $y_{t-1}$ has no effect on $y_t$ (i.e., $d = 0$), then $y_{t-1}$ should not be in the solution (6.5.2) ($\Pi_1 = 0$). By substituting $d = 0$ into both possible solutions:

$$\Pi_1^+ = \frac{1+\sqrt{1-4bd}}{2b} = \frac{1+\sqrt{1-4b\,(0)}}{2b} = \frac{1}{b}, \tag{6.5.10}$$

and:

$$\Pi_1^- = \frac{1-\sqrt{1-4bd}}{2b} = \frac{1-\sqrt{1-4b\,(0)}}{2b} = 0. \tag{6.5.11}$$

From solutions (6.5.10) and (6.5.11), the conclusion is that $\Pi_1^-$ would be a plausible solution consistent with the model.

## Adaptive Learning

At this point a brief introduction to adaptive learning and expectational stability is discussed. These two matters were explicated by Evans (1985, 1989) and Evans and Honkapohja (1995, 2001). For purposes of continuity the example in Section 5.5.2 is further developed using the following a cobweb model (as defined in equation (5.5.41)):

$$p_t = a + bE_{t-1}p_t + dw_{t-1} + e_t'.$$

Solving for the REE:

$$p_t^{RE} = \frac{a}{1-b} + \frac{d}{1-b}w_{t-1} + e_t'.$$

As mentioned earlier in Chapter 5, the REE shows that agents rationally forecast the price level that depends on a constant term, $\frac{a}{1-b}$, and the observable, $w_{t-1}$ with the coefficient of $\frac{d}{1-b}$.

Under the assumption of rational expectations, agents are assumed to be very "smart" and able to make an optimal forecast of $p_t$ using $w_{t-1}$ with an coefficient of $\frac{d}{1-b}$ in the forecasting process. Acquiring all available information immediately, agents are able to form conditional (mathematical) expectations. This is a very strong assumption. Sargent (1993) points out that agents with RE are even more sophisticated than the economist who sets up the economic model.

Instead of assuming agents possess rational expectations, we assume agents *learn* in an adaptive manner by forming expectations as new data becomes available. Further, we analyze the conditions of expectational stability (E-stability) under which the parameters in agents' forecasting rules — perceived law of motion (PLM) — are slowly adjusted to (or mapped to) the parameters in the actual law of motion (which can contain the REE).

This E-stability condition determines if agents are able to learn (locally) the correct forecasting rule — the REE. Evans (1989) and Evan and Honkapohja (1992) show that the mapping from the PLM to the ALM is generally consistent with the convergence to REE under least squares learning. This correspondence is called the E-stability principle. Assuming that agents continuously form the forecast of $p_t$ by estimating the following econometric model as new information of $w_t$ becomes available over time we present:

$$p_t = \alpha_0 + \alpha_1 w_{t-1} + \epsilon_t. \tag{6.5.12}$$

Equation 6.5.12 can also be called the PLM. Determining the condition(s) such that the estimated coefficients, $\alpha_0$ and $\alpha_1$, can converge to the REE, $\frac{a}{1-b}$ and $\frac{d}{1-b}$, respectively (when $t \to \infty$) is the goal in this regard.

This principle has additional attributes. If the equilibrium is E-stable, then the RE method may be an appropriate technique for solving long run equilibria. Moreover, E-stability conditions are important selection criteria in determining stable solutions when a model has multiple equilibria.

Employing advanced technical terms, Evans (1989) defines the E-stability condition in terms of the ordinary differential equation (ODE):

$$\frac{d\theta}{d\tau} = T(\theta) - \theta, \tag{6.5.13}$$

where $\theta$ is a finite dimensional parameter specified in the perceived law of motion, $T(\theta)$ is a mapping (so-called T-mapping) from the perceived to the actual law of motion, and $\tau$ denotes "notional" or "artificial" time. The REE, $\bar{\theta}$, corresponds to fixed points of $T(\theta)$.

The stability condition of $\bar{\theta}$ is given under the following definition:

**Definition 1.** $\bar{\theta}$ is expectationally stable (E-stable) if there exists $\varepsilon > 0$ such that $\theta(\tau) \to \bar{\theta}$ as $\tau \to \infty$, for all $\left\|\theta_0 - \bar{\theta}\right\| < \varepsilon$, where $\theta(\tau)$ is the trajectory that solves 106 subject to the initial condition $\theta(0) = \theta_0$.

Evans and Honkapohja (2001) show that the notional time concept of expectational stability is generally consistent with the stability under real-time least squares learning. Additionally, this correspondence is called the *E-stability principle.* Evans and Honkapohja (2001) mention that E-stability conditions are often easy to develop, but the convergence condition of adaptive learning involves a more technical analysis.

**Least Squares Learning and Stochastic Recursive Algorithms**

To understand the general correspondence between E-stability and adaptive learning it is necessary to outline the least squares learning technique and appropriate convergence conditions[9] (See Bray (1982), Bray and Savin (1986), and Marcet and Sargent (1989a, 1989b)). Assuming that agents use recursive least squares (RLS) for updating their expectations each period up to the period $t-1$ :

$$y_t^e = \psi_{t-1}' x_{t-1}, \tag{6.5.14}$$

and

$$\psi_t = \psi_{t-1} + t^{-1} R_t x_{t-1} \left( y_t - \psi_{t-1}' x_{t-1} \right), \tag{6.5.15}$$

$$R_t = R_{t-1} + t^{-1} \left( x_{t-1} x_{t-1}' - R_{t-1} \right). \tag{6.5.16}$$

where $x_t$ and $y_t^e$ are $m \times 1$ vectors of independent and forecast dependent variables, respectively, $\psi_t$ is a $1 \times m$ coefficient vector updated by the system (6.5.15) and (6.5.16), $R_t$ denotes the moment matrix for $x_t$. Equation (6.5.14) represents agents' PLM generating a corresponding ALM for $y_t$ :

$$y_t = T\left(\psi_{t-1}\right)' x_{t-1} + v_t, \tag{6.5.17}$$

where $v_t \sim iid\left(0, \sigma_v^2\right)$. Substitute equation (6.5.17) into (6.5.15), gives the stochastic recursive system:

$$\psi_t = \psi_{t-1} + t^{-1} R_t x_{t-1} \left( x_{t-1}' \left( T\left(\psi_{t-1}\right) - \psi_{t-1}' \right) + v_t \right), \tag{6.5.18}$$

$$R_t = R_{t-1} + t^{-1} \left( x_{t-1} x_{t-1}' - R_{t-1} \right). \tag{6.5.19}$$

The system (6.5.18) and (6.5.19) can also be formed as a standard stochastic recursive algorithm (SRA) determining the asymptotic stability for linear regression models:

$$\theta_t = \theta_{t-1} + \gamma_t Q\left(t, \theta_{t-1}, X_t\right),$$

where $\theta_t' = \left(vec\left(\psi_t\right), vec\left(R_{t+1}\right)\right), X_t = \left(x_t, x_{t-1}, v_t\right)$ and $\gamma_t = t^{-1}$. This SRA relates

[9] See Evans and Honkapohja (2001) for further mathematical details.

to the ODE:

$$\frac{d\theta}{d\tau} = h(\theta(\tau)), \tag{6.5.20}$$

where the limit of $h(\theta)$ exists as:

$$h(\theta) = \lim_{t\to\infty} EQ(t, \theta, X_t),$$

and $E$ represents the expectation of $Q(\cdot)$ with the fixed value of $\theta$.

Following the set-up of the SRA, $\bar{\theta}$ is an equilibrium point if $h(\theta) = 0$ in equation (6.5.20). This result provides a standard mathematical definition of asymptotic stability for the differential equation:

**Definition 2.** $\bar{\theta}$ is locally stable if for every $\varepsilon > 0$ there exists $\delta > 0$ such that $|\theta(t) - \bar{\theta}| < \varepsilon \ \forall \ |\theta(0) - \bar{\theta}| < \delta$. $\bar{\theta}$ is said to be locally asymptotically stable if $\bar{\theta}$ is stable, and that $\theta(\tau) \to \bar{\theta} \ \forall \ \theta(0)$ is somewhere in the neighborhood of $\bar{\theta}$.

We now show the local stability condition of $\bar{\theta}$ by computing the Jacobian matrix $Dh\left(\bar{\theta}\right)$ and using the following lemma. This lemma is generally consistent with the the E-stability condition:

**Lemma 3.** *If all eigenvalues of $Dh\left(\bar{\theta}\right)$ have negative real parts, then $\bar{\theta}$ is a locally stable equilibrium point of $\frac{d\theta}{d\tau} = h(\theta)$. If some eigenvalues of $Dh\left(\bar{\theta}\right)$ have a positive real part, then $\bar{\theta}$ is not a locally stable equilibrium point of $\frac{d\theta}{d\tau} = h(\theta)$.*

**Application: Deriving Expectational Stability (or E-Stability) Conditions**

With this background we derive the E-stability condition for the cobweb model in equation (5.5.41):

$$p_t = a + bE_{t-1}p_t + dw_{t-1} + e_t'. \tag{6.5.21}$$

Assuming that agents do not know the REE while they are able to update their forecasts (the parameters) over time. The PLM is:

$$p_t = \alpha_0 + \alpha_1 w_{t-1} + \epsilon_t. \tag{6.5.22}$$

Therefore, the forecasts of $p_t$ generated by the agents based on the PLM at time $t-1$ is:

$$E_{t-1}p_t = \alpha_0 + \alpha_1 w_{t-1}.$$

As a result, the actual price $p_t$ will be affected by agents' forecasts in this case:

$$\begin{aligned} p_t &= a + bE_{t-1}p_t + dw_{t-1} + e_t' \\ &= a + b\left(\alpha_0 + \alpha_1 w_{t-1}\right) + dw_{t-1} + e_t' \\ &= \left(a + b\alpha_0\right) + \left(d + b\alpha_1\right) w_{t-1} + e_t'. \end{aligned} \tag{6.5.23}$$

Equation (6.5.23) is the implied ALM showing that the parameters in the model are adjusted given the parameters from the PLM. According to Evans (1989) and Evans and Honkapohja (2001), the E-stability condition determines the stability of the equilibrium in which the PLM parameters $\theta = [\alpha_0, \alpha_1]$ adjust to the implied ALM parameters $[a + b\alpha_0, d + b\alpha_1]$. This is written as the following ODE:

$$\frac{d\theta}{d\tau} = T\left(\theta\right) - \theta,$$

where $\tau$ is a notional time period, and $T\left(\theta\right)$ is a mapping (T-mapping) of the PLM parameters $\theta$, that is, $T\left(\theta\right) = [a + b\alpha_0, d + b\alpha_1]$. The ODE can also be rewritten as follows:

$$\begin{aligned} \frac{d}{d\tau}\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} &= T\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} - \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \\ &= \begin{pmatrix} a + b\alpha_0 \\ d + b\alpha_1 \end{pmatrix} - \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}. \end{aligned} \tag{6.5.24}$$

As a result, the REE corresponds to the fixed points of $T\left(\theta\right)$. To determine the E-stability condition, the associated ODE in equation (6.5.24) can be viewed as the dynamic process of the forecasting parameters:

$$\dot{\alpha}_0 = \frac{d\alpha_0}{d\tau} = \left(a + b\alpha_0\right) - \alpha_0, \text{ and} \tag{6.5.25}$$

$$\dot{\alpha}_1 = \frac{d\alpha_1}{d\tau} = \left(d + b\alpha_1\right) - \alpha_1. \tag{6.5.26}$$

According to equations (6.5.25) and (6.5.26), the E-stability condition is shown as $b < 1$, implying that agents are able to learn the REE over time only if $b < 1$ is satisfied.

Understanding the result mathematically, the REE is attainable when the dynamic process for $\alpha_0$ and $\alpha_1$ are in steady steady, such that $\dot{\alpha}_0 = 0$ and $\dot{\alpha}_1 = 0$. In

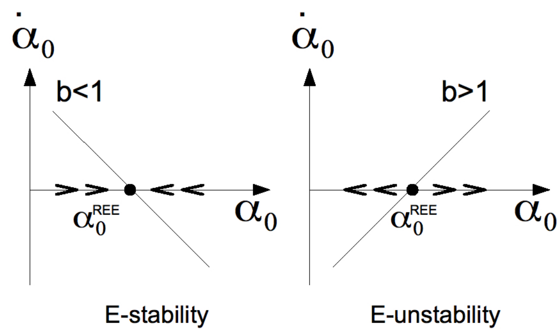


Figure 6.5.1: E-stability versus E-unstability

equations (6.5.25) and (6.5.26), we simplify them as follows:

$$\dot{\alpha}_0 = a + (b-1)\,\alpha_0, \text{ and} \tag{6.5.27}$$

$$\dot{\alpha}_1 = d + (b-1)\,\alpha_1. \tag{6.5.28}$$

An explanation of the E-stability condition is illustrated by the values of $\dot{\alpha}_0$ against $\alpha_0$. This is based on the ODE function (6.5.25) when $b < 1$ and $b > 1$. If $b < 1$, the slope of equation (6.5.27) is negative and can be presented in the left panel of Figure (6.5.1). However, the function is positively sloped if $b > 1$ (the right panel).

On the left panel of Figure (6.5.1), with $b < 1$, any value of $\alpha_{0,t}$ at time $t$ which is less than its REE — $\alpha_{0,t} < \alpha_0^{REE}$ — gives a positive value of $\dot{\alpha}_0$ — $\dot{\alpha}_0 = \frac{d\alpha_0}{d\tau} > 0$. Consequently, $\alpha_{0,t}$ increases over time and approaches $\alpha_0^{REE}$ for $t \to \infty$. Similarly, if $\alpha_{0,t}$ is initially larger than $\alpha_0^{REE}$, the condition, $\dot{\alpha}_0 < 0$, indicates $\alpha_{0,t}$ decreases over time but also approaches $\alpha_0^{REE}$ for $t \to \infty$.

However, considering the case where $b > 1$ (the right panel of Figure (6.5.1) any initial value of $\alpha_{0,t}$ less than $\alpha_0^{REE}$, then under the condition, $\dot{\alpha}_0 < 0$, means $\alpha_{0,t}$ decreases over time and diverges from its REE. Based on the same reasoning, if $\alpha_{0,t} > \alpha_0^{REE}$ for any $t$, then $\dot{\alpha}_0 > 0$ indicates $\alpha_0$ increases over time and moves away from its REE.

The same procedure can be applied to show that $\alpha_{1,t}$ converges (in equation 6.5.28) to $\alpha_1^{REE}$ only if $b < 1$ : the PLM parameters converge to the REE only under the condition of $b < 1$. The conclusion therefore is $b < 1$ satisfies the E-stability condition and $b > 1$ is the E-unstability condition. Consequently, this result demonstrates that, if $b < 1$ is satisfied, agents will obtain the REE if they recursively update their forecasts by the adaptive learning mechanism.

These properties are reinforced further with numeric simulations for the model (6.5.21) – (6.5.23). We assign the values $a = 2$, $d = 3$, and $\sigma_w = 1$ and $\sigma_{e'} = 1$ in

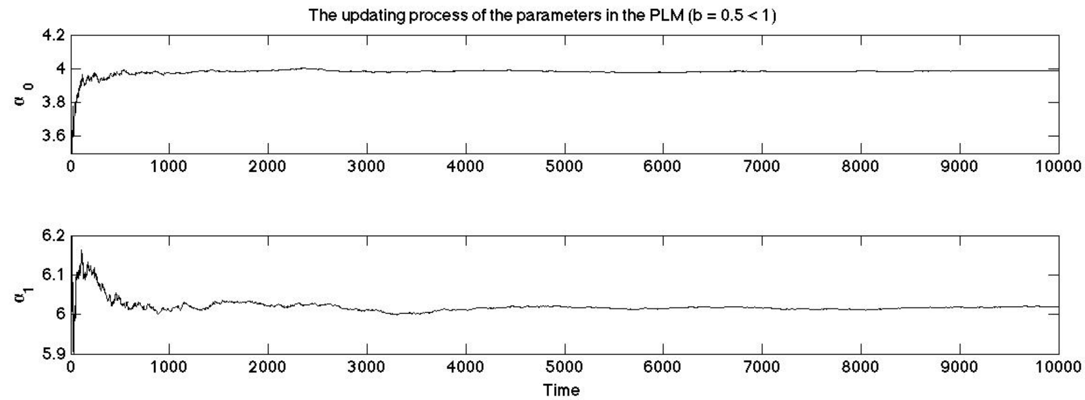


Figure 6.5.2: Simulation with E-stable Condition ($b < 1$)

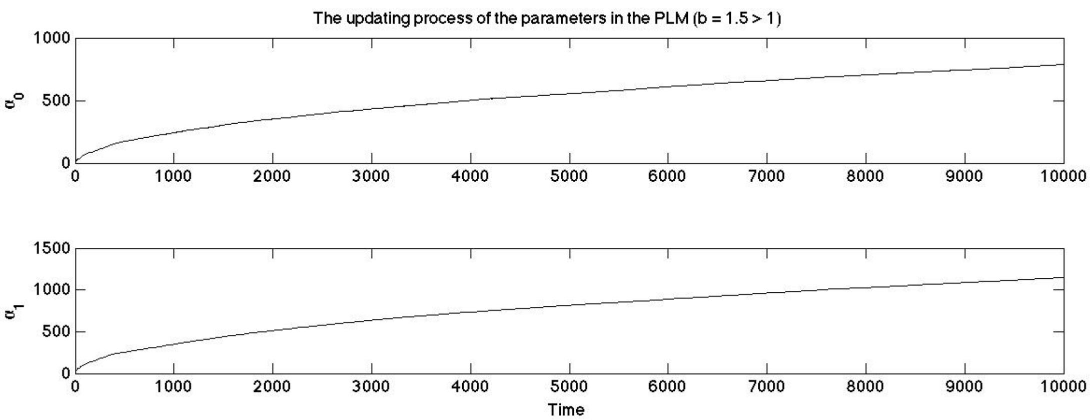


Figure 6.5.3: Simulation with E-unstable Condition ($b > 1$)

the first simulation. By assigning $b = 0.5$, we can solve for the REE: $\alpha_0^{REE} = 4$ and $\alpha_1^{REE} = 6$. Since $b = 0.5$ is less than 1, the model is E-stable and agents are able to learn the REE in the long run. The result is presented in Figure 6.5.2. However, if $b = 1.5$ is assigned (because it is larger than 1) the divergence becomes clear. The REE can be solved for, $\alpha_0^{REE} = -4$ and $\alpha_1^{REE} = -6$, but under this condition for $b$, the REE is not learnable since the parameters diverge in the long run (See Figure 6.5.3).

# Chapter 7

# Information Diffusion

This chapter focuses on information diffusion — the transfer of information from one group to another. It should come as no surprise that information diffusion is an important research area for social scientists. While political scientists have been working on information diffusion processes for many decades (See Lazarsfeld, Berelson, and Gaudet 1944), there is also a very robust tradition in economics (See Chamley 2004). Financial economists, for example, have studied explanations for herding behavior, in which rational investors demonstrate some degree of behavioral convergence (Devenow and Welch, 1996). Most recently studies in monetary economics are exploring how information diffusion influences economic forecasting behavior. Information diffusion, as it pertains to the formation and distribution of expectations, specifically over economic variables such as inflation, has also been examined. Granato and Krause (2000), for example, investigate the possibility of inflation expectations diffusion within the electorate. Using educational differences as a proxy for information heterogeneity, they find the forecasts of the more educated influence the less educated group's forecasts and that the relation is asymmetric.

We are using an example based on Granato, Guse, and Wong (2008) which is extended by Granato, Lo, and Wong (2011) to the diffusion of inflation expectations. This EITM framework is used to examine the consequences of the asymmetric diffusion of expectations. In the spirit of a traditional two-step flow model of communication, less-informed agents learn the expectations of more-informed agents. An important finding is that when there is misinterpretation in the information acquisition process, a *boomerang effect* exists — the less-informed agents' forecasts confound those of more-informed agents.[1]

[1] Research also suggests that agents do not interpret public information in an identical manner (See Kandel and Zilberfarb 1999).

The EITM linkage for this example involves the unification of the behavioral concepts of social interaction, expectations, and learning, with the empirical concepts of simultaneity and prediction error. While expectations are again used, it is the social interaction that is crucial in this chapter.[2] Specifially, the social interaction involves information diffusion from better informed to less informed agents. The formal tools used — expectations and learning — are found in Chapters 4 and 5 and are not put in this chapter's Appendix. The empirical tools include an analogue for predicion error — defined as the mean square error — and is contained in the chapter's text. However, the time series tools used in this chapter dealing with simultaneity (endogeneity) and stationarity are contained in the Appendix.

## 7.1 Step 1: Relating Social Interaction, Expectations, and Learning to Simultaneity and Prediction Error

Information diffusion is characterized as when less informed agents can receive information from more informed agents for the purpose of enhancing their — the less informed agents — forecast accuracy. Further, the relation is not simply one group informing another. Instead, the relation between less- and more-informed agents — social interaction — involves expectations and learning. When these behavioral traits are linked with prediction error (forecast accuracy), the result is a set of distinct predictions based on these behavioral concepts and new equilibrium predictions about behavior. The EITM framework allows for an investigation of a *boomerang effect*, which is defined as a situation where the inaccurate forecasts of a less-informed group confound a more-informed group's forecasts.

[2] Note that under EITM, the empirical concept of simultaneity is used as a modeling and testing attribute and not a statistical nuisance. The simultaneity bias is explicitly modeled via the use of reduced forms and tested.

## 7.2 Step 2: Analogues for Social Interaction, Expectations, Learning, Simultaneity, and Prediction Error

In developing a formal model of inflation's behavior, Granato, Lo, and Wong (2011) link a standard Lucas aggregate supply model (Lucas, 1973) with an aggregate demand function. (Evans and Honkapohja, 2001). The aggregate supply function and demand function, respectively, are:

$$y_t = \bar{y} + \theta\left(p_t - E^*_{t-1}p_t\right) + \epsilon_t, \tag{7.2.1}$$

where $\theta > 0$, and:

$$m_t + v_t = p_t + y_t. \tag{7.2.2}$$

The variables are as follows: $p_t$ and $y_t$ are the price and output level at time $t$, respectively, $\bar{y}$ is the natural rate of output level, $E^*_{t-1}p_t$ is the expectation (may not be rational) of the price level at time $t$. $m_t$ is the money supply, and $v_t$ is a velocity shock. If agents form expectations rationally, it suggests people use all the available information to make the best possible forecasts of the economic variables relevant to them (Lucas 1972). In more technical terms, rational expectations (RE) is an equilibrium condition where the subjective expectations of some variable of interest is equivalent to the objective mathematical expectations *conditional* on all available information at the time the expectation is formed.[3]

It is assumed that velocity depends on some exogenous observables, $w_{t-1}$:

$$v_t = \kappa + \lambda w_{t-1} + \varepsilon_t, \tag{7.2.3}$$

where $\lambda > 0$ and the money supply $(m_t)$ is determined by the following policy rule:

$$m_t = \bar{m} + p_{t-1} + \phi w_{t-1} + \xi_t, \tag{7.2.4}$$

where $\phi > 0$, $\bar{m}$ is a constant money stock, and $\epsilon_t$, $\varepsilon_t$, and $\xi_t$ are iid stochastic shocks.

Using equations (7.2.1) thru (7.2.4) and defining $\pi_t = p_t - p_{t-1}$ and $E^*_{t-1}\pi_t =$

[3]Evans and Honkapohja (2001) argue that the assumption of RE is rather strong. They suggest the assumption can be relaxed by allowing agents to "learn" or update their conditional forecasts over time to obtain RE in the long run. This is called the adaptive learning approach which will be discussed later.

$E^*_{t-1}p_t - p_{t-1}$, gives the inflation dynamics:

$$\pi_t = \alpha + \beta E^*_{t-1}\pi_t + \gamma w_{t-1} + \eta_t, \tag{7.2.5}$$

where:

$$\begin{aligned}
\alpha &= (1+\theta)^{-1}(\kappa + \bar{m} - \bar{y}),\\
\beta &= \theta(1+\theta)^{-1} \in (0,1),\\
\gamma &= (1+\theta)^{-1}(\phi + \lambda),
\end{aligned}$$

and:

$$\eta_t = (1+\theta)^{-1}(\epsilon_t + \varepsilon_t + \xi_t).$$

Equation (7.2.5) is a self-referential model where inflation depends on its expectations $\left(E^*_{t-1}\pi_t\right)$, exogenous variables $(w_{t-1})$, and the stochastic shocks $(\eta_t)$. Since RE is assumed, the unique rational expectations equilibrium (REE) is:

$$\pi = \bar{a}^{REE} + \bar{b}^{REE} w_{t-1} + \eta_t, \tag{7.2.6}$$

where $\bar{a}^{REE} = \frac{\alpha}{1-\beta}$, and $\bar{b}^{REE} = \frac{\gamma}{1-\beta}$. From the equilibrium (7.2.6), agents can make rational forecasts $E_{t-1}\pi_t$ if they have the full information set $w_{t-1}$ at time $t-1$ such that:

$$E_{t-1}\pi_t = \bar{a}^{REE} + \bar{b}^{REE} w_{t-1}. \tag{7.2.7}$$

A body of research suggests forecast accuracy is associated with education, a common proxy for information levels $(w_{t-1})$ (Granato and Krause 2000; Carlson and Valev 2001). Agents possessing more education have more accurate forecasts. An extension of this finding is a second implication relating to information diffusion: more-informed agent forecasts and expectations (e.g., with higher education levels) influence less-informed agent forecasts and expectations (Granato and Krause 2000).

With these findings in mind take equation (7.2.5) and partition the information set $w_{t-1}$ into two parts: $w_{t-1} = \begin{pmatrix} x_{t-1} \\ z_{t-1} \end{pmatrix}$, where $x_{t-1}$ is "common" information, and $z_{t-1}$ represents the "advanced" information:

$$\pi_t = \alpha + \beta E^*_{t-1}\pi_t + \gamma_1 x_{t-1} + \gamma_2 z_{t-1} + \eta_t, \tag{7.2.8}$$

where $\gamma = (\gamma_1, \gamma_2)$. Following Granato, Guse and Wong (2008) populations are separated into two groups of agents. In the spirit of the classic two-step flow model (Lazarsfeld, Berelson, and Gaudet 1944), the groups are separated by the amount of information and interest they possess. Group L signifies the less-informed group. These agents are assumed to be less current on political and economic events. Members of the second group, Group H, are opinion leaders (e.g., issue publics) who are generally up-to-date on political and economic events. Opinion leaders are key in any information diffusion process since they are recognized by the less-informed group as having more and better information.

It follows that these two groups possess different information sets $(x_{t-1,}w_{t-1})$. Group H has the complete information set of $w_{t-1} \equiv \begin{pmatrix} x_{t-1} \\ z_{t-1} \end{pmatrix}$, while Group L only obtains the common information set $x_{t-1}$. The model also assumes there is a continuum of agents located on the unit interval $[0, 1]$ of which a proportion of, $1-\mu$, where $\mu \in [0, 1)$, are agents in Group H who are more informed when forecasting inflation.

Agents are interactive. Group L observes Group H's expectations to make its forecasts (but not vice versa). However, Group L agents may interpret (or even misinterpret) Group H's forecasts differently or may not be able to obtain the exact information from the more-informed agents. The next step is to introduce a distribution of observational errors, $e_{t-1}$, for Group L during the information diffusion process.[4] This gives Group L's forecasting model:

$$\pi_t = a_L + b_L x_{t-1} + c_L \widehat{\pi}_{t-1} + v_t, \tag{7.2.9}$$

and:

$$\widehat{\pi}_{t-1} = E^*_{H,t-1}\pi_t + e_{t-1}, \tag{7.2.10}$$

where $e_{t-1} \sim iid\,(0, \sigma_e^2)$ represents the observational errors which are uncorrelated with $v_t$ and $w_{t-1}$, and $\widehat{\pi}_{t-1}$ is the observed information that Group L obtains from Group H, $E^*_{H,t-1}\pi_t$ (See equation (7.2.12)) with observational error $(e_{t-1})$ at time $t-1$. Since Group L obtains the observed information after Group H forms its

[4]This assumption is supported by Kandel and Zilberfarb (1999). They find that people do not interpret the existing information in an identical way. Using Israeli inflation forecast data, they show that the hypothesis of identical-information interpretation is rejected. In other words, less-informed agents could experience some difficulty in understanding these expectations, and they may interpret the more-informed agents' information differently. It is also intuitively reasonable to believe agents are not able to obtain the exact information from others. Therefore, Granato, Lo, and Wong (2011) use a distribution of observational errors, $e_{t-1}$, to indicate the degree of misinterpretation of others' actions.

expectations, Group L treats the observed information as a predetermined variable.

The forecasting model for Group H is different since this group possesses the full information set to forecast inflation:

$$\pi_t = a_H + b_{1H} x_{t-1} + b_{2H} z_{t-1} + v_t. \tag{7.2.11}$$

In this model, Group L and Group H *do not* directly obtain RE. Instead, Group L and Group H forecast following the process of equations (7.2.9) and (7.2.11), respectively, and have data on the political economic system from periods $t_i = T_i, ..., t-1$, where $i \in \{L, H\}$. The time $t-1$ information set for the less-informed group, Group L, is $\{\pi_i, x_i, \widehat{\pi}_i\}_{i=T_L}^{t-1}$, but the information set for Group H at time $t-1$ is $\{\pi_i, w_i\}_{i=T_H}^{t-1}$.

With analogues for expectations and social interaction established, the analogue for learning is derived (See Evans and Honkapohja, 2001; Granato, Guse, and Wong 2008). Based on the adaptive learning method, agents attempt to learn the stochastic process by updating their forecasts (expectations) as new information becomes available. Both groups use (7.2.12) for their perceived law of motion (PLM) when they forecast the variable of interest (inflation rate):

$$E^*_{i,t-1}\pi_t = \varphi'_i q_{i,t-1}, \tag{7.2.12}$$

where $i \in \{L, H\}$, $q'_{L,t-1} \equiv (1, x_{t-1}, \widehat{\pi}_{t-1})$, $q'_{H,t-1} \equiv (1, x_{t-1}, z_{t-1})$, $\varphi'_L \equiv (a_L, b_L, c_L)$ and $\varphi'_H \equiv (a_H, b_{1H}, b_{2H})$. The inflation expectations, $E^*_{t-1}\pi_t$, in the society can be calculated as the weighted average of the expectations from both groups:

$$E^*_{t-1}\pi_t = \mu E^*_{L,t-1}\pi_t + (1-\mu) E^*_{H,t-1}\pi_t. \tag{7.2.13}$$

Using equations (7.2.8) thru (7.2.11) and (7.2.13), results in the actual law of motion (ALM):

$$\pi_t = \Omega_\alpha + \Omega_x x_{t-1} + \Omega_z z_{t-1} + \Omega_e e_{t-1} + \eta_t, \tag{7.2.14}$$

where:

$$\begin{aligned}
\Omega_\alpha &= \alpha + \beta\mu a_L + \beta(1-\mu) a_H, \\
\Omega_x &= \beta\mu b_L + [\beta\mu c_L + \beta(1-\mu)] b_{1H} + \gamma_1, \\
\Omega_z &= [\beta\mu c_L + \beta(1-\mu)] b_{2H} + \gamma_2,
\end{aligned}$$

and:

$$\Omega_e = \beta\mu c_L.$$

Equations (7.2.5), (7.2.12), and (7.2.14) represent a system that now incorporates adaptive learning. Both Group H and Group L use their PLM's (i.e., equation (7.2.12)) to update their forecasts of inflation ($E^*_{i,t-1}\pi_t$, in equation (7.2.5)) based on information, $q_{i,t-1}$.

Evans (1989) and Evans and Honkapohja (1992) show that mapping the PLM to the ALM is generally consistent with the convergence to REE under least square learning. Further, assuming that agents have a choice of using one of several forecasting models and that there are equilibrium predictions in these models, Guse (2005, 2006) refers to a resulting stochastic equilibrium as a "mixed expectations equilibrium" (MEE).[5] Computing the linear projections on equations (7.2.8), (7.2.12), and (7.2.13), the MEE coefficients results in the following:[6]

$$\bar{\varphi}_L = \begin{pmatrix} \bar{a}_L \\ \bar{b}_L \\ \bar{c}_L \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{1-\beta}\left(1-\bar{c}_L\right) \\ \frac{\gamma_1}{1-\beta}\left(1-\bar{c}_L\right) \\ \frac{\bar{b}_{2H}^2\sigma_z^2}{\bar{b}_{2H}^2\sigma_z^2+(1-\beta\mu)\sigma_e^2} \end{pmatrix} \tag{7.2.15}$$

and

$$\bar{\varphi}_H = \begin{pmatrix} \bar{a}_H \\ \bar{b}_{1H} \\ \bar{b}_{2H} \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{1-\beta} \\ \frac{\gamma_1}{1-\beta} \\ \frac{\gamma_2}{1-\beta+\beta\mu(1-c_L)} \end{pmatrix}, \tag{7.2.16}$$

where $\gamma \equiv (\gamma_1, \gamma_2)$.

The MEE (7.2.15) and (7.2.16) is the equilibrium of the forecasting models for Group L and Group H, respectively. Recall from (7.2.6) that the REE is $\bar{a}^{REE} = \frac{\alpha}{1-\beta}$ and $\bar{b}^{REE} = \frac{\gamma}{1-\beta}$. Both groups can obtain the REE if they are able to receive the same complete information. However, because of the process of information diffusion, Groups L and H fail to obtain the REE.

The observational error $e_{t-1}$ plays a significant role in the model. Whether Group L uses the observed information from Group H depends on how accurately the less-informed group understands information (the expectations) from the more-informed group. The accuracy is represented by the variance of the observational error, $\sigma_e^2$.

Equation (7.2.15) implies that $0 < \bar{c}_L \leq 1$ for $\beta < \frac{1}{\mu}$. If Group L can fully understand and make use of Group H's expectations (i.e., $\sigma_e^2 \to 0$) , then $\bar{c}_L = 1$ (by

[5]In this model, agents have a choice to be either in Group H or in Group L when they form their forecasting models.

[6]To obtain the MEE, one can solve for the orthogonality condition (OC) using ALM (7.2.14) and PLM (7.2.12). For Group H, the OC is $E\left(\pi_t - E^*_{H,t-1}\pi_t\right)(1, x_{t-1}, z_{t-1}) = 0$. For Group L, the OC is: $E\left(\pi_t - E^*_{L,t-1}\pi_t\right)(1, x_{t-1}, \hat{\pi}_{t-1}) = 0$.

solving equations (7.2.15) and (7.2.16) with $\sigma_e^2 = 0$). In addition, $\bar{c}_L \to 0$ as $\sigma_e^2 \to \infty$ and the values of $\bar{c}_L$ affect $\bar{a}_L$ and $\bar{b}_L$ . If $\bar{c}_L \to 0$, $\bar{a}_L \to \frac{\alpha}{1-\beta}$ and $\bar{b}_L \to \frac{\gamma_1}{1-\beta}$, and both $\bar{a}_L$, $\bar{b}_L \to 0$ if $\bar{c}_L \to 1$.

In the case of Group H, under the assumption that the covariance between $x_t$ and $w_{2,t}$ is zero, $\bar{c}_L$ does not affect $\bar{a}_H$ and $\bar{b}_{1H}$. Both will approach the REE,[7] $\left(\bar{a}_H, \bar{b}_{1H}\right) \to \left(\frac{\alpha}{1-\beta}, \frac{\gamma_1}{1-\beta}\right)$. However, equation (7.2.16) shows that $\bar{b}_{2H}$ is affected by $\bar{c}_L$, where $\left|\bar{b}_{2H}\right| \in \left(\frac{|\gamma_2|}{1-\beta(1-\mu)}, \frac{|\gamma_2|}{1-\beta}\right)$ for $\beta \in [0,1)$ and $\left|\bar{b}_{2H}\right| \in \left(\frac{|\gamma_2|}{1-\beta}, \frac{|\gamma_2|}{1-\beta(1-\mu)}\right)$ for $\beta \in (-\infty, 0)$. This latter relation is evidence of *a boomerang effect on expectations*: the observational error of the less-informed group biases the parameter(s) of the highly informed group's forecasting rule.[8]

The applied statistical analogue for prediction (forecast) error is the mean square error (MSE). For the inflation forecast error the mean square error is represented by the following formula: :

$$MSE_i \equiv E\left(\pi_t - E_{i,t-1}^{*}\pi_t\right)^2,$$

for $i \in \{L, H\}$.

## 7.3 Step 3: Unifying and Evaluating the Analogues

The formal model demonstrated that Group L places weight on the observed information from Group H. Group L makes use of Group H's expectations (i.e., higher $\bar{c}_L$) as long as Group L does not face large variation in observation error when interpreting Group H's information (i.e., lower $\sigma_e^2$). Linking the formal and applied statistical analogues shows how expectations, information diffusion, and learning create testable dynamics.

To show this, calculate the mean squared error (MSE) for the forecasts of Groups L and H, respectively:[9]

$$MSE_L = \left[\frac{\gamma_2\left(1-\bar{c}_L\right)}{1-\beta+\beta\left(1-\bar{c}_L\right)\mu}\right]^2 \sigma_z^2 + (1-\beta\mu)^2 \bar{c}_L^2 \sigma_e^2 + \sigma_\eta^2 \qquad (7.3.1)$$

$$MSE_H = \left(\beta\mu\bar{c}_L\right)^2 \sigma_e^2 + \sigma_\eta^2 \qquad (7.3.2)$$

[7] If the $cov\left(x_t, w_{2,t}\right) \neq 0$, then $\bar{b}_{1H}$ can also be affected by the less informed group's forecast errors.

[8] See Granato, Guse, and Wong (2008: 358-360) for details.

[9] For comparison, the MSE's are calculated for situations in which both groups have the same (full) information set and learn independently. Both groups' MSE's are at a minimum when $MSE_L = MSE_H = \sigma_\eta^2$.

where $MSE_i \equiv E\left(\pi_t - E^*_{i,t-1}\pi_t\right)^2$ for $i \in \{L, H\}$ .

Equation (7.3.1), using different values of $\sigma_e^2$, depicts the accuracy of the less-informed group's predictions. If Group L is able to fully understand the expectations from Group H (i.e., without any observation errors $\sigma_e^2 = 0$), the result is that Group L obtains the minimum MSE ($MSE_L = \sigma_\eta^2$). Otherwise, the finite $\sigma_e^2$ reduces the less-informed agents' predictive accuracy (where $MSE_L > \sigma_\eta^2$).

More importantly, due to the information diffusion, Group H fails to obtain the most accurate forecast. If there is no information diffusion process, then both groups form their forecasts independently, Group H obtains the minimum forecast error, $MSE_H = \sigma_\eta^2$. However, when information diffusion exists, with a finite $\sigma_e^2$, Group H has *higher* forecast errors: $MSE_H = (\beta\mu\bar{c}_L)^2\,\sigma_e^2 + \sigma_\eta^2 > \sigma_\eta^2$ in equation (7.3.2). This result is called *the boomerang effect on the MSE.*[10]

The results for Group H indicate that only the two limit points of the variance of the observation errors ($\sigma_e^2 = 0$ or $\sigma_e^2 \to \infty$) produce the most efficient outcome (i.e., when $\sigma_e^2 = 0$, $\bar{c}_L = 1$). Stated differently, Group L uses the expectations from the highly informed group. This implies that Group L's expectations become exactly the same as those of Group H, resulting in both groups forecasting efficiently. However, if $\sigma_e^2 \to \infty$, $\bar{c}_L = 0$. In this case, Group L is unable to interpret Group H's expectations and eventually discards them. Both groups learn independently and the boomerang effect is absent.

Surveyed inflation expectations from the SRC at the University of Michigan are used to test the dynamics embedded in (7.2.5). The tests are directed at two things. First, our theoretical model assumes that information diffusion is asymmetric: the expectations of Group H influence the expectations of Group L. The first test serves as a necessary condition for the second test. The second test examines whether the boomerang effect exists and involves examining whether larger observation errors made by Group L agents ($\sigma_e^2$) results in greater inaccuracy in inflation predictions by Group H agents ($MSE_H$).[11]

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[10] See Proposition 4 in Granato, Guse, and Wong (2008: 360-361)

[11] Inflation expectations surveys are conducted by the SRC at the University of Michigan and the results are published in the Survey of Consumer Attitudes. Since 1978 the center has conducted monthly telephone interviews from a sample of at least 500 households randomly selected to represent all American households, excluding those in Alaska and Hawaii. Each monthly sample is drawn as an independent cross-section sample of households. Respondents selected in the drawing are interviewed once and then re-interviewed six months later. This rotating process creates a total sample made up of 60 percent new respondents and 40 percent prior respondents.

Survey respondents are asked approximately 50 core questions that cover three broad areas of consumer opinions: personal finances, business conditions, and buying conditions. The following questions relate to measuring inflation expectations:

Granger causality tests are used to test what variables are exogenous or endogenous. Since there is evidence that the data possess unit roots, first differences for all classes of inflation forecasts are employed. The Akaike information criterion (AIC) and Lagrange multiplier (LM) test statistics suggesting that the VAR system with lag order of seven is preferable on the basis of a minimum AIC with no serial correlation or heteroskedasticity in the residuals.[12]

Table 7.1 reports results of the Granger causality tests. The null hypotheses that Group H does not Granger-cause Group L2 is rejected (p-value equals 0.030). However, Group H does not Granger-cause Group L1 (p-value equals 0.122). Note too that Group L1 Granger causes Group L2 (p-value equals 0.047). In contrast, Groups L1 and L2 do not Granger-cause Group H. Another finding is that Group L2 does not Granger-cause Group L1's forecasts. Overall, the testing results in Table 7.1 clearly indicate that there is an asymmetric information diffusion: the inflation forecasts of the more-educated affect the less-educated.

To test for the existence of a boomerang effect requires a determination of whether a "positive" relation exists between the size of observation errors of less-informed agents and the size of forecast inaccuracy of more-informed agents. The size of observation error ($e_t$) is based on its variance ($\sigma_e^2$), while the forecast (prediction) accuracy of the more-informed is the size of the mean square error of Group H's forecasts ($MSE_H$).

Using equations (7.2.9) and (7.2.10), it is possible to construct the following regression model:

$$E^*_{Lj}\pi_t = a_{Lj} + b_{Lj}x_{t-1} + c_{Lj}\left(E^*_{H,t-1}\pi_t + e_{Lj,t-1}\right), \tag{7.3.3}$$

1. During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?

2. By about what percent do you expect prices to go (up/down), on the average, during the next 12 months?

If respondents expect that the price level will go up (or down) on question 1, they are asked in the second question to provide the exact percent the price level will increase (or decrease), otherwise the second question is coded as zero percent. Then divide the inflation expectation survey data into different educational categories. To be consistent with the theory, the respondents with college or graduate degrees are put in the highly informed group (Group H) and those without a college degree are categorized as the less-informed group (Group L). Based on the unique characteristics of the data set, it is possible to to further separate Group L into two distinct levels: (1) high school diploma or some college (denoted as "L1"); and (2) less than high school or no high school diploma (denoted as "L2").

[12]The unit root test results are based on both the augmented Dickey-Fuller test (1979), and the Elliott-Rothenberg-Stock test (1996). The results of the unit root tests and of the lag order selection for the VAR are available from the current authors on request.

If forecasts of the *higher* educated group Granger-cause those of the *less* educated group?

| | Null Hypothesis | Chi-sq statistics [*P*-value] |
|---|---|---|
| a. | Group H does not Granger-cause Group L1 | 11.401 [0.122] |
| b. | Group H does not Granger-cause Group L2 | 15.522** [0.030] |
| c. | Group L1 does not Granger-cause Group L2 | 14.253** [0.047] |

If forecasts of the *less* educated group Granger-cause those of the *higher* educated group?

| | Null Hypothesis | Chi-sq statistics [*P*-value] |
|---|---|---|
| d. | Group L1 does not Granger-cause Group H | 3.897 [0.792] |
| e. | Group L2 does not Granger-cause Group H | 7.583 [0.371] |
| f. | Group L2 does not Granger-cause Group L1 | 2.603 [0.919] |

** indicates statistical significance at 5 percent.

Table 7.1: Granger Causality Test Results: Group H, Group L1, and Group L2

where $E^*_{Lj,t-1}\pi_t$ and $E^*_{H,t-1}\pi_t$ represent the inflation forecasts of less and more-informed groups, respectively, $j \in \{1,2\}$ and $x_t$ is the information set for inflation forecasts for Group L, which includes the current and lagged federal funds rate, the current inflation rate, and oil prices.[13] The series, $\sigma^2_{e_{Lj}}$, is constructed using a rolling regression technique in which the regression window of (7.3.3) is set at 12 years and moved forward every quarter.[14]

The observation error generated from equation (7.3.3) for the less-informed groups is:

$$e_{Lj,t-1} = \frac{E^*_{Lj,t-1}\pi_t - a_{Lj} - b_{Lj}x_{t-1} - c_{Lj}E^*_{H,t-1}\pi_t}{c_{Lj}}.$$

This result follows that the variances of the observation error $\left(\sigma^2_{e_{Lj},t}\right)$ for the less-informed groups is:

$$\sigma^2_{e_{Lj},t} = \frac{\sum_t^{t+s} e^2_{Lj,t}}{s-1}, \ \forall t$$

where $s$ represents the number of quarters in rolling windows.

Applying the same rolling regression technique to estimate the mean square error for Group H:

$$MSE_{H,t} = \frac{\sum_t^{t+s}\left(\pi_t - E^*_{H,t-1}\pi_t\right)^2}{s}, \ \forall t.$$

A concern is the long-run (inter-)relation between $MSE_H$ and $\sigma^2_{e_{Lj}}$ and also whether a larger value of $\sigma^2_{e_{Lj}}$ causes $MSE_H$ to increase. This result would support the

[13]The data are from the FRED database provided by the Federal Reserve Bank of St. Louis.

[14]15-year and 10-year rolling regression windows are used in this empirical analysis. However, results from using different choices of regression windows do not show any substantive or statistical difference, indicating the robustness of empirical findings presented in the paper.

A. Data in levels

| Variable | Augmented Dickey-Fuller Test $DF_\mu$ [a] | $DF_\tau$ [b] | Elliott-Rothenberg-Stock Test Optimal Lag | $DF-GLS_\mu$ [c] | $DF-GLS_\tau$ [c] | Conclusion |
|---|---|---|---|---|---|---|
| $MSE_H$ | -2.222 | -0.661 | 3 | -0.305 | -1.690 | I(1) |
| $\sigma^2_{e,L1}$ | -0.826 | -2.797 | 3 | -0.531 | -2.638 | I(1) |
| $\sigma^2_{e,L2}$ | -1.896 | -3.327* | 6 | -0.743 | -3.327** | I(1) |

B. Data in first differences

| Variable | $DF_\mu$ [a] | $DF_\tau$ [b] | Optimal Lag | $DF-GLS_\mu$ [c] | $DF-GLS_\tau$ [c] | Conclusion |
|---|---|---|---|---|---|---|
| $MSE_H$ | -4.536*** | -4.966*** | 2 | -2.041* | -2.371** | I(0) |
| $\sigma^2_{e,L1}$ | -7.616*** | -7.588*** | 2 | -2.957*** | -2.973* | I(0) |
| $\sigma^2_{e,L2}$ | -7.002*** | -6.926*** | 7 | -3.367*** | -3.440** | I(0) |

***, **, and * indicate statistical significance at 1, 5 and 10 percent, respectively.
a. Test allows for a constant; one-sided test of the null hypothesis that the variable is nonstationary. Fuller (1976) 1 and 5 percent critical values for a sample size of 41 equal -3.597 and -2.934, respectively.
b. Test allows for a constant; one-sided test of the null hypothesis that the variable is nonstationary. Fuller (1976) 1 and 5 percent critical values for a sample size of 41 equal -4.196 and -3.522, respectively.
c. Test allows for a constant; one-sided test of the null hypothesis that the variable is nonstationary. The critical values, not reported here, are calculated from the response surface estimates of Table 1, Cheung and Lai (1995).

Table 7.2: Unit Root Test Results for $MSE_H$, $\sigma^2_{\tilde{e},L1}$, and $\sigma^2_{\tilde{e},L2}$

boomerang effect hypothesis. To obtain consistent estimates of the unknown parameters entering the system consisting of $MSE_H$, $\sigma^2_{e_{L1}}$, and $\sigma^2_{e_{L2}}$, first characterize the stochastic properties of these underlying variables.

Table 7.2 presents the augmented Dickey-Fuller (1979) and Elliott-Rothenberg-Stock (1996) test results. Granato, Lo, and Wong find that $MSE_H$, $\sigma^2_{e_{L1}}$, and $\sigma^2_{e_{L2}}$ all contain a unit root. With test results indicating that all variables in the system are non-stationary, the cointegration methodology is useful for exploring the long-run (inter-)relation among the variables and the existence of a boomerang effect.[15]

## 7.4 Leveraging EITM and Extending the Model

Granato, Lo, and Wong (2011) use the Survey Research Center (SRC) inflation expectations data to test the existence of asymmetric information diffusion and the boomerang effect. The quarterly survey data, divided along different educational groups, covers 1978 thru 2000. One of the attributes of using a formal model of social interaction, expectations and learning is the potential of a boomerang effect where more informed agents have the accuracy of their forecasts impaired by their

[15] Granato, Lo, and Wong (2011) use the Johansen test for this particular task is detailed in the application section of the Appendix.

interaction with less informed agents.

Testing for the existence of the boomerang effect, Granato, Lo, and Wong use a cointegration test to estimate the long run relation between the variance of observational errors from the less educated group and the mean square error of the more educated group's expectations. A long-run positive relation is found and there is evidence of a boomerang effect.

This model and test may be extended in several ways. Focusing on the boomerang effect several issues are open for examination. The first is that the model itself assumes the less informed agents are merely passive recipients of the information. There is a point in this model where the variance of the tranmission of the information can lead the less informed to discard the information. However, this process is not modeled and there is no estimation of how long it would take the less informed agents to discard the information.

A second point of examination is adding more information on the characteristics of various groups and their willingness to cooperate and use information. Jasso (2008) for example, has shown how various group characteristics can lead to situations where there is enhanced or reduced cooperation.

A third way of extending the model and test is to use experiments and vary the information but also the proportions of more informed and less informed agents. These population proportions, along with alternative types of information (not related to inflation expectations), and repeated play would contribute to the robustness and the circumstances when various information diffusion scenarios contribute to a boomerang effect.

## 7.5 Appendix

The tools in this chapter are used to create a transparent and testable linkage using social interaction, expectations, learning, and combining these behavioral concepts with empirical concepts — simultaneity and prediction error. The formal analogues for social interaction is mathematical substitution and the analogues for expectations and learning have been presented in Chapters 4 thru 6 and will not be repeated. The applied statistical tools used in this chapter that require further background is the use of advanced time series methods. We then demonstrate the use of these tools in the empirical application for this chapter.

### 7.5.1 Empirical Analogues

The applied statistical tools used in this chapter assist in sorting out issues of simultaneity (endogeneity) and causality. A basic tool to determine what variables are exogenous or endogenous is the Granger causality test (1969). But, Granger causality tests, while important, can be misleading without an accounting of the behavior of time series data.

In the early 1990s, research in applied econometrics went through a major revolution. The revolution was triggered by the fact that a large number of macroeconomic time series are non-stationary: their means and variances change over time. Standard estimation methods assume that the means and variances of the variables are constant and independent of time. Classical estimation methods to estimate relationships with non-stationary variables leads to a spurious regression problem. One way to "correct" spurious regressions is first-differencing (i.e., $\triangle y_t = y_t - y_{t-1}$). However, first differencing removes long-term relationships. The challenge is how to account for spurious regressions and not remove vital long-term equilibrium relations.

Cointegration is important in that it offers an estimation method that captures economic notation of a long-run equilibrium without the spurious regression problem. For example, Johansen stated in 1995 that:

> An economic theory is often formulated as a set of behavioural relations or structural equations between the levels of the variables, possibly allowing for lags as well. If the variables are $I(1)$, that is, non-stationary with stationary differences, it is convenient to reformulate them in terms of levels and differences, such that if a structural relation is modeled by a stationary relation then we are led to considering stationary relations between levels, that is, cointegrating relations (page 5).

Unit root and cointegration together give important implications for the estimation of a model using time series data. There have been various methods to test for the presence of unit roots and to estimate the parameters in cointegrating regressions. For unit-root tests we present:

- the Dickey-Fuller test
- the Augmented Dickey-Fuller test.

The tests for cointegration we include are:

- the Engle-Granger two-step procedure

- the Johansen (maximum likelihood).

## The Granger Causality Test

Implementing single equation Granger Causality tests are straightforward. In the case of single equation estimation, Freeman (1983) gives a clear exposition and we follow it here. The test requires estimating two equations in a manner similar to F-tests for the relevance of a regressor. One unresricted regression (including both variables) and one restricted (including only one variable) are estimated separately and the residuals are compared to see if there is a significant "difference" going from the unrestricted model to the restricted model. To test whether $X_t$ Granger causes $Y_t$ estimate the unrestricted model as:

$$Y_t = \sum_{i=1}^{T} \alpha_{1i} Y_{t-i} + \sum_{i=1}^{T} \beta_{1i} X_{t-i} + v_{1t} \tag{7.5.1}$$

and the restricted model as:

$$Y_t = \sum_{i=1}^{T} \delta_{1i} Y_{t-i} + v_{2t}. \tag{7.5.2}$$

Now construct the tests,

$$\begin{aligned} H_0 &: \sum_{i=1}^{T} \beta_{1i} X_{t-i} = 0 \rightarrow x_t \text{ does not Granger "cause" } y_t \\ H_A &: \sum_{i=1}^{T} \beta_{1i} X_{t-i} \neq 0 \rightarrow x_t \text{ Granger "causes" } y_t \end{aligned}$$

This is tested by saving the residual sum of squares from both models and using the formula:

$$\frac{\frac{RSS_R - RSS_U}{K_R}}{\frac{RSS_U}{(T-K)}} \sim F\left(K_R, T-K\right), \tag{7.5.3}$$

where:

- $T$ = sample size
- $RSS_R$ = residual sun of squares (restricted model)
- $RSS_U$ = residual sum of squares (unrestricted model)

- $K_R$ = number of restrictions (the number of $X_{t-i}$'s removed)
- $K$ = number of regressors (including the intercept) in the unrestricted regression.

As might be expected, it is also necessary to reverse the test and see if $Y_t$ Granger "causes" $X_t$.

## Unit Root Tests

### Dickey-Fuller (DF) Test

The Dickey and Fuller (1979) tests determines whether a unit root is present in an autoregression model. To see the motivation for the DF test procedure, we start with the simplest example, an AR(1) model without an intercept:

$$Y_t = \phi_1 Y_{t-1} + \varepsilon_t, \tag{7.5.4}$$

where $\varepsilon_t$ has a zero mean, and is independently and identically distributed, i.e., $\varepsilon_t \sim iid(0, \sigma^2)$. We can rewrite AR(1) model by subtracting $Y_{t-1}$ from both sides of (7.5.4):

$$\triangle Y_t = \gamma Y_{t-1} + \varepsilon_t, \tag{7.5.5}$$

where $\gamma = \phi_1 - 1$. If $\gamma = 0$ (i.e., $\phi_1 = 1$), there is a unit root since $Y_t = Y_{t-1} + \varepsilon_t$. However, $\triangle Y_t = \varepsilon_t$ is a pure random walk process given that $\varepsilon_t$ is a random variable with zero mean, unit variance, and zero autocorrelations.

The $Y_t$ is integrated (or $Y_t \sim I(1)$) implying that future changes do not depend on the current level. The null hypothesis is straightforward since it corresponds to the existence of a (single) unit root, that is $H_0 : \gamma = 0$ implying $Y_t = Y_{t-1} + \varepsilon_t$ and the series $Y_t$ is $I(1)$. The alternative hypothesis is one-sided alternative $H_a : \gamma < 0$ (i.e., $\phi_1 < 1$) and this result corresponds to $Y_t$ being an $I(0)$ process.

Note that $r > 0$ is not chosen as an alternative hypothesis because it corresponds to $\phi_1 > 1$ and in this case the process generating $Y_t$ will be unstable. The test statistic is referred as $\widehat{\tau}$, and the appropriate critical values depend on the sample size. The critical values can be found in Fuller (1976) or MacKinnon (1991).

Regression (7.5.4) can be extended to include an intercept:

$$Y_t = \mu + \phi_1 Y_{t-1} + \varepsilon_t. \tag{7.5.6}$$

Recall we subtract $Y_{t-1}$ from both sides:

$$\triangle Y_t = \mu + \gamma Y_{t-1} + \varepsilon_t. \tag{7.5.7}$$

Here, the null hypothesis is $H_0 : \gamma = 0$ against the alternative hypothesis of $H_a : \gamma < 0$ and the test statistic is referred as $\widehat{\tau}_{\mu}$.

Similarly, regression (7.5.4) can be extended to include both an intercept and a trend:

$$Y_t = \mu + \beta t + \phi_1 Y_{t-1} + \varepsilon_t, \tag{7.5.8}$$

which can be rewritten as:

$$\triangle Y_t = \mu + \beta t + \gamma Y_{t-1} + \varepsilon_t. \tag{7.5.9}$$

The associated test statistic is referred as $\widehat{\tau}_{\beta}$.

### Augmented Dickey-Fuller (ADF) Test

The Augmented Dickey Fuller test extends the DF test but it allows for the possibility that the difference process is persistent. Consider an actual economic time series, $Y_t$, which can be generated by an AR($p$) process:

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + \varepsilon_t. \tag{7.5.10}$$

However, an AR(1) process is used to fit time time series, $Y_t$, such that:

$$Y_t = \mu + \phi_1 Y_{t-1} + \upsilon_t. \tag{7.5.11}$$

It follows that:

$$\upsilon_t = \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + ... + \phi_p Y_{t-p} + \varepsilon_t. \tag{7.5.12}$$

Note the residual, $\upsilon_t$, is not free of serial correlation because of the presence of lagged $Y_t$ in $\upsilon_t$ and $\upsilon_{t-k}$ for $k \geq 1$. To remove the serial correlations in residual, $\upsilon_t$, a common strategy is to add more lagged values of $Y_t$ (i.e., to increase the order of the AR process) till a higher order of AR process, say AR($p$), can generate white noise residuals in (7.5.11).

Typically, the general-to-specific search strategy can be used to determine $p$. Specifically, the search strategy starts with picking a maximum lag lenght, say $p^*$, that seems likely to ensure white noise residuals in the fitted equation (7.5.11). For example, if $Y_t$ is monthly data, the maximum lag length $p^*$ would be set at 12. One can then start to fit sucessively lower order models, such as AR($p^*-1$), AR($p^*-2$), and so on until a lower order AR model possesses zero autocorrelations in the residuals. An alternative popular approach is to choose $p$ using information criteria, such as Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC).

If $p > 1$ is found, Fuller (1976) suggests the AR($p$) process in (7.5.10) can be rewritten as:

$$\triangle Y_t = \mu + \gamma Y_{t-1} + \sum_{j=1}^{p-1} \alpha_j \triangle Y_{t-j} + \varepsilon_t. \tag{7.5.13}$$

We will use an AR(2) model to illustrate this rewritting. Equation (7.5.14) gives an AR(2) process:

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t. \tag{7.5.14}$$

Note that this is the same as:

$$Y_t = \mu + (\phi_1 + \phi_2) Y_{t-1} - \phi_2 (Y_{t-1} - Y_{t-2}) + \varepsilon_t. \tag{7.5.15}$$

Subtracting $Y_{t-1}$ from both sides of (7.5.15) gives:

$$\triangle Y_t = \mu + \gamma Y_{t-1} + \alpha_1 \triangle Y_{t-1} + \varepsilon_t, \tag{7.5.16}$$

where $\gamma = \phi_1 + \phi_2 - 1$ and $\alpha_1 = -\phi_2$. This shows that $\triangle Y_{t-1}$ should be added to the regression if the appropriate model is an AR(2). We can then test the unit root by testing if $\gamma = 0$ in the AR(2) process. In this particular case, the standard DF model has been "augmented" by $\triangle Y_{t-1}$ (with coefficient of $\alpha_1$) and we refer to this as the ADF(1) model.

In general, the ADF($p$) model (which corresponds to an AR($p+1$) model) can be written as:

$$\triangle Y_t = \mu + \gamma Y_{t-1} + \sum_{j=1}^{p} \alpha_j \triangle Y_{t-j} + \varepsilon_t. \tag{7.5.17}$$

To test whether a unit root is present, we test $H_0 : \gamma = 0$. The associated test statistic is referred as $\widehat{\tau}_\mu$. The model (7.5.17) can also be extended to include a determinstic trend:

$$\triangle Y_t = \mu + \beta t + \gamma Y_{t-1} + \sum_{j=1}^{p} \alpha_j \triangle Y_{t-j} + \varepsilon_t. \tag{7.5.18}$$

In this case, the unit-root test statistic on $H_0 : \gamma = 0$ is referred as $\widehat{\tau}_\beta$.

## Cointegration Tests

### The Engle-Granger (EG) Procedure

The difference between a cointegrating regression and a spurious regression is whether a linear combination of $I(1)$ variables is stationary. Suppose the variables $X_t$ and $Y_t$ are random walk processes, and we would like to determine whether a linear combination of them can be reduced to $I(0)$. First, their linear combination is written as:

$$Y_t = \varphi_1 + \varphi_2 X_t + \xi_t \tag{7.5.19}$$

Next, determining whether $X_t$ and $Y_t$ are cointegrated, the natural focus will be on the properties of the residual, $\xi_t$. If $\xi_t \sim I(0)$, then (7.5.19) is a cointegrating regression; if $\xi_t \sim I(1)$, then (7.5.19) is a spurious regression or a misspecified regression involving omitted relevant $I(1)$ variables.

With this description in mind the Engle-Granger procedure (See Engle and Granger 1987) includes two steps. The first step of the test is to use either the DF or ADF tests discussed earlier to assess the order of integration of each time series in the regression. For simplicity, we continue to consider the bivariate case that involves only two time series of $X_t$ and $Y_t$. The second step is to assess whether the residuals, $\xi_t$, are consistent with $I(1)$ process: If it is $I(1)$, the regression (7.5.19) is not a cointegrating regression; if it is $I(0)$, then (7.5.19) is a cointegrating regression.

In sum, the Engle-Granger procedure has the following steps:

**Step** 1: Ascertain using DF procedures (ADF is recommended here) how many series are $I(1)$. Then create a residual by running a regression on the variable of interest:

$$\underset{I(1)}{Y_t} - \varphi_1 - \underset{I(1)}{\varphi_2 X_t} = \underset{I(0)?}{\xi_t}.$$

**Step** 2: Run an ADF on $\xi_t$ :

$$\Delta\xi_t = \Pi\xi_{t-1} + \sum_{i=2}^{n} \rho_i \Delta\xi_{t-i} + v_t.$$

where the hypotheses are:

$\mathbf{H}_0$:$\Pi = 0 \Rightarrow \xi_t \sim \mathbf{I}(1)$ "not cointegrated."

$\mathbf{H}_A$:$\Pi < 0 \Rightarrow \xi_t \sim \mathbf{I}(0)$ "cointegrated."

As a final matter, the Engle-Granger procedure requires the researcher to choose one of the jointly endogenous variables to put on the left-hand side (i.e., as the dependent variable) of the regression. However, this means one can reach different conclusions on the existence of cointegration.

### The Johansen Procedure

An alternative multivariate — system based — approach, such as the Johansen (1988, 1992) procedure has an advantage over the Engle-Granger procedure since all variables are treated as endogenous. The researcher does not need to choose $X_t$ over $Y_t$ (or vice versa) as the dependent variable when testing for cointegration. With a system of equations, testing for the existence of cointegration is based on the rank of a matrix.

Johansen procedure can be illustrated by considering the multivariate model of a VAR($k$) with $N$ variables:

$$X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + ... + \Pi_k X_{t-k} + \Phi D_t + \epsilon_t, t = 1, ..., T \tag{7.5.20}$$

where $X_t$ is a $N \times 1$ vector, $D_t$ is a $d \times 1$ vector of deterministic terms which can contain a constant, a trend, and (seasonal or event) dummy variables. Equation (7.5.20) can be reparameterised as:

$$\begin{aligned} \triangle X_t &= \Pi_1 X_{t-1} + \Gamma_1 \triangle X_{t-1} + \Gamma_2 \triangle X_{t-2} + ... + \Gamma_{k-1} \triangle X_{t-(k-1)} + \Phi D_t + \epsilon_t \quad (7.5.21) \\ &= \Pi_1 X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \triangle X_{t-i} + \Phi D_t + \epsilon_t, \quad t = 1, ..., T \end{aligned}$$

where $\Pi_1 = \alpha\beta'$ is the coefficient matrix on the lagged level ($X_{t-1}$), and $k$ is the lag length that can be chosen based on various information criteria (e.g., AIC and SIC) — subject to the lag choice passing a test for the absence of serial correlation in the residuals.

The advantage of the parameterization in (7.5.21) is in the intepretation of the coefficients where the effect of the levels is isolated in the matrix $\alpha\beta'$. The row of $\beta'$(i.e., the column of $\beta$) gives cointegrating vectors and the associated hypothesis test is whether any column of $\beta$ is statistically indifferent from zero vectors. In order to determine the number of cointegrating vectors in the system, the cointegration rank of $\Pi_1 = \alpha\beta'$ has to be determined.

The rest of this section follows Johansen's framework (Johansen 1995) closely to derive the formulation and solution for the problem of how to estimate the cointegrating rank of $\Pi_1$.[16]

Using new notation, we can rewrite (7.5.21) as:

$$Z_{0t} = \alpha\beta' Z_{1t} + \Psi Z_{2t} + \epsilon_t, \tag{7.5.22}$$

where $Z_{0t} = \triangle X_t$ with dimension of $N \times 1$, $Z_{1t} = X_{t-1}$ with dimension of $N \times 1$, $Z_{2t} = (\triangle X_{t-1}, \triangle X_{t-2}, ..., \triangle X_{t-(k-1)}, D_t)'$ with dimension of $[N(k-1)+d] \times 1$, and $\Psi = (\Gamma_1, \Gamma_2, ..., \Gamma_{k-1}, \Phi)$ with dimension of $N \times [N(k-1)+d]$.

Maximizing the log likehihood function of $logL(\Psi, \alpha, \beta, \Omega)$ gives the first order conditions for estimating $\Psi$ as:

$$\sum_{t=1}^{T}(Z_{0t} - \alpha\beta' Z_{1t} - \widehat{\Psi} Z_{2t})Z_{2t}' = 0. \tag{7.5.23}$$

Using a product moment matrix, we rewrite (7.5.23) as:

$$M_{02} = \alpha\beta' M_{12} + \widehat{\Psi} M_{22}, \tag{7.5.24}$$

where the product matrix is:

$$M_{ij} = T^{-1}\sum_{t=1}^{T} Z_{it}Z_{jt}'. \quad i,j = 0,1,2 \tag{7.5.25}$$

Postmultiplying (7.5.25) by $M_{22}^{-1}$ and solving for $\widehat{\Psi}$ gives:

$$\Psi(\alpha,\beta) = M_{02}M_{22}^{-1} - \alpha\beta' M_{12}M_{22}^{-1}. \tag{7.5.26}$$

This leads to the definition of the residuals:

[16] For the complete version of the derivation, consult Johansen (1995).

$$\begin{aligned} R_{0t} &= Z_{0t} - M_{02}M_{22}^{-1}Z_{2t}, \\ R_{1t} &= Z_{1t} - M_{12}M_{22}^{-1}Z_{2t}. \end{aligned} \tag{7.5.27}$$

Note that $R_{0t}$ ($R_{1t}$) gives residuals that can be obtained by regressing $\triangle X_t$ ($X_{t-1}$) on $\triangle X_{t-1}$, $\triangle X_{t-2}$,...,$\triangle X_{t-(k+1)}$, and $D_t$. Substitute $\Psi$ from (7.5.26) into (7.5.22), we solve for residuals of $\widehat{\epsilon}_t$:

$$\begin{aligned} \widehat{\epsilon}_t &= Z_{0t} - \alpha\beta^{'}Z_{1t} - \Psi Z_{2t} \\ &= Z_{0t} - \alpha\beta^{'}Z_{1t} - (M_{02}M_{22}^{-1} - \alpha\beta^{'}M_{12}M_{22}^{-1})Z_{2t} \\ &= Z_{0t} - M_{02}M_{22}^{-1}Z_{2t} - \alpha\beta^{'}(Z_{1t} - M_{12}M_{22}^{-1}Z_{2t}) \\ &= R_{0t} - \alpha\beta^{'}R_{1t} \end{aligned} \tag{7.5.28}$$

Now rewrite (7.5.28) as a regression equation which regresses $R_{0t}$ on $R_{1t}$:

$$R_{0t} = \alpha\beta^{'}R_{1t} + \hat{\epsilon}_t. \tag{7.5.29}$$

For any fixed $\beta$, $\alpha$ and $\Omega$ can be estimated by regressing $R_{0t}$ on $\beta^{'}R_{1t}$:

$$\hat{\alpha}(\beta) = S_{01}\beta(\beta^{'}S_{11}\beta)^{-1} \tag{7.5.30}$$

where $S_{ij} = T^{-1}\sum_{t=1}^{T} R_{it}R_{jt}^{'} = M_{ij} - M_{i2}M_{22}^{-1}M_{2j}$, $i, j = 0, 1$.

$$\begin{aligned} \hat{\Omega}(\beta) &= S_{00} - \hat{\alpha}(\beta)(\beta^{'}S_{11}\beta)\hat{\alpha}(\beta^{'}) \;\; (substitute\ for\ \hat{\alpha}) \\ &= S_{00} - S_{01}\beta(\beta^{'}S_{11}\beta)^{-1}\beta^{'}S_{10}. \end{aligned} \tag{7.5.31}$$

Apart from the constant, which disappears when forming ratios, the likelihood function to be maximized is now:

$$L(\beta) = |\hat{\Omega}(\beta)| = |S_{00} - S_{01}\beta(\beta^{'}S_{11}\beta)^{-1}\beta^{'}S_{10}| \tag{7.5.32}$$

Johansen (1995) has shown that the maximum of $L(\beta)$ can be obtained by solving the following eigenvalue problem:

$$|\lambda S_{11} - (S_{11} - S_{10}S_{00}^{-1}S_{01})| = 0 \tag{7.5.33}$$

For the $N$ solutions, $\lambda_1$, $\lambda_2$,..., $\lambda_N$, with corresponding eigenvectors $\nu_1$, $\nu_2$,..., $\nu_N$; each $\lambda_i$ is a scaler and each $\nu_i$ is a $N \times 1$ vector. The $\lambda_i$ are ordered such that $\lambda_1 > \lambda_2 > ... > \lambda_N$. The space spanned by the eigenvectors corresponding to the $r$ largest eigenvalues is the $r$-dimensional "cointegrating space". $\hat{\beta}$ with a dimension $N \times r$ corresponds to the first $r$ eigenvectors ($r = 0 \sim N$). For example, if $r = 2$, $\hat{\beta}$ is $N \times 2$, then the first column is the eigenvector corresponding to the largest eigenvalue, and the second column is the eigenvector corresponding to the second largest eigenvalue. If $r = 0$, then $\Pi_1 = 0$ and all the eigenvalues are zero.

Denote the eigenvalues and eigenvectors as $\hat{\lambda}_i$ and $\hat{\beta}$, respectively. Given the choice of $\hat{\beta}$, the resulting value of $L(\beta)$ in (7.5.32) becomes:

$$\begin{aligned} L(\beta) &= L(H(r)) \\ &= |S_{00}| \frac{|\hat{\beta}'(S_{11} - S_{10}S_{00}^{-1}S_{01})\hat{\beta}|}{|\hat{\beta}'S_{11}\hat{\beta}|} \\ &= |S_{00}| \prod_{i=1}^{r} (1 - \widehat{\lambda}_i), \end{aligned} \tag{7.5.34}$$

where $H(r)$ denotes the hypothesis that the rank of $\Pi_1$ is $r$. The likelihood ratio test statistic for $H(r)$ against $H(N)$ is:

$$\begin{aligned} LR(r|N) &= L(H(r))/L(H(N)) \\ &= \frac{|S_{00}| \prod_{i=1}^{r} (1 - \widehat{\lambda}_i)}{|S_{00}| \prod_{i=1}^{N} (1 - \widehat{\lambda}_i)}. \end{aligned} \tag{7.5.35}$$

The factor $|S_{00}|$ cancels, and the so-called Johansen's trace test statistic is:

$$\begin{aligned} Trace(r|N) &= -2ln[LR(r|N)] \\ &= -T \sum_{i=r+1}^{N} ln(1 - \hat{\lambda}_i). \end{aligned} \tag{7.5.36}$$

A non-rejection of $H(r)$ means the cointegration rank $\leq r$.

To pinpoint the "exact" cointegration rank for the estimation system, Johansen has suggested a test sequence. The test sequence starts with the first test that sets $r = 0$, and the associated test statistic is $Trace(0|N) = -T \sum_{i=1}^{N} ln(1 - \hat{\lambda}_i)$.

If $Trace(0|N)$ is larger than the critial value (which can be found in Johansen and Juselius 1990), then reject $H(0)$ and move onto testing $H(1)$. Otherwise, if $Trace(0|N)$ is smaller than the critial value, then conclude that there is no cointegrating relationship in the system.

For the second test statistic, set $r = 1$, then the test statistic is $Trace(1|N) = -T\sum_{i=2}^{N} ln(1-\hat{\lambda}_i)$. If $Trace(1|N)$ is larger than the critical value, then reject $H(1)$ and move onto testing $H(2)$. Contiune the sequence until $H(N-1)$ with $Trace(N-1|N) = -ln(1-\hat{\lambda}_N)$. If $Trace(N-1|N)$ exceeds the critical value, then reject $H(N-1)$ and the evidence is in favor of $H(N)$.

A test statistic alternative to the trace test is the maximum eigenvalue test known as $\lambda_{max}$. $\lambda_{max}$ test is used to test a cointegrating rank of $r$ against a cointegrating rank of $r+1$:

$$\begin{aligned}\lambda_{max} &= -2ln[LR(r|r+1)] \\ &= -Tln(1-\hat{\lambda}_{r+1}) \end{aligned} \tag{7.5.37}$$

For $\lambda_{max}$ test, a convenient notation to indicate which hypothesis is being tested against which alternative is $H(r|r+1)$. Its test sequence starts with $H(0|1)$, and if $H(0)$ is not rejected the sequence stops. Otherwise, move onto $H(1|2)$ and if necessary continue the testing sequence unitl $H(N-1|N)$.

The Johansen procedure can now be summarized:

1. Pick an autoregressive order $k$ for the variables in the system.

2. Run a regression of $\triangle X_t$ on $(\triangle X_{t-1}, \triangle X_{t-2}, ..., \triangle X_{t-(k+1)}, D_t)'$ and output the residual $\gamma_t^1$.

3. Run a regression of $X_{t-1}$ on $(\triangle X_{t-1}, \triangle X_{t-2}, ..., \triangle X_{t-(k+1)}, D_t)'$ and output the residual $\gamma_t^2$.

4. Compute the squares of the canonical correlations between $\gamma_t^1$ and $\gamma_t^2$, calling these $\varrho_1^2 > \varrho_2^2 > ... > \varrho_n^2$ .

5. Let $T$ denote the number of time periods available in the data, compute the trace test statistic as $-T\sum_{i=k+1}^{N} ln(1-\varrho_i^2)$. The null hypothesis is that there are $k$ or fewer than $k$ cointegrating vectors. Alternatively, one can choose to use the maximum eigenvalue test which gives statistic as $-Tln(1-\varrho_{r+1.}^2)$. The

null hypothesis in this case is that there are $r$ cointegrating vectors and the alternative hypothesis is that there are $r+1$ cointegrating vectors.

6. Compare the test statistic to the appropriate table in Johansen and Juselius (1990).

## Application: Testing for the Boomerang Effect

In this chapter the empirical implication of the model (See equation (7.3.2)) is that when information diffusion exists — with a finite $\sigma_e^2$— Group H will have *higher* forecast error since $MSE_H = (\beta\mu\bar{c}_L)^2\,\sigma_e^2 + \sigma_\eta^2 > \sigma_\eta^2$.[17]

Panel A in Table 7.3 reports the results of the cointegration tests of the long-run relation between $MSE_H$ and $\sigma_{e_{Lj}}^2$ . Columns 1 and 2 in Panel A summarize the results of cointegrating relations for two pairs of variables, $\left(MSE_H, \sigma_{e_{L1}}^2\right)$ and $\left(MSE_H, \sigma_{e_{L2}}^2\right)$. Both the maximum eigenvalues and trace statistics indicate that there are long-run equilibrium relations for both. Using the Johansen cointegration procedure, they find the cointegrating vectors of $\left(MSE_H, \sigma_{e_{Lj}}^2\right)$ are $(1, -29.58)$ and $(1, -21.54)$ for $j \in \{1, 2\}$.

These results show a positive long-run equilibrium relation with the existence of the boomerang effect between $MSE_H$ and $\sigma_e^2$ . It suggests that the mean square error on inflation forecasts for the respondents who hold a college degree or above $(MSE_H)$ are positively related with the measurement errors resulting from the non-degree-holding respondents $(\sigma_e^2)$.

The results in column (3), where the cointegrating system consists of all three variables of $MSE_H$, $\sigma_{e_{L1}}^2$, and $\sigma_{e_{L2}}^2$, provides further evidence to support the boomerang effect found in columns (1) and (2). With an estimated cointegrating vector of $(MSE_H,\ \sigma_{e_{L1}}^2\ ,\ \sigma_{e_{L2}}^2) = (1, -20.52, -0.79)$, this robustness check shows that both $\sigma_{e_{L1}}^2$ and $\sigma_{e_{L2}}^2$ are positively related with $MSE_H$ in the long run; that is, $MSE_H = 20.52\sigma_{e_{L1}}^2 + 0.79\sigma_{e_{L2}}^2$. The results in column (4) also show that the robust cointegrating vector among the three variables is solely the result of the boomerang effect since the variances of the measurement errors in the two levels of Group L are not cointegrated.

Furthermore, they examine if the boomerang effect is robust when both levels of Group L are combined. They determine this case by averaging the inflation expectations from Groups L1 and L2 to obtain $\sigma_e^2$. The cointegration estimation indicates

[17] The Johansen test is applicable here and we report the results in Granato, Lo, and Wong (2011).

A: Rank test and cointegrating relation

| | Variables in the System | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $MSE_H, \sigma^2_{e,L1}$ [a] | | $MSE_H, \sigma^2_{e,L2}$ [b] | | $MSE_H, \sigma^2_{e,L1}, \sigma^2_{e,L2}$ [c] | | $\sigma^2_{e,L1}, \sigma^2_{e,L2}$ [d] | |
| | (1) | | (2) | | (3) | | (4) | |
| Null Hypothesis | $\hat{\lambda}_{\max}$ | Trace | $\hat{\lambda}_{\max}$ | Trace | $\hat{\lambda}_{\max}$ | Trace | $\hat{\lambda}_{\max}$ | Trace |
| No rank | 12.82** | 15.22** | 8.00 | 12.20* | 48.60*** | 87.52*** | 6.52 | 8.80 |
| | [11.44] | [12.53] | [11.44] | [12.53] | [22.00] | [34.91] | [11.44] | [12.53] |
| At most 1 rank | 2.40 | 2.40 | 4.20 | 4.20 | 32.65*** | 38.92*** | 2.28 | 2.28 |
| | [3.84] | [3.84] | [3.84] | [3.84] | [15.67] | [19.96] | [3.84] | [3.84] |
| At most 2 ranks | - | - | - | - | 6.27 | 6.27 | - | - |
| | | | | | [9.24] | [9.24] | | |
| Conclusion | 1 cointegrating relation | | 1 cointegrating relation | | 2 cointegrating relations | | None | |
| Estimated Cointegration Vector | $(MSE_H, \sigma^2_{e,L1}) =$ (1, -29.58) | | $(MSE_H, \sigma^2_{e,L2}) =$ (1, -21.54) | | $(MSE_H, \sigma^2_{e,L1}, \sigma^2_{e,L2}) =$ (1, -20.52, -0.79) | | None | |

B: The direction of causality in VECM

| | Variables in the System | | | |
|---|---|---|---|---|
| | $MSE_H, \sigma^2_{e,L1}$ [a] | $MSE_H, \sigma^2_{e,L2}$ [b] | $MSE_H, \sigma^2_{e,L1}, \sigma^2_{e,L2}$ [c] | $\sigma^2_{e,L1}, \sigma^2_{e,L2}$ [d] |
| | (1) | (2) | (3) | (4) |
| Null Hypothesis | Chi-sq statistics [*P*-value] | Chi-sq statistics [*P*-value] | Chi-sq statistics [*P*-value] | Chi-sq statistics [*P*-value] |
| $\sigma^2_{e,L1}$ does not cause $MSE_H$ | 14.36*** [0.006] | - | 21.04*** [0.007] | - |
| $MSE_H$ does not cause $\sigma^2_{e,L1}$ | 3.82 [0.430] | - | 5.68 [0.682] | - |
| $\sigma^2_{e,L2}$ does not cause $MSE_H$ | - | 19.43*** [0.000] | 30.87*** [0.000] | - |
| $MSE_H$ does not cause $\sigma^2_{e,L2}$ | - | 4.72 [0.194] | 7.15 [0.521] | - |

***, **, and * indicate statistical significance at 1, 5 and 10 percent, respectively. We use the AIC criterion to choose the optimal number of lags to be included in each empirical model. 5 percent critical values, from Osterwald-Lenum (1992), for rank tests are in parentheses.
a. Test allows for a constant but no trend in the data space and 4 lags are included in the system.
b. Test allows for a constant but no trend in the data space and 3 lags are included in the system.
c. Test allows for a constant but no trend in the cointegration space and 8 lags are included in the system.
d. Test allows for a constant but no trend in the data space and 4 lags are included in the system.

Table 7.3: Johansen Cointegration Tests and Granger Causality Tests: $MSE_H$, $\sigma^2_{\tilde{e},L1}$, and $\sigma^2_{\tilde{e},L2}$

(results available from the authors) that the boomerang effect is still robust where $\sigma^2_e$ is positively related with $MSE_H$.

Additional support for a boomerang effect occurs if we see that the direction of causality runs from $\sigma^2_e$ to $MSE_H$ (but not vice versa). Panel B of Table 3 gives the results of the Granger-causality tests. The results from systems (1) and (2) indicate rejection of the null hypotheses that $\sigma^2_{e_{Lj}}$ does not Granger causes $MSE_H$, for $j \in \{1, 2\}$. The respective test statistics are equal to 14.36 and 19.43 and are significant at the 0.05 level. On the other hand, null hypothesis (for reverse causation) is not rejected. Column (3) in Panel B report associated results which are highly consistent with findings in columns (1) and (2).

# Chapter 8

# Political Parties and Representation

The relation between political parties and representation provides a useful window for understanding methodological unification. One well researched area focuses on when and why voters choose one party over the others. This choice is based on the relative political positions of parties on key policies. The work of Kedar (2005) is the focus of this chapter. Here EITM links the behavioral concept of decision making and the applied statistical concept, nominal choice. Empirical tools in this chapter involve discrete choice estimation methods. Formal tools include the application of basic decision theory, including an understanding of random utility models.

## 8.1 Step 1: Relating Decision Theory and Discrete Choice

Voting provides a useful window into methodological unification. Hotelling (1929) and Downs (1957) argue that voters choose one party over the others based on the relative political positions of parties — proximity voting theory. Voters are more likely to vote for a political party if the position of the party is closer to a voters' ideal position. As the party's position further deviates from a voter's ideal position, the voter receives less utility and is less likely to vote for it.[1] While the voting literature finds some empirical support for the proximity model, Kedar (2005) believes this effect would be reduced if the institutional environment involves power-sharing.

[1] Applications of this particular utility function abound. Erikson, Mackuen and Stimson (2002), for example, assume that voters' utility is an inverse function of the squared distance of party political position and the voters' ideal position.

## 8.2 Step 2: Analogues for Decision Making and Nominal Choice

Kedar (2005) asserts that, along with the proximity of parties' positions, voters are also concerned about each party's contribution to the aggregate policy outcome. Beginning with the proximity model:

$$U_{ij} = -\beta_1 \left(v_i - p_j\right)^2, \tag{8.2.1}$$

where $U_{ij}$ is the utility of voter $i$ for party $j$, $v_i$ is the ideal point of voter $i$, $p_j$ is the position of party $j$, and $\beta_1$ is a scalar representing the importance of party-position deviations. In Kedar's analogue for decision making, equation (8.2.1), voter $i$ perceives disutility for party $j$ when the position of party $j$ deviates from voter $i'$s ideal point. On the other hand, if the position of party $j$ is equivalent to his ideal point (i.e., $v_i = p_j$), no disutility is perceived to result from party $j$.

Assuming that party positions can affect policy outcomes, Kedar (2005) specifies the policy outcome as a weighted average of policy positions of the respective parties:

$$P = \sum_{k=1}^{m} s_k p_k, \tag{8.2.2}$$

where there are $m$ parties in the legislature, $0 < s_k < 1$ is the relative share of party $k$, and $\sum_{k=1}^{m} s_k = 1$ for all $k$.

If voters are policy-outcome oriented, and concerned that the policy outcome may deviate from their ideal point if party $j$ is not elected, then the utility of voter $i$ pertaining to party $j$ becomes:

$$U_{ij} = -\beta_2 \left[\left(v_i - P\right)^2 - \left(v_i - P_{-p_j}\right)^2\right], \tag{8.2.3}$$

where:

$$P_{-p_j} = \left(\frac{1}{\sum_{k \neq j} s_k}\right) \sum_{k \neq j} s_k p_k. \tag{8.2.4}$$

Equation (8.2.4) represents the policy outcome if party $j$ is not in the legislature and $\beta_2$ is a scalar weighting the deviations of the policy outcome when party $j$ is excluded.

Equation (8.2.3) provides an important insight on how voters view the contribution of party $j$ to the policy outcome affecting their utility. If party $j$ takes part in policy formulation and makes the policy closer to voter $i$'s ideal point $v_i$, that is, $\left(v_i - P_{-p_j}\right)^2 > (v_i - P)^2$, then voter $i$ will gain positive utility when party $j$ is involved in the policy formation process (i.e., $U_{ij} > 0$). However, if the inclusion of party $j$ makes the policy outcome increase in distance from voter $i$'s idea point such that $\left(v_i - P_{-p_j}\right)^2 < (v_i - P)^2$ , then the utility of voter $i$ for party $j$ is negative.

Now, consider the expectations analogue. Assume voter $i$ has expectations (an expected value) concerning party $j$ based on the weighted average of both the party's relative position and its contribution to policy outcomes. This analogue[2], in the context of voter $i$'s utility for party $j$, can be written as:

$$U_{ij} = \theta \left\{ -\gamma \left(v_i - p_j\right)^2 - (1-\gamma) \left[ (v_i - P)^2 - \left(v_i - P_{-p_j}\right)^2 \right] \right\} + \delta_j z_i, \qquad (8.2.5)$$

where $\theta$ is a scalar, $\delta_j$ is a vector of coefficients on voter $i$'s observable variables $z_i$ for party $j$, and $\gamma \equiv \beta_1 / (\beta_1 + \beta_2)$. When $\gamma \to 1$, the implication is that voters are solely concerned with a party's positions. This situation is called *representational voting behavior.* On the other hand, if $\gamma \to 0$ then voters vote for a party such that the policy outcome can be placed at the voter's desired position(s). This outcome is called *compensational voting behavior.*

From equation (8.2.5) , we obtain voter $i$'s optimal or "desired position for party $j$ by solving the first order condition of $U_{ij}$ with respect to $p_j$:

$$p_j^* = v_i \left[ \frac{\gamma (1 - s_j) + s_j}{\gamma \left(1 - s_j^2\right) + s_j^2} \right] - \frac{(1-\gamma) \left( s_j \sum_{k=1, k \neq j}^{m} s_k p_k \right)}{\gamma \left(1 - s_j^2\right) + s_j^2}. \qquad (8.2.6)$$

When $\gamma \to 1$ (representational voting), we have:

$$p_j^* = v_i. \qquad (8.2.7)$$

But, when $\gamma \to 0$ (compensational voting), we have:

$$p_j^* = \frac{v_i - \sum_{k=1, k \neq j}^{m} s_k p_k}{s_j}, \qquad (8.2.8)$$

[2]Note the analogue here for expectations is not to be confused with conditional expectation analogues discussed in earlier chapters.

and the policy outcome would be:

$$
\begin{aligned}
P\Big|_{\gamma \to 0, p_j = p_j^*} &= \sum_{k=1}^{m} s_k p_k = s_j p_j + \sum_{k=1, k \neq j}^{m} s_k p_k \\
&= s_j p_j^* + \sum_{k=1, k \neq j}^{m} s_k p_k \\
&= s_j \frac{v_i - \sum_{k \neq j}^{m} s_k p_k}{s_j} + \sum_{k=1, k \neq j}^{m} s_k p_k \\
&= v_i. \qquad (8.2.9)
\end{aligned}
$$

## 8.3 Step 3: Unifying and Evaluating the Analogues

In (8.2.7) thru (8.2.9), voters make an optimal voting decision based on representational (proximity) and compensational voting considerations. These two considerations reflect the levels of political bargaining in different institutional systems. In majoritarian systems, where the winning party is able to implement its ideal policy with less need for compromise, voters place greater value on $\gamma$ and vote for the party positioned closest to their ideal position. However, in the case where institutional power sharing ($\gamma$ is small) exists, voters select a party whose position is further from their ideal positions so as to draw the collective outcome closer to the voter's ideal point.

Kedar tests these empirical implications using survey data from Britain, Canada, Netherlands, and Norway:

**Hypothesis 1:** *Voters' behavior in the countries with a majoritarian system follows the proximity model more closely (larger $\gamma$) than those in the countries with a consensual system (smaller $\gamma$).*

**Hypothesis 2:** *The pure proximity model ($\gamma = 1$) does not sufficiently represent voting behavior.*

For Hypothesis 1, Kedar (2005) first identifies the institutional features of Britain, Canada, Norway, and the Netherlands. Using the indicators of majoritarianism and power-sharing in Lijphart (1984), she concludes that Britain and Canada are more unitary whereas the Netherlands and Norway are more consensual.

Methodological unification occurs when Kedar derives an empirical analogue for discrete choice, the log-likelihood multinomial model based on equation (8.2.5), and estimates issue voting in four political systems using three measures: i) seat shares in the parliament; ii) vote shares; and iii) portfolio allocation in government.

The empirical results support the first theoretical hypothesis: voting behavior in the majoritarian systems (i.e., Britain and Canada) is more consistent with the proximity model relative to that in the consensual systems (i.e., the Netherlands and Norway). Hypothesis 2 is tested using a likelihood ratio test. The results show that, in all four political systems, compensational voting behavior exists. The pure proximity model is an insufficient explanation.

## 8.4 Leveraging EITM and Extending the Model

In forming the behavioral mechanism of decision making, Kedar chooses utility maximization as an analogue: voters select their ideal party position and/or policy outcome by maximizing their utility. The author links the theoretical findings of the optimal choice model to multinomial estimation.

One way to build on the formal model is to relax the behavioral assumption that voters' expectations are error free since it is well-known that equilibrium predictions change when expectations are based on imperfect or limited information. The extension would amend the formal model of voter expectations to incorporate modern refinements on how voters adjust and learn from their expectation errors. Leveraging Kedar's EITM design allows drawing (empirical) implications on how voter expectations and learning affect ex-ante model predictions.

## 8.5 Appendix

In contrast to standard regression models which have continuous dependent variables, discrete choice models are statistical procedures that model choice made by individuals among a finite set of discrete alternatives. They can be classified according to the number of available alternatives. A discrete model for explaining the outcome as a choice between two alternatives is referred to as a binomial model (i.e., binary choice model). Discrete choice problems that involve choices between more than two alternatives are termed multinomial models. Examples of the application of multinomial models include the decision regarding which job/occupation to take, which shopping area to go to, which car to buy, which candidate to vote for in an election, and which mode of transportation to use for travel.

Discrete choice models estimate the probability that a person chooses a particular alternative. Sharing the same spirit as regression analysis, that links a dependent variable to a set of factors, a discrete choice model statistically links the choice made by each individual to a set of relevant factors. These factors typically include the characteristics of the individual (such as education level, gender, income level, and marital status) and the attributes of the alternatives (such as travel time, and costs in a study of which mode of transportation to take). A general framework for such probability models can be written as:

$$\begin{aligned} Prob(Individual\ i\ chooses\ alternative\ choice\ j) &\equiv P_{ij} \\ &= F(X_{ij} : \beta), \end{aligned} \tag{8.5.1}$$

where $X_{ij} = [Z_i, x_{ij}, x_{ik}]$, $\forall j \neq k$, $Z_i$ is a vector of characteristics of individual $i$, $x_{ij}$ ($x_{ik}$) is a vector of attributes of alternative $j$ (other alternatives $k$) for individual $i$, and $\beta$ is a set of parameters that are estimated by the choice probability model.

### 8.5.1 Empirical Analogues

Different assumptions about error distributions lead to different statistical choice models. In the binary choices setting, these models are binary probit and binary logit models. For multinomial choice settings, these models include multinomial logit and conditional logit models. Here we briefly discuss each of these statistical choice models.

## Binary Choice Models

In this setting, choices can include (among other things) a decision between engaging in an activity (or not), or a decision between two alternative activities.

#### Probit Models

For a decision between taking on an activity ($Y = 1$) or not ($Y = 0$), as in index function model (8.5.18) we let $Y^*$ be a latent variable representing the net benefit of taking on the activity. Alternatively, one can regard $Y^*$ as the utility of choosing the activity. Assume an individual would take on an activity ($Y = 1$) when the utility of choosing the activity is positive (ie., $Y^* > 0$). We write the specification of the model as:

$$
\begin{aligned}
Y^* &= \beta' X + \epsilon \\
Y &= \begin{cases} 1, & if\, Y^* > 0 \\ 0, & otherwise \end{cases} \\
and\, \epsilon &\sim standard\, normal.
\end{aligned}
\tag{8.5.2}
$$

With the assumption that error terms ($\epsilon$) are distributed standard normal, the probability of choosing the activity can be presented as a probit model[3]:

$$
Prob(Y = 1) = \int_{-\infty}^{\beta' X} \phi(t)dt = \Phi(\beta' X),
\tag{8.5.3}
$$

where the function $\Phi(\cdot)$ denotes the cumulative distribution of standard normal.

If the binary choice is a decision between two alternative activities such as choosing activity 1 ($Y_{i1} = 1$) or activity 2 ($Y_{i1} = 0$), the specification of the model is:

$$
\begin{aligned}
U_{i1} &= \beta' X_{i1} + \epsilon_{i1} \\
U_{i2} &= \beta' X_{i2} + \epsilon_{i2} \\
Y_{i1} &= \begin{cases} 1, & if\, U_{i1} > U_{i2} \\ 0, & otherwise \end{cases} \\
and\, \epsilon_{i1},\ \epsilon_{i2} &\sim standard\, normal,
\end{aligned}
\tag{8.5.4}
$$

where $U_{i1}$ ($U_{i2}$) is the utility that individual $i$ obtains from choosing activity 1 (2). With the assumption that both error terms ($\epsilon_{i1}$ and $\epsilon_{i2}$) are distributed standard normal, the probability of choosing activity 1 is a probit model:

$$
\begin{aligned}
Prob(Y_{i1} = 1) &= Prob(U_{i1} > U_{i2}) \\
&= Prob(\beta' X_{i1} + \epsilon_{i1} > \beta' X_{i2} + \epsilon_{i2}) \\
&= Prob(\epsilon_{i2} - \epsilon_{i1} < \beta' X_{i1} - \beta' X_{i2}) \\
&= \Phi(\beta(X_{i1} - X_{i2})/\sqrt{2}).
\end{aligned}
\tag{8.5.5}
$$

[3] In this appendix we assume the error term ($\epsilon$) is distributed standard normal (i.e., $\epsilon \sim N(0,1)$). We can also consider a case where $\epsilon$ has a general normal distribution, $\epsilon \sim N(0, \sigma^2)$. In this case the probit model can be written as: $Prob(Y = 1) = \Phi(\beta' X/\sigma)$.

### Logit Models

The specification of the model is the same as model (8.5.2) except that the error terms are assumed to have a logistic distribution (i.e., $\epsilon \sim logistic$). Then, the probability of choosing the activity is a logit model:

$$Prob(Y = 1) = \frac{e^{\beta' X}}{1 + e^{\beta' X}}. \tag{8.5.6}$$

Similarly, the specification of the model is the same as model (8.5.4) except that we assume $\epsilon_{i1}, \epsilon_{i2} \sim iid\ extreme\ values$. When both error terms are *iid extreme values*, their difference is distributed logistically. Therefore, the probability of choosing activity 1 is a logit model:

$$Prob(Y_{i1} = 1) = \frac{e^{\beta' X_{i1}}}{e^{\beta' X_{i1}} + e^{\beta' X_{i2}}}. \tag{8.5.7}$$

## Multinomial Choice Models

Here the decision involves more than two available alternatives. It has been pointed out that the probit model has limited use in multinomial choice models because of the need to evaluate multiple integrals of the normal distribution. With the limited use of probit models, we focus our attention on multinomial logit models and conditional logit models.

### Multinomial Logit Models

The multinomial logit (MNL) models are appropriate to use when the data consist of individual-specific characteristics. The general form of MNL models is:

$$\begin{gathered} U_{ij} = \beta_j' Z_i + \epsilon_{ij} \\ and\ \epsilon_{ij} \sim\ iid\ extreme\ values. \end{gathered} \tag{8.5.8}$$

where $U_{ij}$ is individual $i'$s utility of choosing the $j$th alternative, and $Z_i$ gives a vector of values that represent individual's characteristics. The model estimates a set of regression coefficients for each of the alternatives ($\beta_j$). The utility for all alternatives depends on the characteristics of the individual, but the coefficients are different for different alternatives.

Following the concepts introduced in the random utility model (RUM) earlier, suppose that an individual $i$, who is faced with "$J$" alternatives, would choose alternative $j$, then we know they must obtain the maximum utility from alternative $j$. Since only differences in utility matter when choosing an alternative with the highest utility, it is necessary to normalize $\beta_j = 0$ (for one alternative). For convenience, we assume $\beta_1 = 0$. Then, the resulting choice probabilities are:

$$\begin{aligned} Prob(Y=j) &= Prob(U_{ij} > U_{ik}) \quad \forall j \neq k \\ &= \frac{e^{\beta_j' Z_i}}{1+\sum_{k=1}^{J} e^{\beta_k' Z_i}} \quad for\; j = 1,\, 2,\, 3, ..., J. \end{aligned} \tag{8.5.9}$$

**Conditional Logit Models**

When the data consist of choice-specific attributes, the appropriate choice model to use is the conditional logit (CL) models. The general form of the CL is:

$$\begin{aligned} U_{ij} &= \beta' X_{ij} + \epsilon_{ij} \\ X_{ij} &= [x_{ij},\, Z_i] \\ and\; \epsilon_{ij} &\sim\; iid\; extreme\; values. \end{aligned} \tag{8.5.10}$$

where $x_{ij}$ gives a vector of values that represent attributes of alternatives, and $Z_i$ gives a vector of values representing an individual's characteristics. If an individual $i$, who is faced with "$J$" alternatives, ends up choosing alternative $j$, then we know they obtain the maximum utility from alternative $j$ with the resulting choice probabilities:

$$\begin{aligned} Prob(Y=j) &= Prob(U_{ij} > U_{ik}) \quad \forall j \neq k \\ &= \frac{e^{\beta' X_{ij}}}{\sum_{j=1}^{J} e^{\beta' X_{ij}}} \quad for\; j = 1,\, 2,\, 3, ..., J. \end{aligned} \tag{8.5.11}$$

The difference between MNL and CL models is that in CL models the utility for each alternative depends on the attributes of the choice in addition to the characteristics of the individual. Importantly, the attribute ($x_{ij}$) varies across the choices and possibly across the individuals as well.

This unique setup marks Daniel McFadden's most influential contribution in discrete choice models (See Manski 2001). That is, enabling forecasting in a new setting through characterizations of alternatives and individuals. To be more specific, there is no way to predict the choice or demand for a new good or behavior of new individuals because both goods and individuals are qualitatively distinct. McFadden's work departs from standard practice by allowing the utility function to take the following form:

$$U_{ij} = f(x_j,\ Z_i) + \epsilon_{ij,} \tag{8.5.12}$$

where $U_{ij}$ maps the attributes of alternatives and individuals into utility values. This characterization of alternatives and individuals as attribute vectors enables forecasting, as the qualitative distinctions among alternatives and individuals are mapped into quantitative differences in their attributes. In this setting, utility of any alternative to any individual can be determined as long as the attributes ($x_j$, and $Z_i$) are known and the form of utility is also partially known ($\epsilon_{ij}$ captures the unknown part of utility form). Therefore, the individual's choice behavior becomes predictable.

## Estimation and Hypothesis Tests

### Estimation

In nearly all cases, the method of estimation for discrete choice models is maximum likelihood. For binary choice models, each observation is treated as a single draw from a Bernoulli distribution. The resulting general format of the likelihood function (or the joint probability) for a sample of $n$ observations can be written as:

$$L = \prod_{i=1}^{n} \pi_{i1}^{Y_{i1}} \pi_{i2}^{Y_{i2}}. \tag{8.5.13}$$

Or taking the logs, we obtain:

$$log\ L = \sum_{i=1}^{n} \left[ Y_{i1} log(\pi_{i1}) + Y_{i2} log(\pi_{i2}) \right], \tag{8.5.14}$$

where the dependent variable is individual choice such that $Y_{i1} = 1$ if the $i$th individual chose alternative 1 between two available alternatives, and $Y_{i1} = 0$ otherwise; and $\pi_{i1}$ is the probability of individual "$i$" ($i = 1,\ 2,\ 3, ...,\ n$) choosing for alternative 1.

The estimation for multinomial models is an extension of binary choice models. The general format of a likelihood function for a multinomial choice model is:

$$L = \prod_{i=1}^{n} \pi_{i1}^{Y_{i1}} \pi_{i2}^{Y_{i2}} ... \pi_{im}^{Y_{im}}. \tag{8.5.15}$$

Again, taking logs:

$$log\ L = \sum_{i=1}^{n}\sum_{j=1}^{m} Y_{ij} log(\pi_{ij}), \tag{8.5.16}$$

where $\pi_{ij}$ is the probability of individual "$i$" ($i = 1,\ 2,\ 3, ...,\ n$) choosing alternative $j$ ($j = 1,\ 2,\ 3, ...,\ m$), and $\pi_{ij} = 1$ if alternative $j$ is chosen by individual $i$ and 0 if not for the $m$ possible outcomes. Note that for each individual $i$, one and only one of the $\pi_{ij}$'s is 1.

Since this appendix is for reference purpose only, we will not go into a discussion of the various properties and the necessary conditions for the maximum likelihood estimation. Interested readers should consult standard econometrics textbooks (e.g., Greene (2011) and Hendry (1995)) for relevant information. Also, detailed proofs for some of the models discussed above can be found in McFadden (1984).

**Hypothesis Tests**

A relevant hypothesis test mentioned in Chapter 8 (and used in Kedar (2005)) is a likelihood ratio test. The likelihood ratio statistics based on (8.5.16) can be computed as:

$$LR = -2\left[ln\hat{L_r} - ln\hat{L}\right], \tag{8.5.17}$$

where $\hat{L_r}$ and $\hat{L}$ are the log-likelihood functions evaluated at the restricted and unrestricted estimates, respectively. In Kedar (2005), the hypothesis test is whether $\gamma = 1$. The theoretical importance of this coefficient ($\gamma \in (0,\ 1)$) is that it represents components of voter utility such that the more proximity-lead is the voting, the larger the coefficient.

### 8.5.2 Formal Analogues

For an individual to arrive at an actual choice, contemporary choice theory conceptualizes that individuals when faced with choices among a number of alternatives adopt a variety of decision rules. The following is a discussion of two common decision rules: the marginal decision rule and the utility-maximization rule.

### Marginal Decision Rule

One of the simplest decision rules is the marginal decision rule. It states that whether an individual takes on an additional activity ($Y = 1$) or not ($Y = 0$) is based on whether they receive the net benefit ($Y^*$) by opting to do the activity. The net benefit is not observable because it is defined as the difference between the marginal benefit and marginal cost, which can both be derived from an individual's unobservable utility function. Although the net benefit of choosing to do an activity is not observable, the choice outcome of an individual is observed. Consequently, the individual choices can be modeled using an index function model:

$$
\begin{aligned}
Y^* &= \beta' X + \epsilon \qquad (8.5.18)\\
and\ Y &= \begin{cases} 1, & if\ Y^* > 0 \\ 0, & otherwise. \end{cases}
\end{aligned}
$$

The choice probability is:

$$
\begin{aligned}
Prob(Y = 1) &= Prob(Y^* > 0) \\
&= Prob(\beta' X + \epsilon > 0) \qquad (8.5.19)\\
&= Prob(\epsilon > -\beta' X).
\end{aligned}
$$

Greene (2011) noted that discrete dependent variable models are often cast in the form of an index function model. See also Nakosteen and Zimmer (1980) for an index function model used for an application of a discrete choice model.

### Utility-Maximization Rule: Random Utility Models

Another commonly used rule for decision-making is utility maximization. Applying basic utility theory to the problem of discrete choice, McFadden's pioneering work (1973, 1974) introduced the key methodological framework, random utility models (RUM), to modern econometric analysis of discrete choice. RUM supposes that each member of a population of interest faces a finite choice set and selects an alternative that maximizes utility.

To demonstrate RUM, let $U_{ij}$ and $U_{ik}$ be the individual $i$'s utility for two alternative choices: $U_{ij}$ ($U_{ik}$) is the utility that individual $i$ obtains from choosing alternative $j$ $(k)$. Using the rule of utility-maximization, individual $i$ chooses the alternative providing the highest utility. Since an individual's utility from choosing an alternative is not observable, it is modeled in a way to depend on some variables $(X)$ a researcher observes and on some variables $(\epsilon_j)$ the researcher cannot observe:

$$U_{ij} = \beta_j' X + \epsilon_{j,} \tag{8.5.20}$$

where $X = (Z_i,\ x_{ij})$, $Z_i$ is a vector of characteristics of individual $i$, and $x_{ij}$ is a vector of attributes of alternative $j$ of individual $i$.

Although, individual $i's$ utility for either alternative is not observable, observing his choice outcome indicates which alternative between $j$ and $k$ gives higher utility. We designate the indicator variable of the choice outcome $(Y_{ij})$ using a dummy variable. Therefore, the individual choices is modeled using the following RUM:

$$\begin{aligned} U_{ij} &= \beta_j' X + \epsilon_j \\ U_{ik} &= \beta_k' X + \epsilon_k, \end{aligned}$$

$$and\ Y_{ij} = \begin{cases} 1, & if\ U_{ij} > U_{ik},\ \forall j \neq k \\ 0, & otherwise. \end{cases} \tag{8.5.21}$$

The choice probability is:

$$\begin{aligned} Prob(Y_{ij} = 1) &= Prob(U_{ij} > U_{ik}) \\ &= Prob(\beta_j' X + \epsilon_j > \beta_k' X + \epsilon_k) \\ &= Prob(\epsilon_k - \epsilon_j < \beta_j' X - \beta_k' X) \\ &= Prob(\epsilon < -\beta' X). \end{aligned} \tag{8.5.22}$$

In RUM's, utilities are conceptualized as random variables. McFadden interpreted the "randomness" in RUM as arising from cross-sectional variation in utility functions across the population. He emphasized the idea that individual utilities will not always be the same under identical conditions. This heterogeneity is reflected in the random variation of utility assessment and measurement error.

## Details on the Kedar (2005) Application

It is this framework of utility-maximization multinomial response models that was used by Kedar (2005) to setup a voting behavior model for empirical testing. Recall a key equation (8.5) that gives voter $i's$ utility for party $j$:

$$U_{ij} = \theta \left\{ -\gamma \left( v_i - p_j \right)^2 - (1-\gamma) \left[ \left( v_i - P \right)^2 - \left( v_i - P_{-p_j} \right)^2 \right] \right\} + \delta_j z_i,$$

where $\left( v_i - p_j \right)^2$ is the measurement for representational voting behavior affecting voter $i's$ utility for party $j$, and $\left[ \left( v_i - P \right)^2 - \left( v_i - P_{-p_j} \right)^2 \right]$ is the measurement for compensational voting behavior affecting voter $i's$ utility for party $j$. We can rewrite equation (8.5) as:

$$U_{ij} = \theta \left\{ -\gamma (representational_{ij}) - (1-\gamma) \left( compensational_{ij} \right) \right\} + \delta_j z_i, \tag{8.5.23}$$

Since equation (8.5.23) involves data that consist of choice-specific attributes (i.e., $representational_{ij}$ and $compensational_{ij}$) in addition to individual-specific characteristics ($z_i$), with the assumption of logistic error terms (such that $f(a) = exp(a)$), the appropriate statistical model for estimation is the CL model. The resulting choice probability is:

$$Prob(Voter_i = j) \equiv \pi_{ij} = \frac{f(U_{ij})}{\sum_{k=1}^{m} f(U_{ik})}, \tag{8.5.24}$$

where $\pi_{ij}$ denotes the probability of voter $i$ ($i = 1,\ 2,\ 3, ...,\ n$) voting for party $j$ ($j = 1,\ 2,\ 3, ...,\ m$). The following maximum likelihood function can, therefore, be derived for estimation and hypothesis testing:

$$log\ L = \sum_{i=1}^{n} \sum_{j=1}^{m} Y_{ij} log(\pi_{ij}), \tag{8.5.25}$$

where $Y_{ij} = 1$ if the $i$th voter votes for party $j$, and $Y_{ij} = 0$ otherwise.

# Chapter 9

# Voter Turnout

Voter turnout studies have a rich history (e.g., Merriam and Gosnell 1924; Milbraith 1965; Blais 2000). In this chapter we present Achen's (2006) EITM approach to the topic. The EITM linkage is between the behavioral concepts — decision making and learning — and the applied statistical concept of nominal choice. What is also important about Achen's approach is the ability to link formalization with known distribution functions. In prior examples the EITM link was accomplished in ways not involving distribution functions.

The empirical tools in this chapter include discrete choice estimation methods. The background for these empirical tools were presented earlier in Chapter 8. The formal tools involve the use of Bayesian updating.[1] and a basic understanding of distribution functions is also required.

## 9.1 Step 1: Relating Decision Making, Learning, and Discrete Choice

In prior studies of turnout, researchers used discrete choice models to estimate the probability of voting. The explanatory variables in these empirical models include ad-hoc transformations. For example, age, the square of age, education level, and the square of education level are used. However, there is weak theoretical justification for the squared terms. The variables are included typically for the sake of a better statistical fit within sample. Yet, Achen (2006) argues this:

> ...lack of theoretical structure leaves researchers free to specify their sta-

[1] Note that Achen demonstrates an alternative analogue for learning. In previous chapters adaptive learning tools were used as the learning analogue.

> tistical models arbitrarily, so that even closely related research teams make different choices...These modeling choices have substantial implications if we really mean them: If age measures learning, for example, it makes a difference whether over a lifetime, political learning accelerates, decelerates, or is constant. Alas, the theory that would provide the interpretation and structure our specications is missing (pages 2-3).

With this criticism in mind, Achen (2006) provides an estimated model with a theoretical interpretation (i.e., no "squared" variables). The intuition behind his behavioral model, the way he conceptualizes the decision to vote, is that it is an "expressive act"[2] *where potential voters learn* about the candidates via party label or contact from a trusted source. He further asserts that:

> written down formally, these simple ideas generate mathematical implications that map directly onto the behavioral literature and connect closely to what the voters are actually doing...Moreover, the model implies new functional forms for the statistical modeling of voter turnout. The resulting predictions go through the data points, while those from the most widely used statistical specications in the behavioral literature do not (pages 4-5).

Turning now to the analogues for decision making, learning and discrete choice — we discuss how they can be linked.

## 9.2 Step 2: Analogues for Decision Making, Learning, and Discrete Choice

Achen's theoretical model assumes a voter receives positive utility by voting if he expects the true value of the difference between two parties in the next period, $u_{n+1}$, to be different from zero (where $n$ is the number of prior elections that the voter experiences). Achen assumes, too, that the voter does not have perfect foresight on the true value of the party differences. Instead the voter "learns" the expected value based on his information set (updated by a Bayesian mechanism).

[2]Achen defines an expressive act as "a decision to do ones duty or take pleasure in a collective enterprise or cheer for ones team without imagining that one might personally determine the outcome of the game (Milbrath,1965, 12-13)." (page 5).

The subjective (expected) distribution of $u_{n+1}$ can be written as:

$$f\left(u_{n+1}\left|I\right.\right), \tag{9.2.1}$$

where $f(\cdot)$ is the probability density distribution based on the voter's information set $I$ given the period of $n$. The corresponding cumulative distribution function (cdf) from equation (9.2.1) is:

$$F\left(u_{n+1}\left|I\right.\right), \tag{9.2.2}$$

where $F(\cdot)$ is the cdf with the mean $\hat{u}_{n+1}$ and variance $\sigma^2_{n+1}$.

For theoretical convenience, Achen (2006) assumes that $\hat{u}_{n+1}$ is non-negative: the voter only votes for the party valued higher than another. The probability of the voter making a correct decision is when $u_{n+1} \geq 0$, is therefore:

$$\Pr(\text{correct}) = 1 - F\left(0\left|I\right.\right), \tag{9.2.3}$$

whereas the probability of an incorrect decision is:

$$\Pr(\text{incorrect}) = F\left(0\left|I\right.\right). \tag{9.2.4}$$

If we assume a voter will vote only if the probability of making a correct decision exceeds that of making an incorrect decision, then we can present the expected benefit of voting, $E(D_{n+1})$, in the next period given by the difference between the two probabilities:

$$\begin{aligned} E(D_{n+1}) &= \alpha\left[\Pr(\text{correct}) - \Pr(\text{incorrect})\right] \\ &= \alpha\left[1 - F\left(0\left|I\right.\right) - F\left(0\left|I\right.\right)\right] \\ &= \alpha\left[1 - 2F\left(0\left|I\right.\right)\right], \end{aligned}$$

where $\alpha > 0$ represents the weight (importance) of voting.

Following Downs (1957), Achen (2006) suggests that the utility of voting in period $n+1$ is the difference between the expected benefit of voting, $E(D_{n+1})$, and the cost of voting:

$$\begin{aligned} U &= E(D_{n+1}) - c \\ &= \alpha\left[1 - 2F\left(0\left|I\right.\right)\right] - c, \end{aligned} \tag{9.2.5}$$

where $c$ is the cost of voting. Assuming that $u_{n+1}$ is normally distributed, we can transform equation (9.2.5) to:

$$U = \alpha \left[1 - 2\Phi\left(-\hat{u}_{n+1}/\sigma_{n+1}\right)\right] - c, \tag{9.2.6}$$

where $\Phi(\cdot)$ is a standard normal cdf. Since $\Phi(-z) = 1 - \Phi(z)$, we can rewrite equation (9.2.6) as:

$$U = \alpha \left[2\Phi\left(\hat{u}_{n+1}/\sigma_{n+1}\right) - 1\right] - c. \tag{9.2.7}$$

Achen argues that voters use a Bayesian updating procedure (assuming a normal distribution of $u_{n+1}$ ) and voters "learn" the true $u_{n+1}$ based on: i) the difference(s) in party identification (PID) from the last period, $u_n$; ii) the campaign information, $c_{n+1}$; and iii) a trusted information source, $q_{n+1,}$ received from a political party.[3]

The learning process can now be characterized. The posterior mean as it pertains to party identification is:

$$u_t = \delta + v_t, \tag{9.2.8}$$

where $u_t \sim N\left(\delta, w^2\right)$. The voter first updates the posterior mean of his PID up to time $n$ using the standard Bayesian formulation:

$$\hat{\delta}_n = \frac{h_1 \bar{u}_n}{h_0 + h_1}, \tag{9.2.9}$$

where $\bar{u}_n = \frac{\Sigma u_t}{n}$ is the mean of PID based on past voting experience, $h_1 = \left(w^2/n\right)^{-1}$ is the inverse of the sample variance, and $h_0 = \left(\sigma_0^2\right)^{-1}$ represents the inverse of the prior variance, $\sigma_0^2$. In the next period, the voter also receives new information from the party campaign:

$$c_{n+1} = u_{n+1} + \theta_{n+1} + \epsilon_{n+1}, \tag{9.2.10}$$

where $\theta \sim N\left(0, \varphi^2\right)$ and $\epsilon \sim N\left(0, \tau^2/m\right)$.

Based on the posterior mean of PID at time $n$ (i.e., $\hat{\delta}_n$, in equation (9.2.9)), the campaign information, $c_{n+1}$, in equation (9.2.10), and the trusted information source, $q_{n+1}$, at time $n+1$, we can use the same Bayesian updating procedure to update the posterior mean of the PID difference $\hat{u}_{n+1}$:

$$\hat{u}_{n+1} = \frac{h_c \hat{\delta}_n + h_\tau c_{n+1} + h_q q_{n+1}}{h_c + h_\tau + h_q}, \tag{9.2.11}$$

where $h_c \equiv \left[\left(h_0 + h_1\right)^{-1} + w^2\right]^{-1}$, $h_\tau \equiv \left(\varphi^2 + \tau^2/m\right)^{-1}$, and $h_q$ is the inverse of

[3]Achen (2006) also suggests that *trusted information* can also come from the voter's spouse or some interest groups.

known variance of the trusted information source. The posterior variance of $\hat{u}_{n+1}$ is presented as:

$$\sigma_{n+1}^2 = \frac{1}{h_c + h_\tau + h_q}. \tag{9.2.12}$$

To derive the utility function of voting with the feature of Bayesian learning, we substitute equations (9.2.11) and (9.2.12) into equation (9.2.7):

$$\begin{aligned} U &= \alpha \left[ 2\Phi \left( \left( \frac{h_c\hat{\delta}_n + h_\tau c_{n+1} + h_q q_{n+1}}{h_c + h_\tau + h_q} \right) / \left( \frac{1}{(h_c + h_\tau + h_q)^{1/2}} \right) \right) - 1 \right] - c \\ &= \alpha \left[ 2\Phi \left( \frac{h_c\hat{\delta}_n + h_\tau c_{n+1} + h_q q_{n+1}}{(h_c + h_\tau + h_q)^{1/2}} \right) - 1 \right] - c. \end{aligned} \tag{9.2.13}$$

## 9.3 Step 3: Unifying and Evaluating the Analogues

To estimate the determinants of voting turnout, Achen presents the probit model which follows from equation (9.2.13). Let there be a critical level of utility, call it $U^*$, such that if $U > U^*$, the voter will vote, otherwise the voter will not. Given the normality assumption for the utility distribution, we can construct the probability that $U^*$ is less than or equal to $U$ based on the normal cumulative density function (cdf):

$$\begin{aligned} &\Pr(vote = 1 \,|\text{PID, Campaign Information, and Trusted Source}) \\ &= \Pr(U^* \leq U) \\ &= \Phi \left( \alpha \left[ 2\Phi \left( \frac{h_c\hat{\delta}_n + h_\tau c_{n+1} + h_q q_{n+1}}{(h_c + h_\tau + h_q)^{1/2}} \right) - 1 \right] - c \right). \end{aligned} \tag{9.3.1}$$

In equation (9.3.1), we can see that the inner normal cdf represents the Bayesian learning process and the outer normal cdf is used for the purpose of discrete choice estimation. At this point unification is achieved.

Using maximum likelihood estimation, Achen (2006) estimates simultaneously two normally distributed cdf's in equation (9.3.1): a *double-probit*. To interpret the coefficients, we first focus on the inner normal cdf. If the voter does not have accurate information about the party, that is, $(h_c + h_\tau + h_q)^{1/2} = 0$, then $\Phi\left(\frac{h_c\hat{\delta}_n + h_\tau c_{n+1} + h_q q_{n+1}}{(h_c + h_\tau + h_q)^{1/2}}\right) = \Phi(0) = 1/2$. In this case equation (9.3.1) is equivalent to:

$$\begin{aligned}\Pr\left(vote=1\left|I\right.\right)&=\Phi\left(\alpha\left[2\Phi\left(\frac{h_c\hat{\delta}_n+h_\tau c_{n+1}+h_q q_{n+1}}{\left(h_c+h_\tau+h_q\right)^{1/2}}\right)-1\right]-c\right)\\&=\Phi\left(\alpha\left[2\Phi\left(0\right)-1\right]-c\right)\\&=\Phi\left(-c\right).\end{aligned}\tag{9.3.2}$$

Given that $c$ is the z-value which ranges between 2 or 3, then $\Phi\left(-c\right)$ will range between -2 and -3 implying that the probability of voting will be very low.

On the other hand, if the voter is fully informed and the posterior precision of information is quite large, that is, $\left(h_c+h_\tau+h_q\right)^{1/2}\to\infty$, then $\Phi\left(\frac{h_c\hat{\delta}_n+h_\tau c_{n+1}+h_q q_{n+1}}{\left(h_c+h_\tau+h_q\right)^{1/2}}\right)=\Phi\left(\infty\right)\to 1$. Therefore, we have:

$$\begin{aligned}\Pr\left(vote=1\left|I\right.\right)&=\Phi\left(\alpha\left[2(1)-1\right]-c\right)\\&=\Phi\left(\alpha-c\right).\end{aligned}\tag{9.3.3}$$

Given that $\alpha$ can range between 4 and 5, $\Phi\left(\alpha-c\right)$ will range between 2 and 3. This relation shows that the probability of voting will be high and close to 1.

To estimate equation (9.3.1) , Achen uses the variables, systemtime and education, as the proxies for PID, $\hat{\delta}_{n,}$ and campaign information, $c_{n+1}$, respectively. Systemtime is defined as the voter's age subtracted from 18 years. Education is measured as the voter's education level — classified in six categories:

1) No High-School,
2) Some High-School,
3) High-School Degree,
4) Some College,
5) College Degree, and
6) Postgraduate Level.

Achen argues that the age of voters (systemtime) shows the strength of PID while voters' education level are attributes in understanding campaign information.[4] Based on the availability of data, the theoretical model (9.3.1) is used to estimate the following double-probit model:

$$\Pr\left(vote=1\right)=\Phi\left(\lambda_0+\lambda_1\left[2\Phi\left(\beta_1 systemtime+\beta_2 education\right)-1\right]\right),\tag{9.3.4}$$

[4]Note, there is no proxy measure used for trusted source. Therefore $q_{n+1}$ is dropped from equation (9.3.1).

where the empirical component, $\Phi(\beta_1 systemtime + \beta_2 education)$, is theoretically equivalent to the Bayesian learning procedure $\Phi\left(\frac{h_c\hat{\delta}_n+h_\tau c_{n+1}+h_q q_{n+1}}{(h_c+h_\tau+h_q)^{1/2}}\right)$, and $\lambda_0$ and $\lambda_1$ are equivalent to $-c$ and $\alpha$ in equation (9.3.1), respectively.

To test the EITM relation, Achen (2006) uses voter turnout data from the 1998 and 2000 Current Population Surveys (CPS) and the Annenberg 2000 presidential election study. Contrasting the EITM-based model with traditional applied statistical models in the existing literature, he finds that his models have a better fit. Equally important, when the focus turns to the parameters in Achen's model he finds the empirical estimates are consistent with the theoretical predictions of the model (see (9.3.1)). For example, he finds that the estimated values of $c$ and $\alpha$ range between 1.212 and 2.424 and between 3.112 and 4.865, respectively. These values are statistically indistinguishable from the values predicted in the model.

## 9.4 Leveraging EITM and Extending the Model

Achen uses the behavioral concepts of rational decision making and learning. His behavioral analogues are basic utility maximization and Bayesian learning respectively. He links these behavioral analogues with the applied statistical analogue for discrete choice: probit. To accomplish this EITM linkage he assumes that the voting decision and Bayesian learning are normally distributed *events*. With that assumption in place the formal model is tested using two probit regressions simultaneously.

Achen's EITM model can be leveraged in a number of ways. One of the more important extensions is to take advantage of the dynamic properties in his theory and model. Retrospective evaluations are assumed in the model but there is no specification or test on how long these evaluations persist or how long a voter's memory lasts. We know, for example, that in matters of policy, retrospective judgments by the public can have a profound influence on policy effectiveness. Equally important, there are analogues for persistence that can be linked to a formal extension of the model.

## 9.5 Appendix

The applied statistical analogue for discrete choice is located in Chapter 8. In this appendix we provide some background on distribution functions since they aid in prediction. For the formal tools we include a short presentation of Bayesian updating methods to estimate the mean and variance.

### 9.5.1 Empirical Analogues

Probability distributions are typically defined in terms of the probability density function (PDF). In probability theory, a PDF of a random variable describes the relative frequencies (or likelihood) of different values for that variable. If the random variable ($X$) is discrete, the PDF provides the probability associated with each outcome:

$$f(x) = Prob(X = x)$$

If the random variable ($X$) is continuous, the probability associated with any single point is zero. In this case, the PDF is expressed in terms of an integral between two points:

$$f(x) = Prob(a \leq X \leq b) = \int_a^b f(x)dx$$

The PDF of a continuous variable is a continuous function of $f(x)$, and the area under $f(x)$ provides the probability for a range of outcomes. Note that probabilities are always positive or zero and that they should total 1 (i.e., $\sum_X f(x) = 1$ (if $X$ is discrete) and $\int_{-\infty}^{+\infty} f(x) = 1$ (if $X$ is continuous)).

The cumulative distribution function (CDF) gives the probability that a random variable ($X$) takes a value of less than or equal to $x$:

$$F(x) = Prob(X \leq x)$$

Conceptually, the CDF accumulates the probabilities (PDF) of single events $x_j$ such that $x_j \leq x$. For a discrete distribution, the CDF is:

$$\begin{aligned}
F(x) &= Prob(X \leq x) \\
&= Prob(X = x_1) + Prob(X = x_2) + ... + Prob(X = x_j) \\
&= f(x_1) + f(x_2) + ... + f(x_j) \\
&= \sum_{X \leq x} f(x)
\end{aligned}$$

For a continuous distribution, the CDF is:

$$
\begin{aligned}
F(x) &= Prob(X \leq x) \\
&= \int_{-\infty}^{x} f(t)dt \\
and \quad f(x) &= \frac{dF(x)}{dx}
\end{aligned}
$$

Note that $F(x)$ has the following properties: (1) $0 \leq F(x) \leq 1$, (2) If $x_2 > x_1$, then $F(x_2) \geq F(x_1)$, and (3) $F(-\infty) = 0$ and $F(+\infty) = 1$.

## The Normal PDF and CDF

In the following Figure 9.5.1, we plot the normal PDF and CDF.

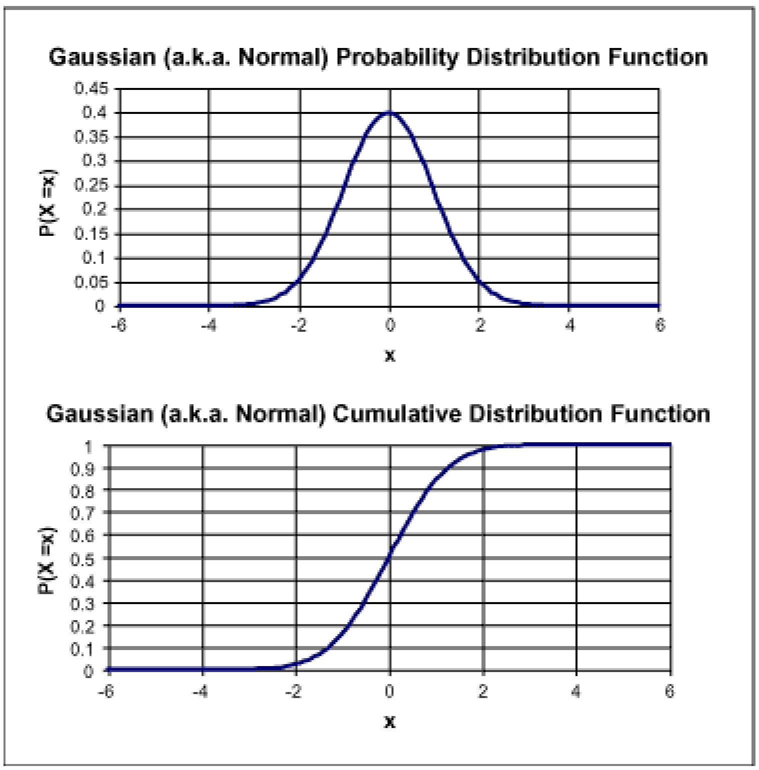



Figure 9.5.1: Normal PDF and CDF

The CDF of the standard normal distribution is denoted with the capital Greek letter $\Phi$. For a generic normal random variable with mean of $u$ and variance of $\sigma^2$, its CDF is denoted as $\Phi(\frac{x-u}{\sigma})= F(x;\, u;\, \sigma^2)$. An important property of the standard normal CDF is 2-fold rotationally symmetric around point $(0, \frac{1}{2})$: $\Phi(-x) = 1 - \Phi(x)$

### Details on the Achen (2006) Application

Achen uses the statistical concept of CDF from a normal distribution to derive the utility function for voting as expressed in equation (9.2.7). Specifically, using the notations discussed above, $F(u_{n+1} = 0|I)$ gives the CDF that the voter decided to vote when the PID takes a value of less than or equal to zero. Since Achen argues that the voter only vote for the party valued higher than another (i.e., $u_{n+1} > 0$) , the CDF of $F(0|I)$ (equation (9.2.4)) gives the probability that voter is making incorrect decision. Adding the assumption that $u_{n+1}$ is normally distributed, he then derived the utility of voting as in equation (9.2.7).

### 9.5.2 Formal Analogues

Bayesian statistics is a system for describing uncertainty using the language of probability. Bayesian methods start with existing "prior" beliefs and update these beliefs using new sample information to give "posterior" beliefs.

To derive the utility function of voting with the feature of Bayesian learning (shown in equation (9.2.13)), Achen applies Bayesian updating methods to the utility function of equation (9.2.7). In more detail, Achen uses Bayesian methods to update voter's prior belief that the PID ($u_{n+1}$) is a normal distribution. This prior belief (distribution) expresses the uncertainty about the mean value of the process. The mean of this prior is the mean of the process, while the variance is the process variance divided by the sample size. Voters update the prior belief (distribution) to generate the posterior distribution based on the sample of new information when they becomes available. The updated distribution can be characterized in terms of its mean and variance (which are referred to as posterior mean, and posterior variance, respectively). The required formulations to obtain the posterior mean ($\mu''$) and posterior variance ($\sigma''^2$) from Bayesian updating scheme with a prior normal distribution can be expressed as:

$$\mu'' = \frac{\frac{1}{\sigma'^2}\mu' + \frac{1}{\sigma^2/n}\bar{x}}{\frac{1}{\sigma'^2} + \frac{1}{\sigma^2/n}},$$

$$\sigma''^2 = \frac{1}{\frac{1}{\sigma'^2} + \frac{1}{\sigma^2/n}}.$$

where $\bar{x}$ =sample mean, $s^2 = \sigma^2$ for a sample variance with a sufficiently large sample; $\mu'$ is the prior mean, and $\sigma'^2$ is the prior variance. Equation (9.5.6) is the basis for expression (9.2.11), while equation (9.5.7) is for expression (9.2.12).

# Chapter 10

# International Conflict and Cooperation

This chapter explicates the EITM linkage between the behavioral concepts of decision making, bargaining, and strategic social interaction and the empirical concept of discrete choice with random utility.[1] This linkage — termed quantal response equilibrium (QRE) — was developed in a series of papers by McKelvey and Palfrey (1995, 1996, 1998). The approach has also been used in studies of political choice (Carson, 2003), international conflict (Signorino and Tarar 2006), and other areas (e.g., McLean and Whang 2010; Carter 2010; Helmke 2010). Carson (2003) examines the likelihood that Congress members choose to seek reelection or retire based on the decisions of potential challengers. Signorino and Tarar (2006) study the determinants of extended immediate deterrence in the context of strategic interaction between attackers and defenders.

An early and important application of QRE is Signorino's (1999) work on international conflict and cooperation. Signorino used QRE and contrasts it with stand alone discrete choice estimation. In particular, Signorino demonstrates that discrete choice estimations, such as logit and probit or Heckman selection models, do not incorporate a situation where the observed outcome, followed by an agent's choice, depends on decisions strategically made by another. Ignoring the possibility of strategic decision makings, these non-strategic choice models can lead to incorrect

[1] Note, in Chapter 7 social interaction was non-strategic — one group accepted information from another group. The form of social interaction in this chapter is strategic: taking other decision-makers choice of action as given, a decision-maker chooses the best action (among all the actions available) in accordance with her preferences. This form of social interaction requires the use of game theory.

inferences (Signorino 2002; Signorino and Yilmaz 2003).[2]

Leblang's (2003) study of speculative currency attacks is used as an example. Traditional studies on speculative exchange rate attacks examine the role of economic and political conditions that lead a country to be more or less likely to experience currency attack by international financial markets (Krugman 1979; Eichengreen, Rose, and Wyplosz 1996; Obstfeld 1994; Drazen 2000; Grier and Lin 2010). Leblang refines these arguments and suggesets that currency crises may not merely depend on domestic conditions in an economy. Instead the observed outcome of currency attack can also be the result of strategic interaction between speculators and domestic government. Speculators are likely to attack a currency if the policymakers are unwilling and unable to defend the currency peg. Yet, if speculators expect that the government is willing and able to defend its currency peg, they would not attack the currency in the first place since the expected cost of initiating the attack is high enough. The exchange rate status quo prevails. In sum, the relations and reactions Leblang discusses involve behavioral concepts — decision making, bargaining, strategic social interaction and nominal choice.

## 10.1 Step 1: Relating Decision Making, Bargaining, Strategic Social Interaction, and Nominal Choice

The concepts decision making, bargaining, strategic social interaction and nominal choice are related in the following way. Leblang (2003) assumes there are two players in an economy: international financial markets (*markets*) and policymakers in government (*governments*). The model can be summarized in Figure 10.1.1. The financial markets have two choices: (1) they can initiate a speculative attack against a currency peg, or (2) they can choose not to attack. If *markets* choose not to attack, then the exchange rate situation will remain status quo ($SQ$) in the economy and the game is over. On the other hand, if *markets* choose to attack, then *governments* must choose either devaluing the currency ($DV$) or defending the currency peg ($DF$).

[2] Along the same lines, Bas, Signorino, and Walker (2008:22) argue that "the 'indirect' statistical tests of formal models generally fail to properly characterize the hypothesized relationships in statistical testing."

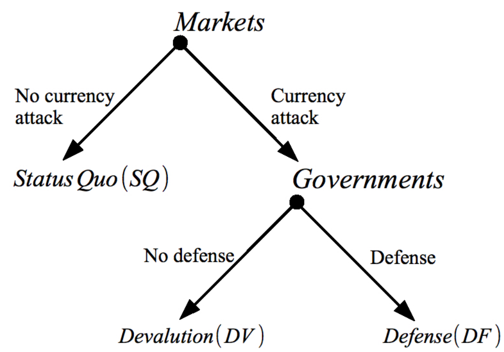



Figure 10.1.1: Extensive-Form Game of Currency Attacks

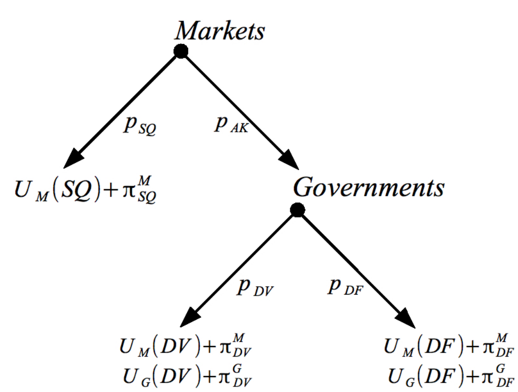



Figure 10.1.2: Extensive-Form Game with True Utility

## 10.2 Step 2: Analogues for Decision Making, Bargaining, Strategic Social Interaction, and Nominal Choice

The model assumes players are concerned about the utility over each outcome (See Figure 10.1.2). Let the markets' utility for $SQ$, $DV$, and $DF$ be defined as $U_M(SQ)$, $U_M(DV)$, and $U_M(DF)$, where $U_M(\cdot)$ represents the utility function of each observed outcome for *markets*. We also define $U_G(DV)$ and $U_G(DF)$ as the utility of devaluation and currency defense for *governments*, respectively.

Leblang (2003) assumes the *true* utility for an outcome for each player $i$ can be represented as consisting of an observable component $U_i(m)$ and an unobservable (or private) component $\pi_m^i$, where $i \in \{markets, governments\}$, and $m \in \{SQ, DV, DF\}$. $\pi_m^i$ is defined as a random variable which has a normal distribution with mean 0 and variance $\sigma^2$ (i.e., $\pi_m^i \sim N(0, \sigma^2)$). For instance, we interpret $\pi_{DV}^M$ as private information for devaluation for the markets but it is unobservable to the governments and to the analysts: the governments and analysts can only know a statistical distribution of $\pi_{DV}^M$.

Following Signorino (2003), Leblang derives equilibrium choice probabilities for each of the actions in the model. Let $p_{AK}$ denote the probability *markets* attack the currency, and $p_{DF}$ represents the probability *governments* defend the currency peg. The government decision calculus is as follows: *governments* defends the currency peg only if the expected utility of defending the currency is larger than the expected utility of devaluation.

We can derive the corresponding probability of defending the currency as:

$$p_{DF} = \Phi\left(\frac{U_G(DF) - U_G(DV)}{\sigma\sqrt{2}}\right), \tag{10.2.1}$$

and the corresponding probability of devaluation as $p_{DV} = 1 - p_{DF}$, where $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF).[3]

Similarly, *markets* attack the currency only if the expected utility of attacking the currency is greater than the expected utility of status quo. The corresponding

[3] See the Appendix for the theoretical background and the estimation procedure in detail.

probability of attacking against the currency peg is:

$$p_{AK} = \Phi\left(\frac{p_{DV}U_M(DV) + p_{DF}U_M(DF) - U_M(SQ)}{\sigma\sqrt{1 + p_{DV}^2 + p_{DF}^2 + 1}}\right), \tag{10.2.2}$$

and the corresponding probability of the economic status quo is $p_{SQ} = 1 - p_{AK}$.

## 10.3 Step 3: Unifying and Evaluating the Analogues

Given the derivation of equations (10.2.1) and (10.2.2), we can construct a likelihood equation based on these probabilities to obtain maximum likelihood estimates. For each observation $n$, let $y_{AK,n} = 1$ if *markets* attacks against the currency peg in observation $n$, and zero if the currency crisis remains in a status quo outcome. Let $y_{DF,n} = 1$ if *governments* defends the currency peg, and zero otherwise. The log-likelihood function to be maximized is:

$$\begin{aligned}\ln L &= \sum_{n=1}^{N}\left[(1 - y_{AK,n})\ln p_{SQ} + y_{AK,n}(1 - y_{DF,n})(\ln p_{AK} + \ln p_{DV})\right.\\ &\quad \left. + y_{AK,n}y_{DF,n}(\ln p_{AK} + \ln p_{DF})\right], \end{aligned} \tag{10.3.1}$$

where $N$ is the total number of observations.

With a sample of 90 developing countries, in the period 1985-1998, Leblang defines a set of factors determining the market's utility and the government's utility according to the previous studies in the literature. For the market's utility, Leblang considers the following factors which affect the probability of a speculative attacks: expansionary monetary policy, overvaluation, large external imbalances, banking sector conditions, and the capital account conditions. For the government's utility, Leblang groups the factors into two areas: 1) the factors that influence the willingness to defend the currency peg, and 2) the factors that reflect the ability to defend.

The factors that influence the willingness to defend are: electoral timing, constituent interests, and partisanship. The factors reflecting the ability to defend are: foreign exchange reserves, interest rates, capital controls, and policy decisiveness. Figure 10.3.1 shows the general specification of the utilities employed in the data analysis. We also summarize the estimation system, called a *strategic probit model*, as follows:

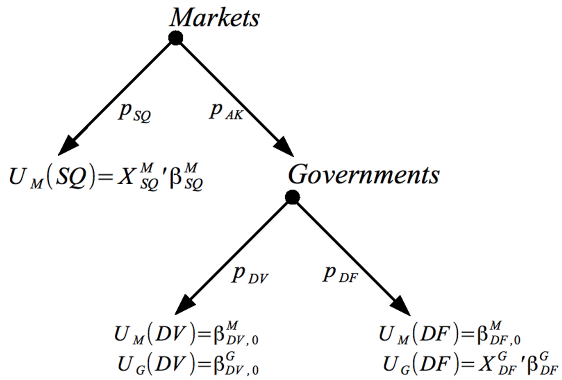



Figure 10.3.1: Leblang (2003) Model with Regressors and Parameters

$$U_M(SQ) = \sum_{k=1}^{K} \beta_{SQ,k}^{M} x_{SQ,k}^{M} \quad (10.3.2)$$
$$U_M(DV) = \beta_{DV,0}^{M} \quad (10.3.3)$$
$$U_M(DF) = \beta_{DF,0}^{M} \quad (10.3.4)$$
$$U_G(DV) = \beta_{DV,0}^{G} \quad (10.3.5)$$
$$U_G(DF) = \sum_{h=1}^{H} \beta_{DF,h}^{G} x_{DF,h}^{G}, \quad (10.3.6)$$

where:

- $\beta_{DV,0}^{M}$, $\beta_{DF,0}^{M}$, and $\beta_{DV,0}^{G}$ are constant terms.
- $X_{SQ}^{M}$ = [Captial control$_{t-1}$, Log(reserve/base money)$_{t-1}$, Real exchange rate overvaluation, Domestic credit growth$_{t-1}$, US domestic interest rate$_{t-1}$, External debt service$_{t-1}$, Contagion, Number of prior speculative attacks], $X_{DF}^{G}$ = [Unified government, Log(exports/GDP)$_{t-1}$, Campaign and election period, Post-election period, Right government, Real interest rate$_{t-1}$, Capital controls$_{t-1}$, Log(reserves/base money)$_{t-1}$],
- $\beta_{SQ}^{M}$ and $\beta_{DF}^{G}$ are the corresponding vector of coefficients on $M_{SQ}^{M}$ and $M_{DF}^{G}$, respectively.

For the specification of the dependent variable, there are several ways that the dependent variable can be specified in the dataset. These include:

- $y = 1$ if there is no currency attack (SQ),

- $y = 2$ if the government devalues the currency in response to an attack (DV),
- $y = 3$ if the government defenses the currency in response to an attack (DF).

We estimate the model using the **games** package in the **R** program.[4] The result of the strategic probit model is provided in Table 10.1. Leblang finds speculative attacks are more likely to occur when the economic fundamentals, such as, relative reserves, overvaluation and domestic credit are weak, or when there is uncertainty in the governmen's ability to defend the currency peg reflected by institutional, electoral and partisan incentives.

## 10.4 Leveraging EITM and Extending the Model

Signorino (1999, 2003) introduces a method in unifying theory and empirical test in the context of game-theoretical framework. Leblang (2003) applies the unified framework to test the model of speculative currency attacks. He uses the behavioral concepts of rational decision making with strategic interaction. His behavioral analogue is expected utility maximization and game theory. This behavioral analogue is linked with an applied statistical analogue for discrete choice: probit model with random utility — QRE.

Leblang's EITM model can be leveraged in several ways. One of the extensions is to include monetary theory in the model since exchange rate policy largely depends on the stance of monetary policy making. Further, because the current model is based on a bilateral strategic interaction between speculators and the government a revised model could incorporate self-fulfilling expectations to allow for the possibility of currency crisis contagion (Keister 2009).

## 10.5 Appendix

The tools in this chapter are used to establish a transparent and testable relation between strategic social interaction and discrete choice. The formal tools include a presentation of basic game theory. The applied statistical tools are slightly different in that the estimation method already incorporates fusing game theory with discrete choice estimation — Quantal Response Equilibrium (QRE). The last section of this appendix provides the code and data in in applying QRE to Leblang (2003).

[4] The **games** package is written by Brenton Kenkel and Curtis S. Signorino (See Kenkel and Signorino 2012).

| | The Market (*Markets*) | | | The Government (*Governments*) |
|---|---|---|---|---|
| | $U_M\,(SQ)$ | $U_M\,(DV)$ | $U_M\,(DF)$ | $U_G\,(DF)$ |
| Intercept | | -3.6648*** | -3.1385*** | 0.4269 |
| | | (0.3855) | (0.4057) | (1.7442) |
| Capital control$_{t-1}$ | -0.4525** | | | 0.0656 |
| | (0.3352) | | | (1.7098) |
| Log(reserve/base money)$_{t-1}$ | 0.2292*** | | | 0.3099* |
| | (0.0629) | | | (0.2046) |
| Real exchange rate overvaluation | -0.4413*** | | | |
| | (0.1400) | | | |
| Domestic credit growth$_{t-1}$ | -0.0648** | | | |
| | (0.0380) | | | |
| US domestic interest rate$_{t-1}$ | -0.0505 | | | |
| | (0.0507) | | | |
| External debt service$_{t-1}$ | -0.0288 | | | |
| | (0.0401) | | | |
| Contagion | -0.1159** | | | |
| | (0.0435) | | | |
| Number of prior speculative attacks | -0.1218** | | | |
| | (0.0457) | | | |
| Unified government | | | | -0.3568 |
| | | | | (0.3862) |
| Log(exports/GDP)$_{t-1}$ | | | | -0.1997 |
| | | | | (0.1891) |
| Campaign and election period | | | | 1.6632** |
| | | | | (1.9215) |
| Post-election period | | | | 1.0623* |
| | | | | (0.9031) |
| Right government | | | | -0.9358** |
| | | | | (0.5176) |
| Real interest rate$_{t-1}$ | | | | 1.7955*** |
| | | | | (1.0693) |
| Log-likelihood | -482.0155 | | | |
| Number of observations | 7240 | | | |

*Notes:* Standard errors in brackets. *** significant at 1%, ** significant at 5%, * significant at 10%.

Table 10.1: Leblang's (2003) Results

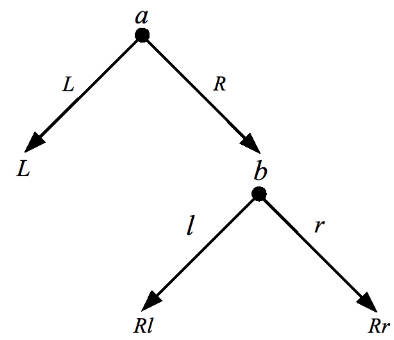



Figure 10.5.1: Two-player, Three-Outcome Game

### 10.5.1 Formal Analogues

### A Simple Game Theoretic Modeal

This appendix describes a basic idea of strategic choice analysis using a simple strategic model with two players and three observable outcomes. In this model, as depicted in Figure 10.5.1, there are two players, $a$ and $b$, who make one of the following two choices: left or right. Assuming player $a$ is a first-mover (or leader) and player $b$ is a second-mover (or follower), who must choose either left ($L$) or right ($R$). If player $a$ chooses $L$, the game is over and player $b$ does not choose. Then the final outcome is: $L$. On the other hand, if player $a$ chooses $R$, then player $b$ chooses either left ($l$) or right ($r$). There are two possible outcomes in this case: $Rl$ or $Rr$, respectively.

Assuming players are rational and well-informed, we determine via backward induction the equilibrium of a finite extensive game — the subgame perfect Nash equilibrium (SPE). We find which the optimal decision players $a$ and $b$ choose by starting at the end of the game (the end decision nodes) and work backward.

Consider a two-player, three-outcome game in Figure 10.5.1. Given the three possible outcomes, suppose player $a$'s preferences are $Rl \succ L \succ Rr$, and player $b$'s are $Rl \succ Rr$.[5] We start with the decision made by player $b$ since she makes the final decision in the game. If player $a$ chooses $R$, player $b$ will choose $l$ over $r$ as player $b$ prefers the outcome of $Rl$ to $Rr$. When player $a$ is well-informed of player $b$'s decision, player $a$ will evaluate the outcome between $L$ and $Rl$ and make a decision of choosing $R$ at the beginning. Therefore, the subgame perfect Nash equilibrium is $Rl$, where $a$ chooses $R$ and $b$ chooses $l$ as their optimal decision and they will not deviate from their optimal decision. We illustrate this result in Figure 10.5.2(a). We

[5] We denote the symbol $\succ$ to mean that one option (or bundle) is strictly preferred to another. For example, $x \succ y$ implies that $x$ is preferred to $y$. Furthermore, $x \succ y \succ z$ implies that $x \succ y$ and $y \succ z$, and $x \succ z$ according to the property of transitivity.

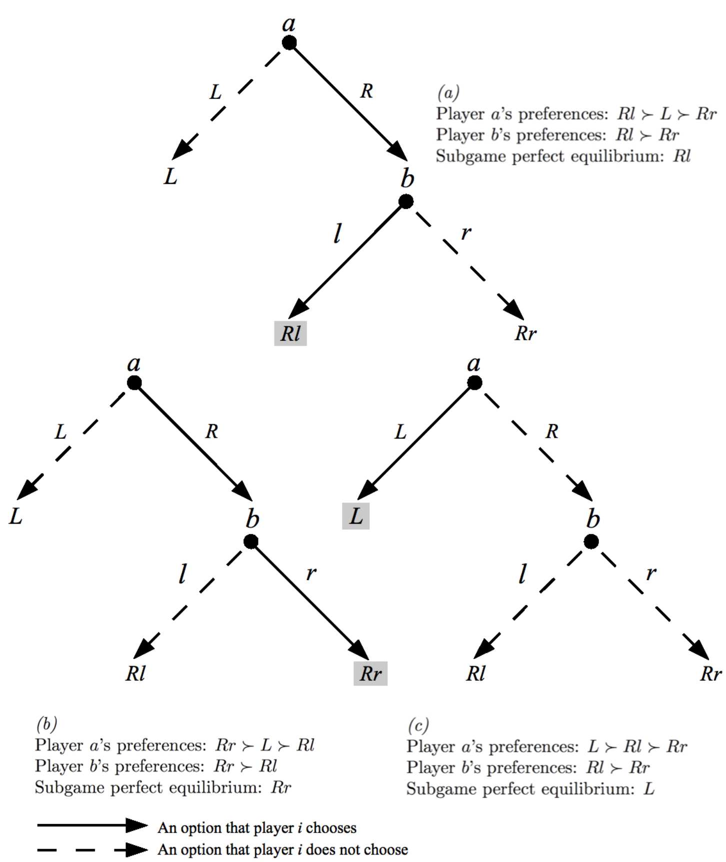



Figure 10.5.2: Subgame Perfect Equilibrium in a Two-Player, Three-Outcome Game

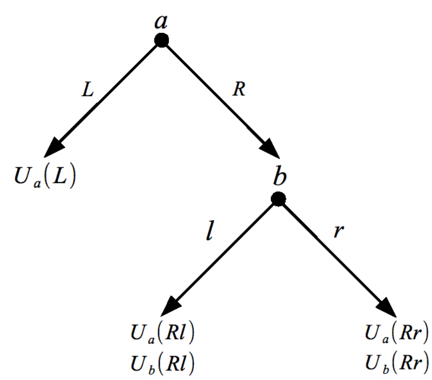



Figure 10.5.3: A Two-Player, Three-Outcome Game with Utility Functions

also present other possible SPE when we alter the preferences for players $a$ and $b$ in Figures 10.5.2(b) and 10.5.2(c).

To simplify preference description, we use a utility function $U(\cdot)$ to assign a number to every possible outcome such that more-referred outcomes get assigned larger values than less-preferred outcomes. According to our previous example, player $b$ prefers $Rl$ to $Rr$ if and only if the utility of $Rl$ is larger than the utility of $Rr$: in symbols, $Rl \succ Rr$ if and only if $U_b(Rl) > U_b(Rr)$, where $U_i(\cdot)$ represents the utility function for player $i$. Similarly, we describe player $a$'s preferences in terms of utility as follows: $U_a(Rl) > U_a(L) > U_a(Rr)$. We revise the extensive game in Figure 10.5.3.

## Quantal Response Equilibrium (QRE)

The assumptions of rationality and perfect information provide greater ease in computation. The (Nash) equilibrium can be derived with certainty. However, Signorino (1999: 281) argues that, "[t]raditional equilibrium concepts prove problematic in statistical analysis primarily because of the zero-likehood problem." One solution to this challenge can be found in McKelvey and Palfrey (1995, 1996, 1998). They develop a random utility model for normal form and extensive form games. They assume that agents' utilities and best responses are no longer deterministic. Instead, certain stochastic processes are assumed in utility and best response functions so that the equilibrium derived from the model can have meaningful statistical properties. As a result, the equilibrium can be estimated empirically. This equilibrium is called the quantal response equilibrium (QRE) — a game-theoretic equilibrium under the assumption of random utility. McKelvey and Ralfrey (1995) use maximum likelihood estimation to test the goodness of fit of the model using a variety of experimental data.

Signorino (1999) extends QRE. He derives a statistical strategic discrete choice model and applies it to a study of international conflict. In contrast to prior work in this research area, he incorporates the structure of the strategic interdependence. More importantly, Signorino's statistical models are directly derived from formal game-theoretic models. This work fits within the EITM framework since every strategic model is associated with a specific game form and solution concept, we introduce a two-player, 3-outcome game.[6]

We noted earlier that rationality and perfect information assumptions leave "zero-likelihood" for statistical analysis. Signorino (1999, 2003) relaxes these assumptions and assumes one of the two forms of uncertainty: 1) agent error and 2) private information, for strategic choice estimations. The assumptions of uncertainty impose a certain structure of the stochastic process, a process which is crucial for the estimation of utility parameters in the game-theoretic model. In the case of agent error, each player's utility over outcomes is fixed and observable for other players. However, player $i$ might not be able to choose the utility-maximizing option correctly. There exists an unobservable stochastic stock, $\alpha^i$, which can lead player $i$ to pick an alternative option, where the stochastic shock is assumed to be either normally or logistically distributed.

Figure 10.5.4 illustrates the basic structure of a strategic choice model with agent error. We assume player $a$ chooses $L$ and $R$ with the probability of $p_L$ and $p_R$, respectively. Similarly, the probabilities for player $b$ to choose $l$ and $r$ are $p_l$ and $p_r$, respectively. Each player's utility depends on the strategic outcome in the game. We also assume that the utility of each outcome $m$ for player $i$ is a linear function of $X_m^i$ explanatory variables:

$$U_i\left(m\right) = X_m^i \beta_m^i,$$

where $i \in \{a, b\}$, $\beta_m^i$ is a vector of coefficient to be estimated, and $m \in \{L, Rl, Rr\}$ is the possible outcome in the game. Each player chooses an option to maximize her expected utility, $E\left[U_i\left(m\right)\right]$.

Given that player $a$ chooses $R$, the expected utility for player $b$ to choose $l$ is $E\left[U_b\left(Rl\right)\right] = U_b\left(Rl\right) = X_{Rl}^b \beta_{Rl}^i$. Similarly, the expected utility for player $b$ to choose $r$ is $E\left[U_b\left(Rr\right)\right] = U_b\left(Rr\right) = X_{Rr}^b \beta_{Rr}^i$ when player $a$'s action is $R$. On the other hand, player $a$'s expected utility of choosing $R$ depends on the action from player $b$. It can be calculated as a weighted average of utilities from two player $b$'s actions, that is,

[6] We refer readers to Signorino (1999, 2003) and Signorino and Kenkel (2012) for other strategic models.

$E[U_a(R)] = p_l U_a(Rl) + p_r U_a(Rr) = p_l X^a_{Rl}\beta^a_{Rl} + p_r X^a_{Rr}\beta^a_{Rr}$. The expected utility of choosing $L$ for player $a$ is $E[U_a(L)] = X^a_L \beta^a_L$.

To determine the probabilities that players $a$ and $b$ choose specific actions, we assume that there exists a stochastic shock $\alpha^i$ for player $i$, which is normally distributed. In this case, given player $a$'s action is $R$, player $b$ chooses $r$ over $l$ with the following *ex ante* probability:

$$\begin{aligned} p_r &= Prob\left(E[U_b(Rr)] + \alpha^b_r \geq E[U_b(Rl)] + \alpha^b_l\right) \\ &= Prob\left(X^b_{Rr}\beta^b_{Rr} + \alpha^b_r \geq X^b_{Rl}\beta^b_{Rl} + \alpha^b_l\right) \\ &= Prob\left(\alpha^b_{Rl} - \alpha^b_{Rr} \leq X^b_{Rr}\beta^b_{Rr} - X^b_{Rl}\beta^b_{Rl}\right) \\ &= \Phi\left(\frac{X^b_{Rr}\beta^b_{Rr} - X^b_{Rl}\beta^b_{Rl}}{\sigma\sqrt{2}}\right), \end{aligned} \tag{10.5.1}$$

where $\alpha^b_r$ and $\alpha^b_l$ are distributed as normal with mean 0 and variance $\sigma^2$, and $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF).[7] The probability for player $b$ to choose $l$ is $p_l = 1 - p_r$. We can apply a similar method for computing the probabilities of player $a$'s actions. The *ex ante* probability for player $a$ choosing $R$ over $L$ is:

$$\begin{aligned} p_R &= Prob\left(E[U_a(R)] + \alpha^a_R \geq E[U_a(L)] + \alpha^a_L\right) \\ &= Prob\left(p^b_l U_a(Rl) + p^b_r U_a(Rr) + \alpha^a_R \geq E[U_a(L)] + \alpha^a_L\right) \\ &= Prob\left(\alpha^a_L - \alpha^a_R \leq p^b_l U_a(Rl) + p^b_r U_a(Rr) - U_a(L)\right) \\ &= Prob\left(\alpha^a_L - \alpha^a_R \leq p^b_l X^a_{Rl}\beta^a_{Rl} + p^b_r X^a_{Rr}\beta^a_{Rr} - X^a_L\beta^a_L\right) \\ &= \Phi\left(\frac{\alpha^a_L - \alpha^a_R \leq p^b_l X^a_{Rl}\beta^a_{Rl} + p^b_r X^a_{Rr}\beta^a_{Rr} - X^a_L\beta^a_L}{\sigma\sqrt{2}}\right). \end{aligned} \tag{10.5.2}$$

It follows that the probability of taking option $L$ is $p_L = 1 - p_R$.

[7] The CDF of a random variable $X$ is the probability that takes a value less than or equal to $x_0$, where $x_0$ is some specified numerical value of $X$, that is, $\Phi(X = x_0) = Prob(X \leq x_0)$. For a variable $X$, which follows the normal distribution with mean $\mu$ and variance $\sigma^2$, its probability density function (PDF) is:

$$\phi(X) = \frac{1}{\sqrt{2\sigma^2\pi}} \exp\left(-(X-\mu)^2/2\sigma^2\right),$$

and its CDF is:

$$\Phi(X) = \int_{-\infty}^{X_0} \frac{1}{\sqrt{2\sigma^2\pi}} \exp\left(-(X-\mu)^2/2\sigma^2\right).$$

See the Appendix in Chapter 8 for a detailed discussion of probit and logit models and the Appendix in Chapter 9 for a discussion of CDF's and PDF's.

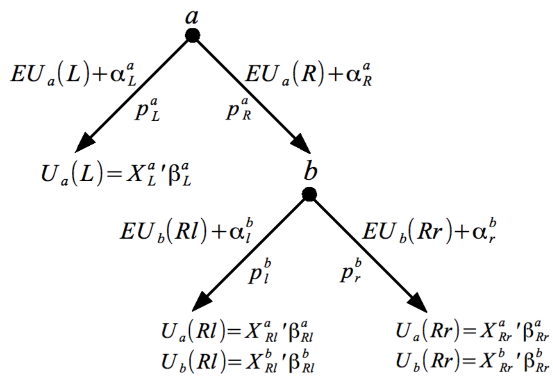



Figure 10.5.4: The Case of Agent Error

Assuming that there are $N$ repeated plays in the game. In each play $n$, we define $y_{an}=1$ if the observable action of player $a$ is $R$ and $y_{an}=0$ if the observable action is $L$. Similarly, we define $y_{bn}=1$ if the observable action of player $b$ is $r$ and $y_{bn}=0$ if the observable action is $l$. We present the following likelihood function for all $N$ plays:

$$L=\Pi_{n=1}^{N}p_L^{(1-y_{an})}\left(p_Rp_l\right)^{y_{an}(1-y_{bn})}\left(p_Rp_r\right)^{y_{an}y_{bn}}. \tag{10.5.3}$$

We can then maximize the log of the likelihood function (10.5.3) with respect to the coefficients $\beta_m^i$:

$$\ln L=\sum_{n=1}^{N}\left[(1-y_{an})\ln p_L+y_{an}\left(1-y_{bn}\right)\left(\ln p_R+\ln p_l\right)+y_{an}y_{bn}\left(\ln p_R+\ln p_r\right)\right]. \tag{10.5.4}$$

Since the log-likehood function is non-linear, we do not obtain a closed-form solution for $\beta_m^i$. However, since the log-likelihood function is globally concave we can perform numerical maximization. While we assume the stochastic shocks $\alpha$'s are normally distributed for probit-type maximum likelihood estimations, the stochastic shocks can also be distributed as Type I Extreme Value for logistic-type estimations.[8]

[8] For the logistic-type estimation, we can maximize the log-likelihood function (10.5.4) according to the following probabilities of actions for player $b$:

$$p_r=\frac{e^{U_b(Rr)}}{e^{U_b(Rl)}+e^{U_b(Rr)}}$$

and:

$$p_l=\frac{e^{U_b(Rl)}}{e^{U_b(Rl)}+e^{U_b(Rr)}}$$

For player $a$:

$$p_L=\frac{e^{U_a(L)}}{e^{U_a(L)}+e^{p_lUa(Rl)+p_rUa(Rr)}}$$

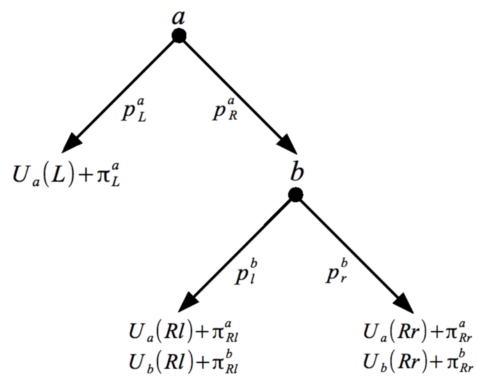



Figure 10.5.5: The Case of Private Information

Next, instead of relaxing the assumption of perfect rationality in the case of agent error model, Signorino (2002) suggests an alternative approach to include an unobservable private component or shock (denoted as $\pi^i$), which limits the analyst or the player to evaluate the other player's preferences over outcomes. Figure 10.5.5 summarizes the basic concept of private information. We now calculate the probabilities of actions for both players according to their expected utilities. For player $b$, the *ex ante* probability of choosing $r$ is:

$$\begin{aligned}
p_r &= Prob\left(U_b\left(Rr\right)+\pi^b_{Rr}\geq U_b\left(Rl\right)+\pi^b_{Rl}\right)\\
&= Prob\left(\pi^b_{Rl}-\pi^b_{Rr}\leq U_b\left(Rr\right)-U_b\left(Rl\right)\right)\\
&= Prob\left(\pi^b_{Rl}-\pi^b_{Rr}\leq X^b_{Rr}\beta^b_{Rr}-X^b_{Rl}\beta^b_{Rl}\right)\\
&= \Phi\left(\frac{X^b_{Rr}\beta^b_{Rr}-X^b_{Rl}\beta^b_{Rl}}{\sigma\sqrt{2}}\right).
\end{aligned}$$

For choosing $l$, the probability is $p_l = 1 - p_r$. On the other hand, when player $a$ chooses $R$, the expected utility is $EU_a\left(R\right)=p^b_l\left(X^a_{Rl}\beta^a_{Rl}+\pi^a_{Rl}\right)+p^b_r\left(X^a_{Rr}\beta^a_{Rr}+\pi^a_{Rr}\right).$ Therefore, player $a$'s probability for choosing $R$ over $L$ is:

$$\begin{aligned}
p_R &= Prob\left(EU_a\left(R\right)\geq EU_a\left(L\right)\right)\\
&= Prob\left(p_l\left(X^a_{Rl}\beta^a_{Rl}+\pi^a_{Rl}\right)+p_r\left(X^a_{Rr}\beta^a_{Rr}+\pi^a_{Rr}\right)\geq X^a_L\beta^a_L+\pi^a_L\right)\\
&= Prob\left(\pi^a_L-p_l\pi^a_{Rl}-p_r\pi^a_{Rr}\leq p_lX^a_{Rl}\beta^a_{Rl}+p_rX^a_{Rr}\beta^a_{Rr}-X^a_L\beta^a_L\right)\\
&= \Phi\left(\frac{p_lX^a_{Rl}\beta^a_{Rl}+p_rX^a_{Rr}\beta^a_{Rr}-X^a_L\beta^a_L}{\sigma\sqrt{1+p_l^2+p_r^2}}\right),
\end{aligned}$$

and:

$$p_R=\frac{e^{p_lUa(Rl)+p_rUa(Rr)}}{e^{U_a(L)}+e^{p_lUa(Rl)+p_rUa(Rr)}}.$$

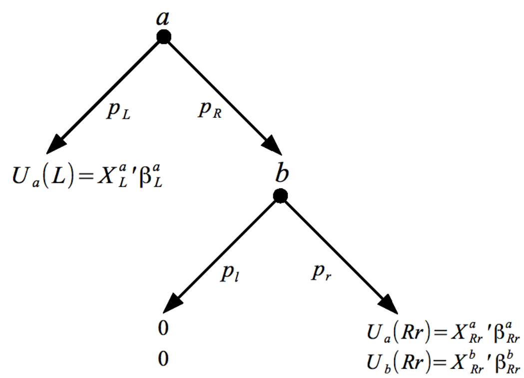



Figure 10.5.6: Identified Model with Regressors and Parameters

and the probability for $L$ is $p_L = 1 - p_R$, where $\pi_m^i$ is normally distributed with mean 0 and variance $\sigma^2$.

While the strategic systems presented in Figures 10.5.4 and 10.5.5 are theoretically sound, they present an identification problem (Lewis and Schultz 2003). Following traditional practice it is necessary to impose some exclusion restrictions in the system. Kenkel and Signorino (2012) suggest that one way to identify a strategic model is to set each player's utility to zero for one of the strategic outcomes.[9] In this particular two-player, three-outcome model described in Figure 10.5.3, the identification condition can be satisfied when $U_a(Rl) = 0$ for player $a$ and $U_b(Rl) = 0$ for player $b$ . Figure 10.5.6 illustrates a possible structure of a system which satisfies the identification condition for estimating the model. We impose the following specification:

$$
\begin{aligned}
U_a(L) &= \beta^a_{L,0} + \beta^a_{L,1} x_1 \\
U_a(Rl) &= 0 \\
U_a(Rr) &= \beta^a_{Rr,0} + \beta^a_{Rr,1} x_1 + \beta^a_{Rr,2} x_2 \\
U_b(Rl) &= 0 \\
U_b(Rr) &= \beta^b_{Rr,0} + \beta^b_{Rr,2} x_2 + \beta^b_{Rr,3} x_3,
\end{aligned}
$$

and define:

$X^a_L = [1, x_1]$
$X^a_{Rr} = [1, x_1, x_2]$
$X^a_{Rr} = [1, x_2, x_3]$

[9]Kenkel and Signorino (2012) indicate that, in general, a necessary condition for identification in a strategic model is that no regressor, including the constant term, is included in all of a player's utility functions of the strategic outcomes (Lewis and Schultz 2003).

$\beta_L^a = \left[\beta_{L,0}^a, \beta_{L,1}^a\right]$
$\beta_{Rr}^a = \left[\beta_{Rr,0}^a, \beta_{Rr,1}^a, \beta_{Rr,2}^a\right]$
$\beta_{Rr}^b = \left[\beta_{Rr,0}^b, \beta_{Rr,2}^b, \beta_{Rr,3}^b\right].$

## Details on the Leblang (2003) Application

In 2001, Signorino developed **STRAT**, a program for analyzing statistical strategic models written in Gauss language. Since then, Signorino and Kenkel further developed the program and implemented a new package, called **games**, in **R** language. The **games** package provides estimation and analysis for different forms of game-theoretic models. In this appendix, we illustrate the usage of **egame12** in the package for the estimation of the two-player, three-outcome model based on the Leblang's (2003) estimation results as an example.

Here we present the estimation procedure used in the chapter. According to Figure 10.3.1, the specification of the model is:

$$U_M(SQ) = \sum_{k=1}^{K} \beta_{SQ,k}^M x_{SQ,k}^M \tag{10.5.5}$$
$$U_M(DV) = \beta_{DV,0}^M \tag{10.5.6}$$
$$U_M(DF) = \beta_{DF,0}^M \tag{10.5.7}$$
$$U_G(DV) = \beta_{DV,0}^G \tag{10.5.8}$$
$$U_G(DF) = \sum_{h=1}^{H} \beta_{DF,h}^G x_{DF,h}^G, \tag{10.5.9}$$

where:

- $\beta_{DV,0}^M$, $\beta_{DF,0}^M$, and $\beta_{DV,0}^G$ are constant terms.

- $X_{SQ}^M$ = [Captial control$_{t-1}$, Log(reserve/base money)$_{t-1}$, Real exchange rate overvaluation, Domestic credit growth$_{t-1}$, US domestic interest rate$_{t-1}$, External debt service$_{t-1}$, Contagion, Number of prior speculative attacks], $X_{DF}^G$ = [Unified government, Log(exports/GDP)$_{t-1}$, Campaign and election period, Post-election period, Right government, Real interest rate$_{t-1}$, Capital controls$_{t-1}$, Log(reserves/base money)$_{t-1}$],

- $\beta_{SQ}^M$ and $\beta_{DF}^G$ are the corresponding vector of coefficients on $M_{SQ}^M$ and $M_{DF}^G$, respectively.

For the specification of the dependent variable, there are several ways that the dependent variable can be specified in the dataset. These include:

- $y = 1$ if there is no currency attack (SQ),
- $y = 2$ if the government devalues the currency in response to an attack (DV),
- $y = 3$ if the government defenses the currency in response to an attack (DF).[10]

The **games** package also allows other specifications for defining the dependent variable.[11] To estimate the model using **R**, first-time **R** users need to install the **games** package in **R** by typing:

```
install.packages("games", dependencies=TRUE)
```

**R** uses a specific package that can be linked to other packages. If the `dependencies=TRUE` argument is specified, then **R** uses the full capacity of the specific package by downloading and installing other packages. Then we load the **games** package from the library by typing:

```
library("games")
```

Since the data set we use is in **STATA** format (Figure 10.5.7), we import the **STATA** data file (leblang2003y.dta) into **R** using a package called **foreign**. We first install the **foreign** package by typing:

```
install.packages("foreign", dependencies=TRUE)
```

and then load the package:

```
library("foreign")
```

Now we can import the Leblang's data into **R** by typing:

```
leblang2003y.stata <- read.dta("file_path/leblang2003y.dta")
```

where *file_path/* is the location where the STATA file is saved. After importing, the data file in **R** is now called leblang2003y.stata. We can view the data set as presented in Figure 10.5.8 using the following command:

```
View(leblang2003y.stata)
```

[10] The data set can be downloaded from the website of Hobby Center for Public Policy at the University of Houston. URL: http://www.uh.edu/class/hcpp/EITM/EITMBook.html.

[11] We refer interested readers to the tutorial of **games** packages by Signorino and Kenkel (2012).

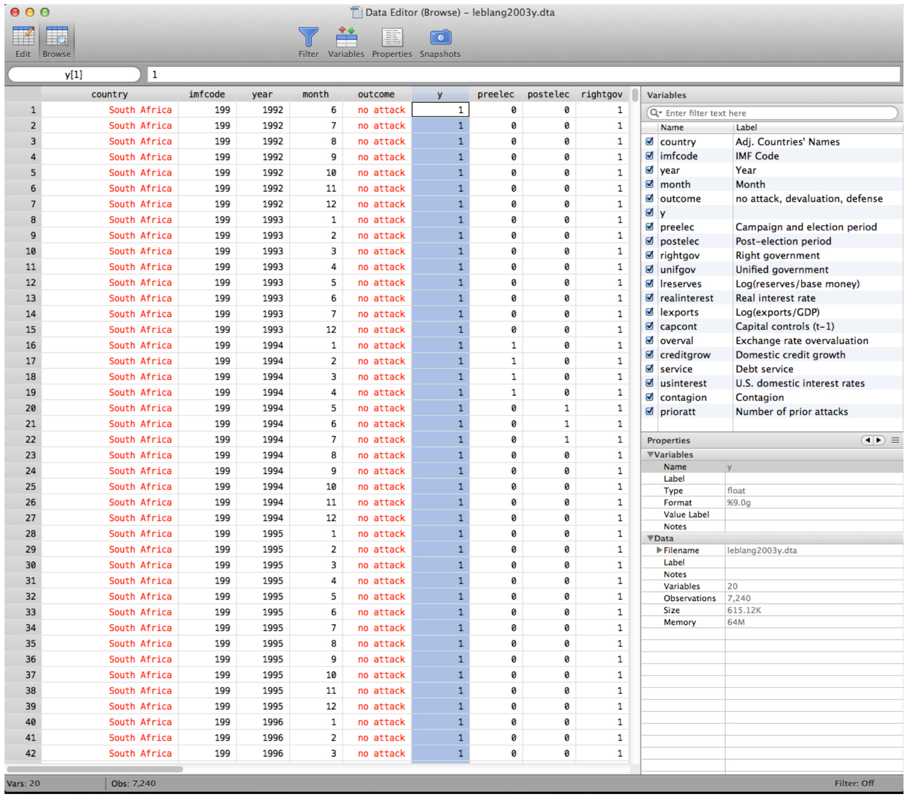



Figure 10.5.7: Leblang (2003) Data Set in **STATA**

Now estimate Leblang's model by typing:

```
m1 <- egame12(y ~ capcont + lreserves + overval + creditgrow + usinterest
+ service + contagion + prioratt - 1 | 1 | 1 | unifgov + lexports + preelec
+ postelec + rightgov + realinterest + capcont + lreserves,
data = leblang2003y.stata, link = "probit", type = "private")
```

where the result is saved as `m1`.

The command **egame12** estimates a two-player, three-outcome extensive-form game. `y` is the dependent variable which is defined as follows: $y = 1$ if the outcome $=$ SQ, $y = 2$ if the outcome $=$ DV, and $y = 3$ if the outcome $=$ DF. The first expression after ~, that is, `capcont + lreserves + overval + creditgrow + usinterest + service + contagion + prioratt - 1`, represents the linear model for estimating the utility of the market when the situation is the status quo (i.e., $U_M(SQ)$ in equation (10.5.5)).

The "minus 1" in the expression excludes model's constant term. The expression "1" followed by the first vertical stroke represents the constant term for the market's

Data: leblang2003y.stata

| | country | imfcode | year | month | outcome | y | preelec | postelec | rightgov | unifgov | lreserves | realinterest | lexports | capcont | overval | creditgrow | service | usinterest | contagion | prioratt |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | South Africa | 199 | 1992 | 6 | no attack | 1 | 0 | 0 | 1 | 1 | -1.72201 | -0.03193252 | -0.7599892 | 1 | -0.08613571 | 0.001540228 | -1.256589 | -0.795279 | -0.6209899 | -0.580608 |
| 2 | South Africa | 199 | 1992 | 7 | no attack | 1 | 0 | 0 | 1 | 1 | -1.795451 | -0.06711444 | -0.7599892 | 1 | -0.2932265 | -0.08322284 | -1.233665 | -0.9530413 | 1.680503 | -0.580608 |
| 3 | South Africa | 199 | 1992 | 8 | no attack | 1 | 0 | 0 | 1 | 1 | -1.647746 | -0.05853848 | -0.7599892 | 1 | -0.3191003 | 0.1270348 | -1.211101 | -1.110804 | -0.6209899 | -0.580608 |
| 4 | South Africa | 199 | 1992 | 9 | no attack | 1 | 0 | 0 | 1 | 1 | -1.592829 | -0.04523288 | -0.7599892 | 1 | -0.2699956 | -0.1565843 | -1.188896 | -1.237014 | -0.6209899 | -0.580608 |
| 5 | South Africa | 199 | 1992 | 10 | no attack | 1 | 0 | 0 | 1 | 1 | -1.81224 | -0.02527315 | -0.7599892 | 1 | -0.07437183 | -0.05376235 | -1.167052 | -1.363223 | 0.5297567 | -0.580608 |
| 6 | South Africa | 199 | 1992 | 11 | no attack | 1 | 0 | 0 | 1 | 1 | -1.781472 | 0.02245984 | -0.7599892 | 1 | 0.1780567 | -0.03625854 | -1.145565 | -1.457881 | 0.5297567 | -0.580608 |
| 7 | South Africa | 199 | 1992 | 12 | no attack | 1 | 0 | 0 | 1 | 1 | -1.953253 | 0.01383657 | -0.7599892 | 1 | 0.1706461 | 8.77e-05 | -1.124437 | -1.489433 | -0.6209899 | -0.580608 |
| 8 | South Africa | 199 | 1993 | 1 | no attack | 1 | 0 | 0 | 1 | 1 | -2.223773 | 0.05001831 | -0.6918341 | 1 | 0.2636068 | -0.2299386 | -1.103669 | -1.520986 | 1.680503 | -0.580608 |
| 9 | South Africa | 199 | 1993 | 2 | no attack | 1 | 0 | 0 | 1 | 1 | -2.237314 | 0.04919934 | -0.6918341 | 1 | 0.3464799 | -0.05375612 | -1.083259 | -1.552538 | 0.5297567 | -0.580608 |
| 10 | South Africa | 199 | 1993 | 3 | no attack | 1 | 0 | 0 | 1 | 1 | -2.27498 | 0.03918188 | -0.6918341 | 1 | 0.4336363 | -0.05996571 | -1.063207 | -1.584091 | 0.5297567 | -0.580608 |
| 11 | South Africa | 199 | 1993 | 4 | no attack | 1 | 0 | 0 | 1 | 1 | -2.442966 | 0.02381511 | -0.6918341 | 1 | 0.3447184 | -0.2807164 | -1.043513 | -1.615643 | 0.5297567 | -0.580608 |
| 12 | South Africa | 199 | 1993 | 5 | no attack | 1 | 0 | 0 | 1 | 1 | -2.275859 | -0.01142069 | -0.6918341 | 1 | 0.3087931 | -0.09980238 | -1.024179 | -1.647196 | -0.6209899 | -0.580608 |
| 13 | South Africa | 199 | 1993 | 6 | no attack | 1 | 0 | 0 | 1 | 1 | -2.419145 | -0.00224795 | -0.6918341 | 1 | 0.4189818 | 0.03391557 | -1.005207 | -1.678748 | -0.6209899 | -0.580608 |
| 14 | South Africa | 199 | 1993 | 7 | no attack | 1 | 0 | 0 | 1 | 1 | -2.603186 | 0.01772677 | -0.6918341 | 1 | 0.6274543 | -0.1780191 | -0.9865898 | -1.678748 | 1.680503 | -0.580608 |
| 15 | South Africa | 199 | 1993 | 12 | no attack | 1 | 0 | 0 | 1 | 1 | -2.6018 | 0.009820695 | -0.6918341 | 1 | 0.4051592 | -0.01199046 | -0.8988943 | -1.678748 | 0.5297567 | -0.580608 |
| 16 | South Africa | 199 | 1994 | 1 | no attack | 1 | 1 | 0 | 1 | 1 | -2.158254 | -8.79e-05 | -0.6889023 | 1 | 0.4287886 | -0.2157717 | -0.8824333 | -1.678748 | 5.132743 | -0.580608 |
| 17 | South Africa | 199 | 1994 | 2 | no attack | 1 | 1 | 0 | 1 | 1 | -2.158871 | -0.007079079 | -0.6889023 | 1 | 0.4688512 | 0.2739864 | -0.8663264 | -1.678748 | -0.6209899 | -0.580608 |
| 18 | South Africa | 199 | 1994 | 3 | no attack | 1 | 1 | 0 | 1 | 1 | -2.273804 | -0.00969763 | -0.6889023 | 1 | 0.4263609 | 0.3383416 | -0.8505848 | -1.678748 | -0.6209899 | -0.580608 |
| 19 | South Africa | 199 | 1994 | 4 | no attack | 1 | 1 | 0 | 1 | 1 | -2.702813 | 0.01338147 | -0.6889023 | 1 | 0.6759833 | -0.09858315 | -0.8351971 | -1.678748 | 0.5297567 | -0.580608 |
| 20 | South Africa | 199 | 1994 | 5 | no attack | 1 | 0 | 1 | 1 | 1 | -2.767383 | 0.06291377 | -0.6889023 | 1 | 0.7282443 | 0.01025976 | -0.8201666 | -1.647196 | 0.5297567 | -0.580608 |
| 21 | South Africa | 199 | 1994 | 6 | no attack | 1 | 0 | 1 | 1 | 1 | -2.959381 | 0.06068723 | -0.6889023 | 1 | 0.6677387 | -0.121273 | -0.8055046 | -1.615643 | -0.6209899 | -0.580608 |
| 22 | South Africa | 199 | 1994 | 7 | no attack | 1 | 0 | 1 | 1 | 1 | -2.90479 | 0.05203898 | -0.6889023 | 1 | 0.6994337 | -0.014078 | -0.791193 | -1.584091 | -0.6209899 | -0.580608 |
| 23 | South Africa | 199 | 1994 | 8 | no attack | 1 | 0 | 0 | 1 | 1 | -2.648558 | 0.03209635 | -0.6889023 | 1 | 0.4878819 | 0.1373961 | -0.7772431 | -1.520986 | -0.6209899 | -0.580608 |
| 24 | South Africa | 199 | 1994 | 9 | no attack | 1 | 0 | 0 | 1 | 1 | -2.347538 | 0.002440165 | -0.6889023 | 1 | 0.3383529 | -0.03843405 | -0.763651 | -1.457881 | -0.6209899 | -0.580608 |
| 25 | South Africa | 199 | 1994 | 10 | no attack | 1 | 0 | 0 | 1 | 1 | -2.312238 | 0.01187266 | -0.6889023 | 1 | 0.2479478 | -0.08242782 | -0.7504198 | -1.394776 | -0.6209899 | -0.580608 |
| 26 | South Africa | 199 | 1994 | 11 | no attack | 1 | 0 | 0 | 1 | 1 | -2.048452 | 0.01946746 | -0.6889023 | 1 | 0.1574674 | 0.02958985 | -0.7375401 | -1.306429 | -0.6209899 | -0.580608 |
| 27 | South Africa | 199 | 1994 | 12 | no attack | 1 | 0 | 0 | 1 | 1 | -2.077594 | 0.01703532 | -0.6889023 | 1 | 0.184236 | -0.1559412 | -0.7250311 | -1.195995 | 0.5297567 | -0.580608 |
| 28 | South Africa | 199 | 1995 | 1 | no attack | 1 | 0 | 0 | 1 | 1 | -1.875734 | 0.01793026 | -0.7006083 | 1 | 0.0798269 | -0.2150954 | -0.7128735 | -1.085562 | -0.6209899 | -0.580608 |
| 29 | South Africa | 199 | 1995 | 2 | no attack | 1 | 0 | 0 | 1 | 1 | -1.822265 | 0.02145002 | -0.7006083 | 1 | 0.07230689 | 0.05002331 | -0.7010764 | -0.9435755 | -0.6209899 | -0.580608 |
| 30 | South Africa | 199 | 1995 | 3 | no attack | 1 | 0 | 0 | 1 | 1 | -1.767273 | 0.04564172 | -0.7006083 | 1 | 0.1021282 | 0.028086 | -0.6896378 | -0.8015894 | -0.6209899 | -0.580608 |
| 31 | South Africa | 199 | 1995 | 4 | no attack | 1 | 0 | 0 | 1 | 1 | -1.784583 | 0.03559718 | -0.7006083 | 1 | 0.05005851 | 0.1030155 | -0.6785594 | -0.6596033 | -0.6209899 | -0.580608 |
| 32 | South Africa | 199 | 1995 | 5 | no attack | 1 | 0 | 0 | 1 | 1 | -2.118047 | 0.01399166 | -0.7006083 | 1 | 0.1202623 | 0.03312818 | -0.667839 | -0.5491695 | -0.6209899 | -0.580608 |
| 33 | South Africa | 199 | 1995 | 6 | no attack | 1 | 0 | 0 | 1 | 1 | -1.779573 | 0.02140822 | -0.7006083 | 1 | 0.0658888 | -0.1901199 | -0.6574807 | -0.4387361 | -0.6209899 | -0.580608 |
| 34 | South Africa | 199 | 1995 | 7 | no attack | 1 | 0 | 0 | 1 | 1 | -1.833756 | 0.06689336 | -0.7006083 | 1 | -0.04697711 | -0.1467837 | -0.6474723 | -0.3283023 | -0.6209899 | -0.580608 |
| 35 | South Africa | 199 | 1995 | 8 | no attack | 1 | 0 | 0 | 1 | 1 | -1.840776 | 0.09648433 | -0.7006083 | 1 | -0.1058159 | -0.0481494 | -0.6378328 | -0.2494212 | -0.6209899 | -0.580608 |
| 36 | South Africa | 199 | 1995 | 9 | no attack | 1 | 0 | 0 | 1 | 1 | -1.899917 | 0.1336004 | -0.7006083 | 1 | -0.1278624 | -0.1330035 | -0.6285439 | -0.1705398 | 0.5297567 | -0.580608 |
| 37 | South Africa | 199 | 1995 | 10 | no attack | 1 | 0 | 0 | 1 | 1 | -1.919831 | 0.1635781 | -0.7006083 | 1 | -0.2218219 | -0.2387499 | -0.6196191 | -0.09165874 | -0.6209899 | -0.580608 |
| 38 | South Africa | 199 | 1995 | 11 | no attack | 1 | 0 | 0 | 1 | 1 | -1.847081 | 0.1613403 | -0.7006083 | 1 | -0.2977539 | 0.08779578 | -0.6110558 | -0.03801963 | -0.6209899 | -0.580608 |
| 39 | South Africa | 199 | 1995 | 12 | no attack | 1 | 0 | 0 | 1 | 1 | -1.712618 | 0.1618695 | -0.7006083 | 1 | -0.3343251 | -0.03211524 | -0.6028435 | -0.00646692 | 0.5297567 | -0.580608 |
| 40 | South Africa | 199 | 1996 | 1 | no attack | 1 | 0 | 0 | 1 | 1 | -1.555987 | 0.151418 | -0.4965486 | 1 | -0.4566668 | -0.2417529 | -0.5949957 | 0.02445425 | 0.5297567 | -0.580608 |
| 41 | South Africa | 199 | 1996 | 2 | no attack | 1 | 0 | 0 | 1 | 1 | -1.571113 | 0.1509145 | -0.4965486 | 1 | -0.2994504 | 0.0361495 | -0.5875055 | 0.008678253 | 0.5297567 | -0.580608 |
| 42 | South Africa | 199 | 1996 | 3 | no attack | 1 | 0 | 0 | 1 | 1 | -1.788955 | 0.1570476 | -0.4965486 | 1 | 0.01260642 | 0.1407844 | -0.5803736 | -0.007098104 | -0.6209899 | -0.580608 |
| 43 | South Africa | 199 | 1996 | 4 | devaluation | 2 | 0 | 0 | 1 | 1 | -1.914186 | 0.1641468 | -0.4965486 | 1 | 0.5189038 | 0.08556628 | -0.573598 | -0.02287446 | -0.6209899 | -0.580608 |

Figure 10.5.8: Leblang (2003) Data Set in **R**

utility when the government devalues the currency (i.e., equation (10.5.6)). Similarly, another expression of "1" followed by the second vertical stroke presents the constant term for the market's utility when the government defends the currency (i.e., equation (10.5.7)).

Finally, the last expression after the third stroke presents the linear model for estimating the government's utility in taking action to defend the currency. `data` is to retrieve the data file for the estimation. `link` defines the distribution of the stochastic shocks in the game, where `‘‘probit’’` is set as the default for normally distributed stochastic shocks, and `‘‘logit’’` can be imposed for the logistic distribution of the stochastic shocks. `type` is determines if the stochastic structures fall under the assumption of agent-error (`‘‘agent’’` as default) or private information (`‘‘private`"). To retrieve the result of the estimation, we type:

```
summary(m1)
```

Figure 10.5.9 shows the results in **R** which is equivalent to those in Leblang (2003). Figure 10.5.10 depicts all codes for the estimation.

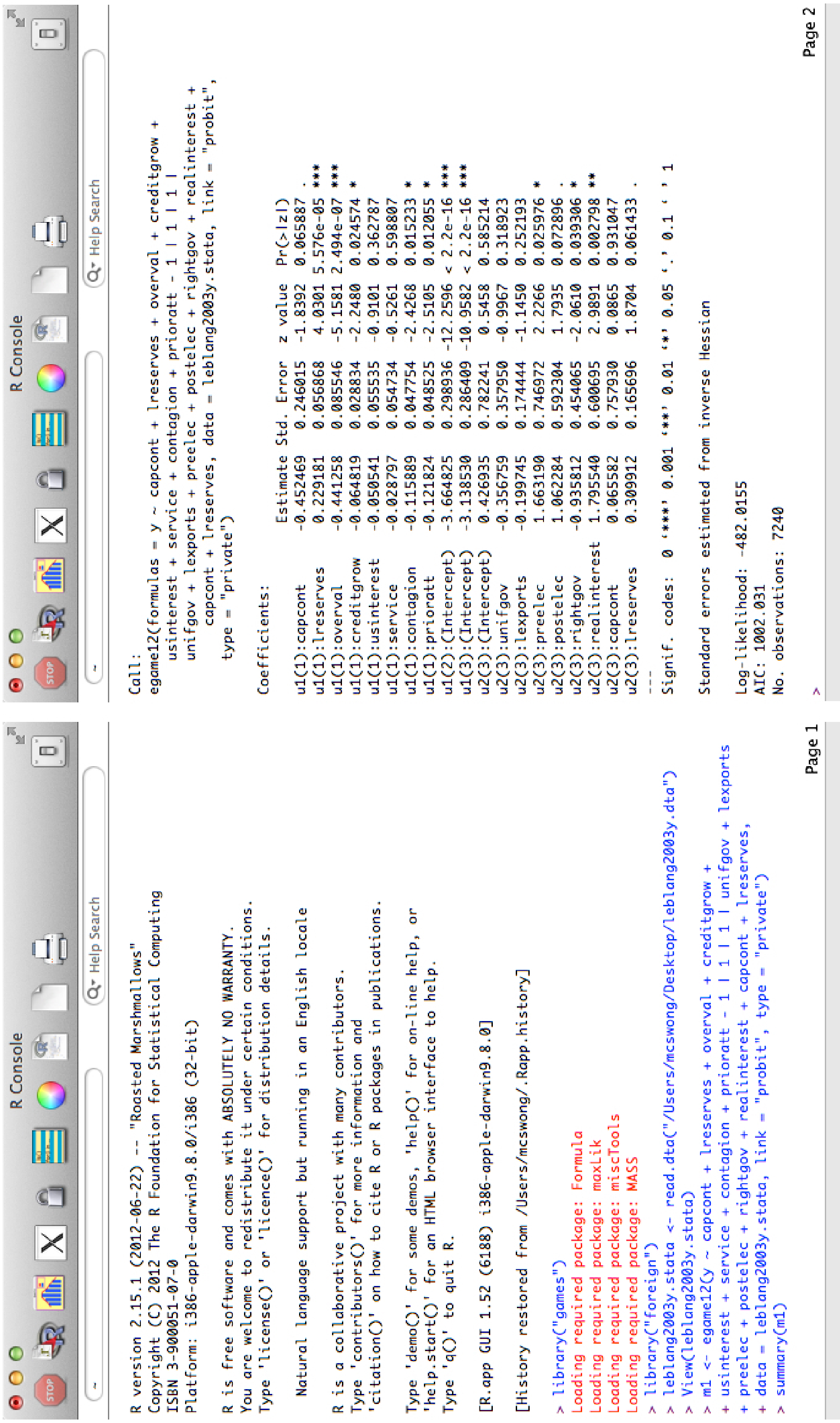
```
Call:
egame12(formulas = y ~ capcont + lreserves + overval + creditgrow +
    usinterest + service + contagion + prioratt - 1 | 1 | 1 |
    unifgov + lexports + preelec + postelec + rightgov + realinterest +
        capcont + lreserves, data = leblang2003y.stata, link = "probit",
    type = "private")

Coefficients:
                   Estimate Std. Error  z value  Pr(>|z|)
u1(1):capcont     -0.452469   0.246015  -1.8392  0.065887 .
u1(1):lreserves    0.229181   0.056868   4.0301 5.576e-05 ***
u1(1):overval     -0.441258   0.085546  -5.1581 2.494e-07 ***
u1(1):creditgrow  -0.064819   0.028834  -2.2480  0.024574 *
u1(1):usinterest  -0.050541   0.055535  -0.9101  0.362787
u1(1):service     -0.028797   0.054734  -0.5261  0.598807
u1(1):contagion   -0.115889   0.047754  -2.4268  0.015233 *
u1(1):prioratt    -0.121824   0.048525  -2.5105  0.012055 *
u1(2):(Intercept) -3.664825   0.298936 -12.2596 < 2.2e-16 ***
u1(3):(Intercept) -3.138530   0.286409 -10.9582 < 2.2e-16 ***
u2(3):(Intercept)  0.426935   0.782241   0.5458  0.585214
u2(3):unifgov     -0.356759   0.357950  -0.9967  0.318923
u2(3):lexports    -0.199745   0.174444  -1.1450  0.252193
u2(3):preelec      1.663190   0.746972   2.2266  0.025976 *
u2(3):postelec     1.062284   0.592304   1.7935  0.072896 .
u2(3):rightgov    -0.935812   0.454065  -2.0610  0.039306 *
u2(3):realinterest 1.795540   0.600695   2.9891  0.002798 **
u2(3):capcont      0.065582   0.757930   0.0865  0.931047
u2(3):lreserves    0.309912   0.165696   1.8704  0.061433 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Standard errors estimated from inverse Hessian

Log-likelihood: -482.0155
AIC: 1002.031
No. observations: 7240
```

Figure 10.5.9: Leblang's (2003) Results in **R**



```
R version 2.15.1 (2012-06-22) -- "Roasted Marshmallows"
Copyright (C) 2012 The R Foundation for Statistical Computing
ISBN 3-900051-07-0
Platform: i386-apple-darwin9.8.0/i386 (32-bit)

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  Natural language support but running in an English locale

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Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.52 (6188) i386-apple-darwin9.8.0]

[History restored from /Users/mcswong/.Rapp.history]

> library("games")
Loading required package: Formula
Loading required package: maxLik
Loading required package: miscTools
Loading required package: MASS
> library("foreign")
> leblang2003y.stata <- read.dta("/Users/mcswong/Desktop/leblang2003y.dta")
> View(leblang2003y.stata)
> m1 <- egame12(y ~ capcont + lreserves + overval + creditgrow +
+ usinterest + service + contagion + prioratt - 1 | 1 | 1 | 1 | unifgov + lexports
+ preelec + postelec + rightgov + realinterest + capcont + lreserves,
+ data = leblang2003y.stata, link = "probit", type = "private")
> summary(m1)
```

```
Call:
egame12(formulas = y ~ capcont + lreserves + overval + creditgrow +
    usinterest + service + contagion + prioratt - 1 | 1 | 1 | 1 |
    unifgov + lexports + preelec + postelec + rightgov + realinterest +
        capcont + lreserves, data = leblang2003y.stata, link = "probit",
    type = "private")

Coefficients:
                    Estimate Std. Error  z value  Pr(>|z|)
u1(1):capcont      -0.452469   0.246015  -1.8392  0.065887 .
u1(1):lreserves     0.229181   0.056868   4.0301 5.576e-05 ***
u1(1):overval      -0.441258   0.085546  -5.1581 2.494e-07 ***
u1(1):creditgrow   -0.064819   0.028834  -2.2480  0.024574 *
u1(1):usinterest   -0.050541   0.055535  -0.9101  0.362787
u1(1):service      -0.028797   0.054734  -0.5261  0.598807
u1(1):contagion    -0.115889   0.047754  -2.4268  0.015233 *
u1(1):prioratt     -0.121824   0.048525  -2.5105  0.012055 *
u1(2):(Intercept)  -3.664825   0.298936 -12.2596 < 2.2e-16 ***
u1(3):(Intercept)  -3.138530   0.286409 -10.9582 < 2.2e-16 ***
u2(3):(Intercept)   0.426935   0.782241   0.5458  0.585214
u2(3):unifgov      -0.356759   0.357950  -0.9967  0.318923
u2(3):lexports     -0.199745   0.174444  -1.1450  0.252193
u2(3):preelec       1.663190   0.746972   2.2266  0.025976 *
u2(3):postelec      1.062284   0.592304   1.7935  0.072896 .
u2(3):rightgov     -0.935812   0.454065  -2.0610  0.039306 *
u2(3):realinterest  1.795540   0.600695   2.9891  0.002798 **
u2(3):capcont       0.065582   0.757930   0.0865  0.931047
u2(3):lreserves     0.309912   0.165696   1.8704  0.061433 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Standard errors estimated from inverse Hessian

Log-likelihood: -482.0155
AIC: 1002.031
No. observations: 7240
>
```

Figure 10.5.10: **R** Codes for Leblang's (2003) Results

# Chapter 11

# Social Behavior and Evolutionary Dynamics

In this chapter we introduce an alternative way to create EITM linkages — Agent-based modeling (ABM). ABM has been considered as a bottom-up approach modeling behaviors of a group of agents, rather than a representative agent, in a system. One important element of ABM is that it allows the possibility of agents' interactions in micro levels with the assumption of bounded-rationality or imperfect information. Given agents' heterogenous characteristics and their interactions at the micro level, researchers can simulate the system and observe changes in the macro level over time according to the system-simulated data. As a result, the simulated data can be compared with real-world empirical data so that researchers are able to make statistical inferences.

While the example in this chapter is economic in nature, ABM has many other uses in the social sciences. Consider voter turnout. Unlike the EITM approach that Achen takes using Bayesian updating (as described in Chapter 9), Bendor, Diermeier and Ting (2003) and Fowler (2006) use ABM. Bendor, Diermeir, and Ting (2003) (hereafter BDT) set up a computational model by assuming that voters are adaptively rational — voters learn to vote or to stay home in a form of trial-and-error. Voters are reinforced to repeat an action (e.g., vote) in the future given a successful outcome today. The reverse also holds. Based on the simulation with the assumption of adaptive behavior, BDT find the turnout rate is substantially higher than the predictions in rational choice models. This result can be observed even in electorates where voting is costly. Fowler (2006) revises the BDT model by removing the feedback in the probability adjustment mechanism and instead includes habitual voting behavior. Fowler finds his behavioral model is a better fit to the same data

BDT use, but a point not to be lost is Fowler leverages the EITM approach to build on BDT's model and test.

While there are various approaches of ABM,[1] in this chapter, we present the work of Arifovic (1994) on genetic algorithm (GA) learning. GA, developed by Holland (1970), has been recognized as an important methodology for computational optimization (Holland 1975; Goldberg 1989). It is used for solving optimal solutions numerically in mathematical systems when deriving closed-form solutions is technically difficult.

ABM has also been used with cobweb models. Recall Cobweb models have been used in the macro models. Muth (1961), for example, formulates the cobweb model with rational expectations. [2] The seminal work of Arifovic (1994) investigates if the macro-level stability condition (the cobweb theorem) is necessary for a stable cobweb economy under GA. Arifovic simulates the cobweb model using the genetic algorithm and compares the GA-simulated results with those based on adaptive learning mechanism and Wellford's experiments. The EITM linkage is that the behavioral concepts – imitation, invention, communication and examination — are mimicked by the genetic operators — reproduction, mutation, crossover and election, respectively, such that the GA-simulated data are empirically compared with human-subject experiments, and other forecasting mechanisms.

## 11.1 Step 1: Relating Elements of Social Interaction: Imitation, Invention, Communication, and Examination to Prediction

The GA, developed by Holland (1970), is considered one of the evolutionary algorithms inspired by natural evolution with a core concept of "survival of the fittest". The GA describes the evolutionary process of a population of genetic individuals (called chromosomes) with heterogeneous beliefs in response to the rules of nature (called fitness function). Genetic individuals form different beliefs or take different actions based on their combination of genes (or bits). Based on the concept of "sur-

[1] See de Marchi (2005) for different computational modeling approaches for details.

[2] Other studies investigate the stability conditions of the model using different learning mechanisms. See, for example, Evans and Honkapohja (1995, 1996), Brock and Hommes (1997), Heinemann (2001), Evans and Branch (2006), Granato, Guse, and Wong (2008). There is also work involving experiments with human subjects (Holt and Williamil 1986; Wellford 1989; Hommes, Sonnemans, Tuinstra, and van de Velden 2007).

vival of the fittest", a genetic individual is more likely to be reproduced in the next generation (that is, a higher survival rate) through a process of natural selection if the individual performs "better" given the rules of nature. This process is called "reproduction". Further, in order to create new ideas or new actions in the next generation, the possibilities of "mutations" (*within*-chromosome recombinations) or "crossovers" (*between*-chromosome recombinations) are imposed in the evolutionary process. The process of reproduction, and recombination will take place repeatedly for each generation until certain conditions are met in the nature (for example, the average fitness value for all genetic individuals in a generation converges to a certain level).

While imposing the bounded rationality assumption has become a standard practice, researchers fail to relax the assumption of *homogeneous agents* due to the difficulty of obtaining closed-form solutions. Arifovic considers an alternative by simulating the cobweb model under genetic algorithm where firms are allowed to make *heterogenous production decisions* by imitating peers' successful production strategies, and innovating new ideas on its own and with other firms. Interestingly, she finds that the stability condition for convergence is not a necessary condition. The GA converges to the rational expectations equilibrium (REE) for an expanded parameter space.

The cobweb model, presented in the Appendix of Chapter 5, is an example of a supply-demand model which demonstrates the dynamic process of a market economy. The cobweb model is summarized in equation (5.5.10):

$$p_t = A + Bp_t^e + \xi_t,$$

where $A = \frac{\alpha-\gamma}{\beta}$, $B = -\frac{\lambda}{\beta}$, and $\xi_t = \frac{\left(\epsilon_t^d - \epsilon_t^s\right)}{\beta}$. The equation shows that the expected price level $p_t^e$ at time $t$ formed at $t-1$ *determines* the actual price level $p_t$ at time $t$. Assuming that $p_t^e$ is replaced with $p_{t-1}$, in the long run, the price level $p_t$ is in a stationary equilibrium.[3]

Arifovic (1994) uses the cobweb model and assumes each firm $i$ chooses a production level, $q_{i,t}$, to maximize its expected profit $\pi_{i,t}^e$. This calculation is based on the production cost function $C_{it}(q_{it})$ and the expectations of the market price $P_t^e$ that will prevail at time $t$. Formally, this production level can be expressed in the following way.

[3] $p_t$ converges to $p^*$ as $t \to \infty$, provided the ratio of supply and demand slopes are less than one (i.e., $\lambda < \beta$). However, if the ratio is larger than one, the price level diverges away from its long run equilibrium. This is called the *cobweb theorem* (Ezekiel 1938).

First, the quadratic cost function for firm $i$ is:

$$C_{it} = aq_{it} + \frac{1}{2}bmq_{it}^2, \tag{11.1.1}$$

where $a, b > 0$, $C_{it}$ represents the cost of production given the level of output $q_{it}$ at time $t$, and $m$ is the number of firms in the market. Firm $i$'s expected profit as the difference between expected total revenue and total cost:

$$\begin{aligned}\pi_{it}^e &= P_t^e q_{it} - C_{it}(q_{it}) \\ &= P_t^e q_{it} - aq_{it} - \frac{1}{2}bmq_{it}^2,\end{aligned} \tag{11.1.2}$$

where $P_t^e$ is the expected price of the good in the market at time $t$. Each firm maximizes the expected profit function by setting the level of production $q_{it}$. The first order condition can be written as:

$$\begin{aligned}\frac{\partial \pi_{it}^e}{\partial q_{it}} &= 0 \\ \Rightarrow P_t^e - a - bmq_{it} &= 0.\end{aligned}$$

The following optimal production level for firm $i$ based on the first order condition is:

$$q_{it} = \frac{P_t^e - a}{bm}. \tag{11.1.3}$$

Assuming all firms are identical in the market, such that $q_{it} = q_t$, we have: $Q_t = \sum_{i=1}^m q_{it} = mq_t$. We can derive the market supply curve by summing up $m$ firms' optimal output level from equation (11.1.3):

$$\begin{aligned}Q_t &= \sum_{i=1}^m q_{it} \\ &= mq_t = \frac{(P_t^e - a)}{b}.\end{aligned} \tag{11.1.4}$$

To determine the price level in the economy, we assume a linear market demand curve:

$$P_t = \gamma - \theta Q_t, \tag{11.1.5}$$

where $Q_t = \sum_{i=1}^m q_{it}$ is the total output demanded in the market. Finally, one can derive the market equilibrium of price level by equating market demand (11.1.5) and market supply (11.1.4):

$$\begin{aligned}\frac{\gamma - P_t}{\theta} &= \frac{P_t^e - a}{b} \\ \Rightarrow P_t &= \frac{\gamma b + a\theta}{b} - \frac{\theta}{b} P_t^e. \end{aligned} \tag{11.1.6}$$

Equation (11.1.6) represents the cobweb model in Arifovic (1994). This expression is equivalent to equation (5.5.10). According to the Cobweb theorem, the price level in the model converges to the REE in the long run (i.e., $P_t^e = P_t$) only if $\frac{\theta}{b}$ is less than 1. If $\frac{\theta}{b} > 1$, the model will be unstable and the sequence of market prices diverge away from the equilibrium.

To investigate whether this particular stability condition is a necessary condition for convergence under a genetic algorithm, Arifovic (1994) compares the market behavior in the genetic environment with three additional learning algorithms as well as the cobweb experiments (Wellford 1989).[4] Those three learning algorithm are:

- Static expectations (i.e., $P_t^e = P_{t-1}$),
- Simple adaptive expectations by averaging the past prices $\left(P_t^e = \frac{1}{t}\left(\sum_{s=0}^{t-1} P_s\right)\right)$ from the initial period up to time $t$,[5] and
- Expectations formed by least squares updating mechanism.[6]

## 11.2 Step 2: Analogues for Social Interaction and Prediction

Arifovic (1994) simulates the cobweb model based on three basic genetic operators in the GA simulations: (1) *reproduction*, (2) *mutation*, and (3) *crossover*. She also introduces a new operator, called *election*, in the simulations. We describe the genetic operators and the procedure of the GA in detail in the Appendix of this chapter.

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[4]The following stability conditions hold:

- Static expectations: the model is stable only if $\theta/b < 1$.
- Simple adaptive expectations: the model is stable if $\theta/b < 1$ *and* $\theta/b > 1$ (See Carlson 1968).
- Least squares learning: the model is stable only if $\theta/b < 1$ (See Bray and Savin 1986).

[5]This expression is similar to equation (5.5.22) where the current price level is a average of all price levels observed in the past with an *equal* weight.

[6]See the Appendix of Chapter 6 for details.

Reproduction is a genetic operator in which an individual chromosome is copied from the previous population to a new population: agents imitate the strategies from better-performing agents.

Mutation is a genetic operator in which one or more gene within an individual chromosome changes value randomly: agents may change their strategies suddenly through innovations.

Crossover is the third basic genetic operator and the crossover which occurs when two randomly drawn chromosomes exchange parts of their genes: agents work with others to innovate or develop a new strategy.

Election is introduced by Arifovic (1991, 1994). It is an operator to "examine" the fitness of newly generated (or offspring) chromosomes and then compare them with their parent chromosomes (the pair of chromosomes before crossover). Both offspring chromosomes *are elected* to be in the new population at time $t+1$ if their potential fitness values evaluated at time $t$ is higher than their parents' fitness values. However, if only one new chromosome has a higher fitness values than their parents, the one with lower value will not enter the new population, but one of the parents with a higher values stays in the new population. If both new chromosomes have lower values than their parents, they cannot enter but their parents stay in the new population.

One can see how the election operator can apply to many dynamic processes involving the change and replacement of the status quo. Arifovic (1994) goes further and interprets this operator as an evaluation of new proposed strategies. Only those promising new ideas can be implemented in the production process. Combining the first three basic GA operators (reproduction, crossover and mutation) with a new election operator, Arifovic considers this simulation process as the augmented GA approach.

We simulate the model using the basic GA approach (i.e., without the election operator) and the augmented GA approach (that is, with election operator) according to the values of parameters suggested in Arifovic (1994). In particular, we focus on 2 cases: 1) the parameter values of a stable cobweb model, and 2) those of an unstable cobweb model used in Wellford's (1989) experiments.

Table 11.1 presents the numerical parameters for the simulations. For the case of the stable cobweb model, we assign $\theta = 0.0152$ and $b = 0.016$, such that $\frac{\theta}{b} = 0.95 < 1$. On the other hand, for the unstable case, the ratio of supply and demand slopes is

| Parameters | Stable Case $\left(\frac{\theta}{b} < 1\right)$ | Unstable Case $\left(\frac{\theta}{b} > 1\right)$ |
|---|---|---|
| $\gamma$ | 2.184 | 2.296 |
| $\theta$ | 0.0152 | 0.0168 |
| $a$ | 0 | 0 |
| $b$ | 0.016 | 0.016 |
| $m$ | 6 | 6 |
| $P^*$ | 1.12 | 1.12 |
| $Q^*=mq^*$ | 70 | 70 |

Table 11.1: Cobweb Model Parameters

| Set | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Crossover rate: $\kappa$ | 0.6 | 0.6 | 0.75 | 0.75 | 0.9 | 0.9 | 0.3 | 0.3 |
| Mutation rate: $\mu$ | 0.0033 | 0.033 | 0.0033 | 0.033 | 0.0033 | 0.033 | 0.0033 | 0.033 |

Table 11.2: Crossover and Mutation Rates

$\frac{\theta}{b} = 0.0168/0.016 = 1.05 > 1$. We also vary the rates of crossover and mutation rates in the simulations as presented in Table 11.2.

## 11.3 Step 3: Unifying and Evaluating the Analogues

The results from the GA are compared with those from three learning mechanisms (i.e, static expectations, adaptive expectations, and least squares learning) as well as human-subject experiments (Wellford 1989). Figure 11.3.1 presents simulations of the stable cobweb model with 8 sets of crossover and mutation parameters corresponding to Table 11.2. We simulate the model using both basic GA and augmented GA approaches. The simulations show that the price level is more volatile when the basic GA is applied. Its movement is bounded around the rational expectations equilibrium (REE), ranging between 0.6 and 1.8 for all simulations. On the other hand, if the election operator, suggested by Arifovic (1994), is included in the GA simulations, the price level converges more quickly to the REE.

Arifovic (1994) provides a detailed discussion of the movements of price and quantity in the cobweb model under the augmented GA approach. In a nutshell, the election operator serves an important component of behavioral convergence in the GA environment. The convergence to an equilibrium happens under GA when all chromosomes (agents' strategies) become identical so that there is no strategy in the genetic population deviating from the optimal quantity ($q^*$) which maximizes profit at an equilibrium price ($P^*$). After equilibrium is achieved, the variance of prices

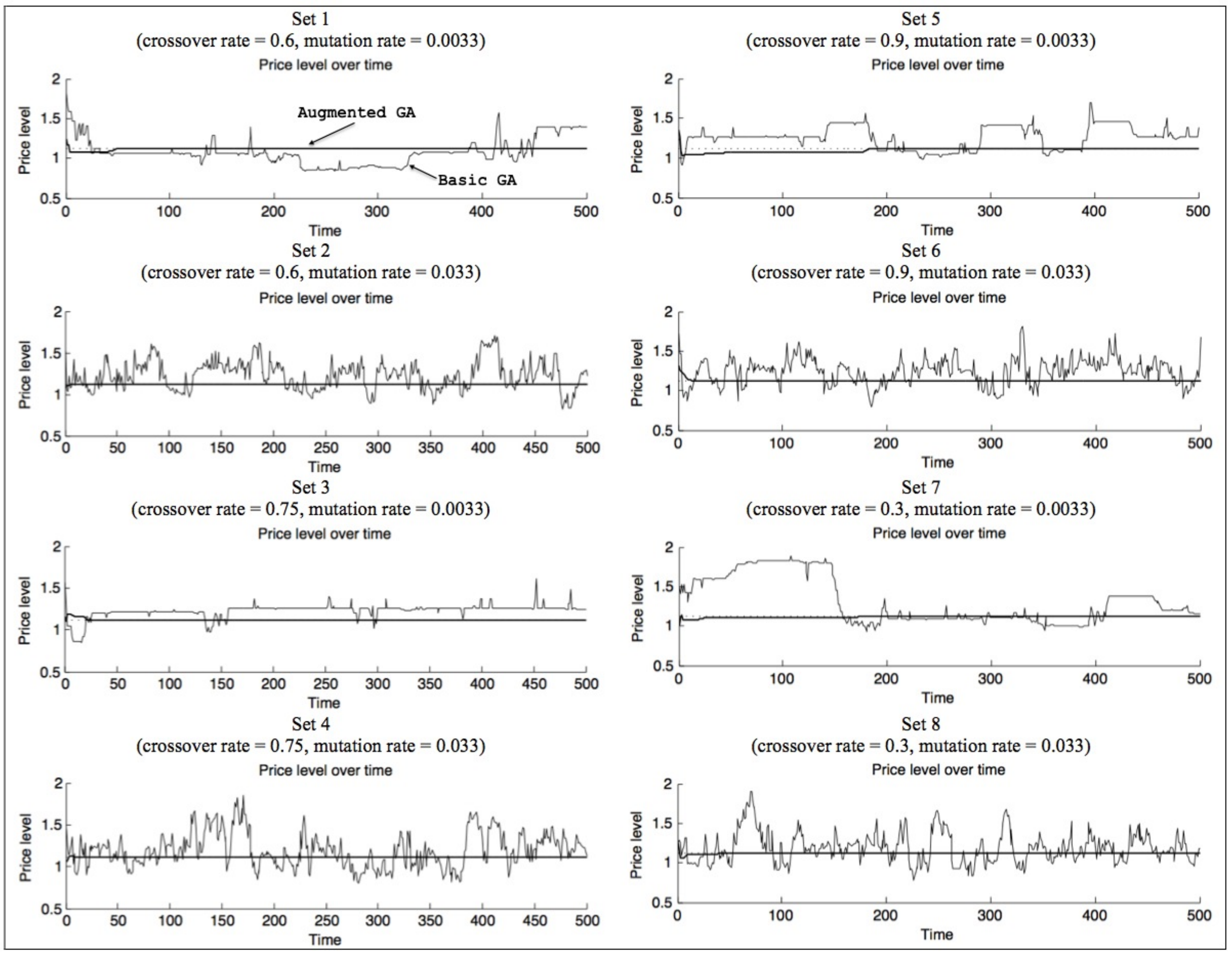


Figure 11.3.1: GA Simulations (Stable Case: $\frac{\theta}{b} = 0.95 < 1$)

and quantities in the population approaches zero. While the reproduction operator attempts to replicate the better performing strategies to reduce the variance in the population, crossover and mutation dynamics allow creation of new strategies which increases the degree of variance. Hence, according to basic GA simulations, the price level cannot be stabilized at a certain level since crossover and mutation happens in every generation (See Figure 11.3.1).

When we extend the basic GA and use an election operator. The election operator "pre-screens" the performance of off-spring strategies (i.e., strategies after crossover and mutation). Off-spring strategies are either elected or eliminated and, via the previous period's fitness function, either enter or do not enter a new generation. This "quality control" helps to keep better strategies in the new population and therefore reduces the variance quickly as the algorithm converges to an equilibrium value. The behavior is confirmed by the augmented GA simulations in Table 11.3.1.

Arifovic (1994) introduces the GA procedure as an alternative learning mechanism, a mechanism that is used to determine convergence to an equilibrium. This alternative learning mechanism mimics social behavior — observed in reality — namely imitation, communication, experiment, and examination. While it can be difficult to test GA empirically, the GA simulated data is compared with the data generated in human-subject experiments. She finds that, in an unstable case of the cobweb model, the divergent patterns *do not* happen under both GA learning and human-subject experiments. Instead, price and quantity fluctuate around the equilibrium values as shown in basic GA learning and Wellford's (1989) human-subject experiments.

More importantly, the unstable model converges to the REE if the election operator is imposed in the algorithm (See Figure 11.3.2). Arifovic demonstrates the features of the GA are consistent with characteristics of human behavior as observed in experiments. In reality, agents might not rely on sophisticated forms of learning, such as adaptive learning or econometric learning, for solving their decision problems. Rather an alternative or more expansive view of human adaptation and change can consider how agents generate new beliefs and how the best beliefs survive. In this more general perspective these latter factors play important roles in adapting new decisions and strategies.

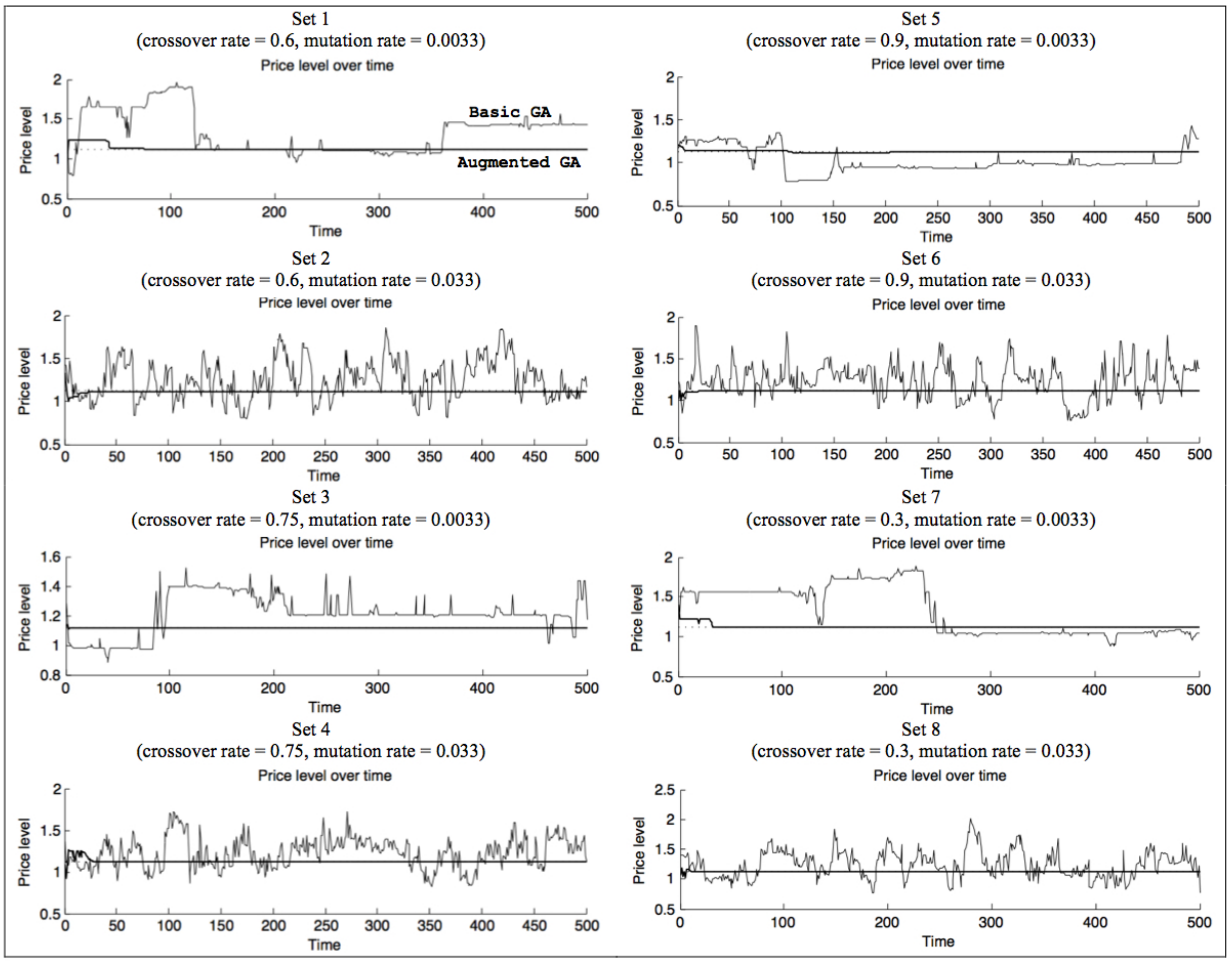


Figure 11.3.2: GA Simulations (Unstable Case: $\frac{\theta}{b} = 1.05 > 1$)

## 11.4 Leveraging EITM and Extending the Model

Arifovic (1994) provides an EITM connection by comparing the results using computational experiments with those in real human-subject experiments. She links elements of social interaction — imitation, innovation, communication, and examination with genetic operators — computational (empirical) analogues (reproductions, mutation, crossover and election).

While this example makes comparisons to experiments and other learning algorithms, GA simulation can be contrasted with "real world data" as well. An example is Arifovic and Maschek's (2006) data based simulation on an agent-based currency crisis model. Using various parameter values in the model, they find that the simulated data share some similar properties from human-subject experimental data and empirical data from emerging markets.

## 11.5 Appendix

ABM has been an important element of understanding complex economic and social systems. ABM also introduces an alternative way to unify formal and emprical anlaysis, but it also has sufficient analytic power that go beyond traditional modeling practice assumptions. In economics, for example, the assumptions of utility maximization, perfect information, and market clearing are firmly rooted as the mainstream micro-foundations. Furthermore, traditional practice also makes use of the representative-agent hypothesis since it allows for greater ease in solution procedures.

Yet, these modeling assumptions and practices come at a price. Consider that fully-rational agents are not bounded by any information capacity; heterogeneity becomes irrelevant; and interactions among agents are unnecessary. When they examine these shortcomings, LeBaron and Tesfatsion (2008) state that:

> "[p]otentially important real-world factors such as subsistence needs, incomplete markets, imperfect competition, inside money, strategic behavioral interactions, and open-ended learning that tremendously complicate analytical formulations are typically not incorporated" (page 246).[7]

For these reasons ABM serves as an alternative method investigating the interactions among agents in different sectors.[8]

[7] Page (2004) points out the importance of ABM for macro-level research. He argues that aggregation models without taking an ABM approach into consideration may disregard important information leading to errors or inaccurate conclusions, especially when a model is complex.

[8] Along this line of argumnet, and after the financial crisis in 2007-2008, Farmer and Foley (2009)

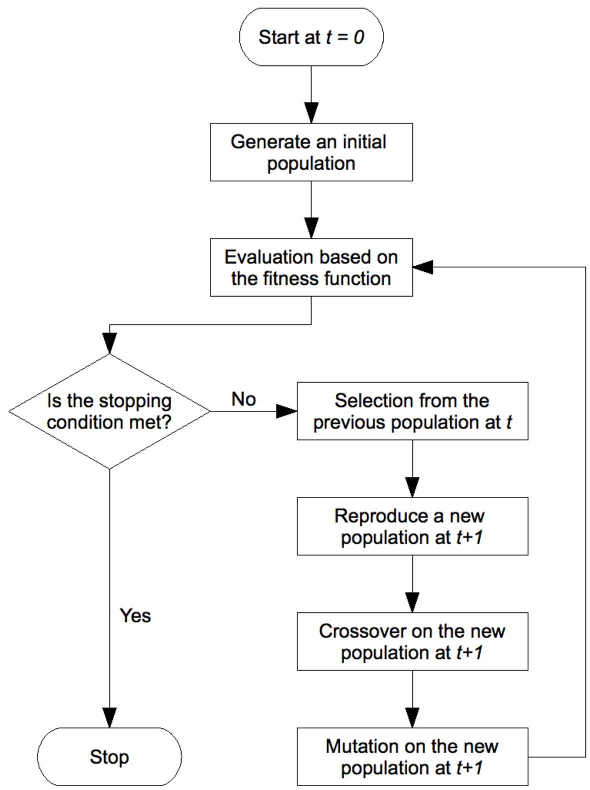


Figure 11.5.1: The Structure of the Genetic Algorithm Process

Miller (1986) and Arifovic (1991) were early users of the GA approach to economic questions. Miller (1986), for instance, develops a model of adaptive economic behavior based on the GA operation with applications — utility and profit maximization, technological innovation, and demographic transitions.[9] Figure 11.5.1 represents a standard structure of the GA operation.[10] In the economic arena —although this can be extended to all manner of social science questions — researchers can consider the GA process akin to a micro-level social learning system and interpret chromosomes as *economic agents*, reproduction as *imitation*, mutation as *experiment*, crossover as *communication*, and evaluation as *market performance* (Riechmann 2001).

The GA is an alternative approach to study dynamic processes that includes

asserted that ABM can be a "better way" to "assemble the pieces and understand the behaviour of the whole economic system" (page 685).

[9] Miller argues that standard optimization approach and adaptive approach are not mutually exclusive. Some studies adopt GA to determine selection criteria (i.e., determining stable solutions) when a model has multiple equilibria (Arifovic 1996; Miller 1989; Bullard and Duffy 1998; Riechmann 1999, 2001; Geisendorf 2011).

[10] This appendix presents the basic GA approach. Election operator, suggested by Arifovic (1991, 1994), is not illustrated. We refer readers to Arifovic (1994) for the implementation of election operator in the GA.

social interaction between heterogeneous agents. Arifovic (1991: 2) argues that:

> [a] genetic algorithm describes the evolution of a population of rules, representing different possible beliefs, in response to experience. ... Rules whose application has been more successful are more likely to become more frequently represented in the population, through a process similar to natural selection in population genetics. Random mutations also create new rules by changing certain features of rules previously represented in the population, thus allowing new ideas to be tried.

Thus, we can test whether agents obtain an REE under a series of genetic processing scenarios (See Bullard and Duffy 1998a, 1998b, 1999; Arifovic 1994, 1995, 1996, 1998; Arifovic, Bullard, and Duffy 1997).

### 11.5.1 Formal Analogues

### Decison Theory

To explain how the basic GA works,[11] we consider a simple profit maximization problem where there exists a group of sellers who sell distinctive products in the market.[12] Assume each seller faces the same individual market demand curve:

$$p = p\left(q\right), \tag{11.5.1}$$

where $p$ is the price the seller receives, $q$ is the level of production, and $p'\left(q\right) = dp/dq < 0$. Every seller also obtains the same production technology so that the cost function is identical for all sellers:

$$c = c\left(q\right), \tag{11.5.2}$$

where $c\left(\cdot\right)$ represents the cost function and $c'\left(q\right) = dc/dq > 0$. Assume further that each seller attempts to maximize profit by choosing an optimal level of output, $q^*$, according to the following profit function:

$$\max_q \pi = p \times q - c\left(q\right). \tag{11.5.3}$$

[11] This appendix provides a basic idea of the GA procedure. See Goldberg(1989) and Riechmann (2001) for further analytical details of the genetic algorithm.

[12] For simplicity, we assume each seller is a monopolist.

We derive the first order condition from equation (11.5.3), and solve the optimal level of output, $q^*$ by equating marginal revenue and marginal cost:

$$\begin{aligned} \left.\frac{d\pi}{dq}\right|_{q=q^*} &= 0 \\ \Rightarrow p'(q^*) &= c'(q^*). \end{aligned} \tag{11.5.4}$$

To simplify the GA simulation, the demand and cost functions are assumed to be linear:

$$p = a - bq, \tag{11.5.5}$$

and:

$$c = d + eq, \tag{11.5.6}$$

where $a, b, d, e > 0$. Thus, the maximization of the profit function can be written as:

$$\max_q \pi = (a - bq)\, q - (d + eq). \tag{11.5.7}$$

The optimal level of output $q$ for each seller is:

$$q^* = \frac{a-e}{2b}, \tag{11.5.8}$$

where $a > e$. Given $q^*$ presented in equation (11.5.8), we can obtain the optimal level of price $p^*$ and profit $\pi(q^*)$ from equations (11.5.5) and (11.5.7), respectively.

In standard practice, agents can determine the optimal level of output immediately and obtain the maximum level of profit. However, we relax this strong form of "rationality" and instead assume agents go through a process of trial and error, and learning from experience to obtain the optimal solution in the model.

### 11.5.2 Computational and Empirical Analogues

## The Operation of the Genetic Algorithm

#### Genes, Chromosomes, and Populations

The GA process is illustrated in Figure 11.5.1: an initial population with $M$ chromosomes (i.e., there are $M$ individuals in the setting (e.g., society, economy) is first generated at time $t = 0$. Each chromosome $C_i$ consists of $L$ genes formed in binary structure with either `'0'` or `'1.'` For example, a representative chromosome (agent) $i$ with length $L = 10$, can be written as: $C_i =$ `0100101110`. Each chromosome is read as a binary number with a maximum value $B^{\max} = 2^L - 1$. Given $L = 10$, the maximum value of a chromosome is: $B(C_i^{\max}) = B^{\max} = B(\texttt{1111111111}) = 2^{10} - 1 = 1023$, where $B(\cdot)$ is a binary operator of converting a binary number into a decimal number.

To understand the binary operator $B(\cdot)$, we consider a binary number $C_i = \texttt{0100101110}$, the decimal number can be represented as:

$$
\begin{aligned}
B(0100101110) &= 0 \times 2^9 + 1 \times 2^8 + 0 \times 2^7 + 0 \times 2^6 + \\
&\quad 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + \\
&\quad 1 \times 2^1 + 0 \times 2^0 = 302.
\end{aligned}
$$

Since there are $m$ chromosomes with length of $L$ in the population, the initial population $P_0$ can be represented as an $m \times L$ matrix where every gene in each chromosome is coded either 0 or 1 with equal probability. Assuming there are 8 genetic individuals (i.e., $M = 8$) in a society or economy at $t = 0$, a possible population $P_0$ can be written as:

$$
P_0 = \begin{matrix}
0100101110 \\
1110101010 \\
0101110100 \\
0100001010 \\
1110101000 \\
0101101101 \\
1100101010 \\
0100011100
\end{matrix}
$$

Note, a chromosome with $L = 10$ can have a possible value between 0 and 1023. Its maximum value might be either too large or too small for a economic variable of interest. For the profit maximization example, suppose the parameters in the demand and cost functions are: $a = 200$, $b = 4$, and $e = 40$ in equation (11.5.8), and the optimal output level $q^*$ is 20. If we are interested in the output level evolution in this genetic economy, we might restrict the output level range between 0 and $U^{\max}$, where $U^{\max}$ is an upper bound. We can also interpret $U^{\max}$ as the production capacity for all sellers in the society or economy. Under these conditions, the possible value of an economic variable of interest can be written as:

$$
V(C_i) = \frac{U^{\max}}{B^{\max}} \times B(C_i), \tag{11.5.9}
$$

where $V(C_i) \in [0, U^{\max}]$ for $B(C_i) \in [0, B^{\max}]$. Therefore, given a chromosome $C_i =$ `0100101110`, the genetic individual $i$ produces an output level $q_i$:

$$q_i = V(C_i) = \frac{100}{1023} \times 302 = 29.52 \approx 30,$$

where $U^{\max}$ is assumed to be 100 in this example.

On the other hand, if the upper bound of the economic variable of interest is larger than the maximum value of the binary number, one can increase the maximum value of the binary number by increasing the length of the chromosomes ($L$) in the population, and then apply equation (11.5.9) to restrict a possible range of the economic variable of interest. As the population contains a total of $m$ individuals with different production strategies, the GA procedure demonstrates the evolution of production strategies when heterogeneity, social interactions, and learning are assumed in the behavioral theory.

### Fitness Function

To determine if a genetic "individual" is more likely to "survive" or be reproduced in the next period, we need to evaluate performance based on the fitness function. In our profit maximization example, equation (11.5.7) can be considered the fitness function $F(C_i)$:

$$\begin{aligned} F(C_i) &= \pi(V(C_i)) \\ &= \pi(q_i) = (a - bq_i)\, q_i - (d + eq_i). \end{aligned} \tag{11.5.10}$$

For instance, with the values of $a = 200$, $b = 4$, $d = 50$, and $e = 40$, the fitness value of $C_i =$ `0100101110` is:

$$F(C_i) = \pi(V(C_i)) = (200 - 4(29.52))\, 29.52 - (50 + 40(29.52)) = 1187.48.$$

If the optimal level of output is $q^* = 20$, the maximum fitness level is:

$$F^{\max} = (200 - 4(20))\, 20 - (50 + 40(20)) = 1550. \tag{11.5.11}$$

Note that fitness function does not apply to minimization problems (i.e., cost minimization problems). For these class of problems an appropriate transformation of the economic values is needed for generating fitness values.

### Reproduction

Reproduction is a genetic operator where an individual chromosome is copied from the previous population to a new population. Given a probability of being drawn for each chromosome based on the fitness value, chromosomes are repeatedly drawn with replacement from the pool of the previous population. They are then put into the new population until the size of the new population equals that of the previous population (i.e., $M_t = M_{t+1}$).

Now, let the probability of reproduction be determined by the relative fitness function:

$$R\left(C_{i,t}\right) = \frac{F\left(C_{i,t}\right)}{\sum_{m=1}^{M} F\left(C_{m,t}\right)}, \tag{11.5.12}$$

where $\sum_{i \in M} R\left(C_{i,t}\right) = 1$. For $F\left(C_{i,t}\right) \geq 0$ and for all $i$, $R\left(C_{i,t}\right)$ is bounded between zero and one for all $i$. The relative fitness value $R\left(C_{i,t}\right)$ gives us the probability chromosome $i$ is copied to the new population at time $t+1$. The larger the fitness value $F\left(C_{i,t}\right)$, the higher likelihood the chromosome survives $R\left(C_{i,t}\right)$ in the next period. One potential limitation in equation (11.5.12) is that $R\left(C_{i,t}\right)$ can be negative if the fitness value $F\left(C_{i,t}\right)$ is negative (See equation (11.5.10)). If the probability value is bounded between zero and one, then this is limitation is not a threat.

Goldberg (1989) proposes a scaled relative fitness function:

$$\begin{aligned} S\left(C_{i,t}\right) &= \frac{F\left(C_{i,t}\right) + A}{\sum_{m=1}^{M} \left[F\left(C_{m,t}\right) + A\right]} \\ &= \frac{F\left(C_{i,t}\right) + A}{\sum_{m=1}^{M} F\left(C_{m,t}\right) + MA}, \end{aligned} \tag{11.5.13}$$

where $A$ is a constant such that $A > -\min_{C_i \in P_t} F\left(C_{i,t}\right)$. Introducing $A$ into the relative fitness function prevents negative probability values of being drawn for some chromosomes in the population (i.e., $S\left(C_{i,t}\right) > 0$ for all $i$.).

The reproduction operator is important in the social sciences. Since well-performed genetic agents are likely to be reproduced or survive in the next period, reproduction can be considered *imitation.* Agents are more likely to imitate or learn from their peers who have done better in a given setting or market.

### Crossover

Crossover occurs when two randomly drawn chromosomes exchange parts of their genes. This is called the *inter*-chromosome recombination. In the GA environment,

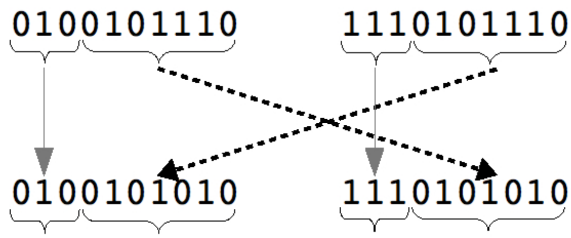


Figure 11.5.2: An Example of Crossover

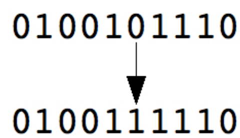


Figure 11.5.3: An Example of Mutation

crossover is a process where parents are replaced by their children in the population. To explain the complete procedure of crossover, assume there are two “parent” chromosomes which are randomly selected (without replacement) from the population. A crossover point will be randomly chosen to separate each chromosome into two sub-strings. Finally, two “offspring” chromosomes will be formed by swapping the right-sided parents’ substrings with probability $\kappa$. Figure 11.5.2 shows the procedure of crossover where the crossover point is the 7th position from the right.

In the social sciences crossover is a process of *communication*. Agents obtain new strategies by exchanging information from each other. Note that crossover may not improve fitness values. The lack of improvement can be interpreted as agents making mistakes in communication.

### Mutation

Mutation is a genetic operator in which one or more gene within an individual chromosome changes value randomly. Every gene within a chromosome has a small probability, $\mu$, changing in value, independent of other positions. It is an *intra*-chromosome recombination. Figure 11.5.3 illustrates an example of the mutation process. In figure 11.5.3, mutation occurs on the 5th gene from the right.

Mutation can be interpreted as *invention* or *an experiment* in new strategies. Firms may not improve their performance by merely imitating other firms. Instead, they may need to invent new products or experiment with new production processes. In a political science context the analogy could extend to any topic involving strategic messages, whether it is campaigns, negotiations between rival nations, and the like.

### Application: Details of the GA Simulation for Profit Maximization

The GA simulation is based on the example of profit maximization presented in the previous section. Again, assume the parameters in the demand function are $a = 200$ and $b = 400$, and those in cost functions are: $d = 50$ and $e = 40$. The optimal level of output is $q^* = 20$.

To investigate if agents in the GA environment are able to choose the optimal production level in the long run, we assume that there are 200 genetic agents ($M = 200$) in the economy with the length of 16 genes ($L = 16$). The maximum production capacity for each seller is $U^{\max} = 50$. Assume the fitness value for each agent is determined by the profit level presented in equation (11.5.10).[13] We simulate the GA economy with 500 iterations (generations) using MATLAB.[14] The output and code of the MATLAB program for the simulation are included at the end of the chapter.

Figure 11.5.4 presents the movement of average output levels over time. We obtain the figure by taking an average of the output levels from 200 agents in each generation, and then plotting the averages over 500 generations. Figure 11.5.4 shows there is a large production level adjustment, on average, in the first 60 generations ranging between 18 and 27. After the first 60 generations, the production level becomes more stable and is set around the optimal level of $q^* = 20$. In this simulation, the average output level over the whole time period is 19.93 with variance of 0.39. This all-time average is very close to the optimal level $q^*$. It implies that, on average, genetic agents are able to learn and determine the optimal output level over time.

We further investigate the distribution of production levels in each generation. Figure 11.5.5 demonstrates the standard deviation of output level within each generation over time. Not surprisingly, there are larger variations of output levels in the early generations. After the process of imitation, experiment and communication, agents have a similar production pattern with the standard deviations becoming smaller in latter generations.

## 11.6 MATLAB Output

```
This is a Genetic Algorithm Simulation.
```

[13] Given that the likelihood of reproduction comes from the scaled relative fitness function (11.5.13) and the probabilities of crossover ($\kappa$) and mutation ($\mu$) are 0.3 and 0.0033, respectively.

[14] We implement the stopping condition to terminate the simulation process after 500 iterations.

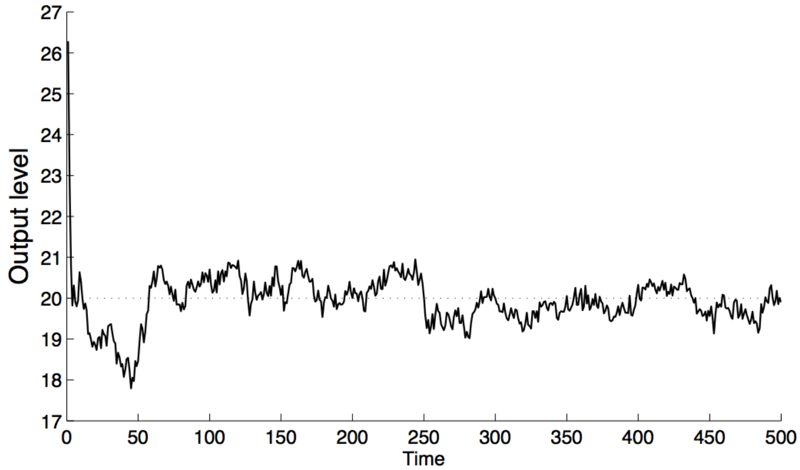


Figure 11.5.4: The Output Level Over Time

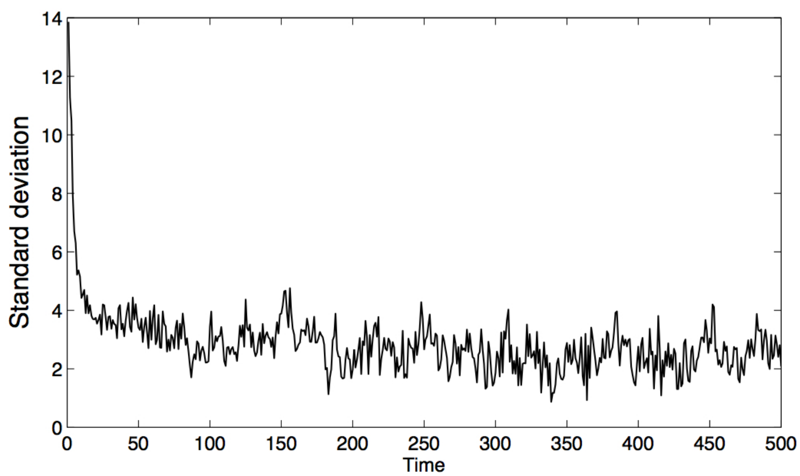


Figure 11.5.5: The Standard Deviation of the Output Level Over Time

```
The simple profit function is:  profit = (a-bq)q - (d+eq).
Given the parameters:  a=200, b=4, d=50,and e=40,
the optimal level of output is:  q*=20
In this simulation, you have:
a) 200 agents in each population;
b) 16 genes for each agent;
c) 50 as the maximum economic value;
d) 0.3 as the probability of crossover;
e) 0.0033 as the probability of mutation;
f) 500 generations in this simulation.
Hit any key to start running the simulation.
The simulation is now running...
Thank you for waiting.  The process of simulation is over.
Please hit any key to get the results.
The mean of the output level in all generations is 19.931.
The variance of the output level is 0.38739.
This is the end of the simulation.
```

## 11.7 MATLAB Code

```
%Genetic algorithm for a simple profit maximization
%
%Initial Population Parameters:
%ind = number of agents(chromosomes) in a population
%bit = number of genes in each agent(chromosome)
%Lmax = the upper bound of the real economic values
%epsilon = the value for the scaled relative fitness
%kappa = Probability of Crossover
%mu = Probability of Mutation
%time = number of generations(simulations)
ind = 200;
bit = 16;
Umax = 50;
epsilon = .1;
kappa = 0.3;
mu = 0.0033;
```

```
time = 500;
%
%Profit function parameters
% Demand function:  p = a - bq
% Cost function:  c = d + eq
% Profit function:  profit = (a-bq)q - (d+eq)
% Optimal level of output:  q* = (a-e)/2b
a = 200;
b = 4;
d = 50;
e = 40;
qstar = (a-e)/(2*b);
disp(' ')
disp(' ')
%
disp('This is a Genetic Algorithm Simulation.')
disp(' ')
disp('The simple profit function is:  profit = (a-bq)q - (d+eq).')
disp(' ')
stra = ['Given the parameters:  a=' num2str(a) ', b=' num2str(b) ', d='
num2str(d) ',and e=' num2str(e) ','];
strb = ['the optimal level of output is:  q*=' num2str(qstar)];
disp(stra);
disp(strb);
disp(' ');
disp(' ');
%
disp('In this simulation, you have:')
disp(' ')
str1 = ['a) ' num2str(ind) ' agents in each population;'];
str2 = ['b) ' num2str(bit) ' genes for each agent;'];
str3 = ['c) ' num2str(Umax) ' as the maximum economic value;'];
str4 = ['d) ' num2str(kappa) ' as the probability of crossover;'];
str5 = ['e) ' num2str(mu) ' as the probability of mutation;'];
str6 = ['f) ' num2str(time) ' generations in this simulation.'];
disp(str1);
```

```
disp(str2);
disp(str3);
disp(str4);
disp(str5);
disp(str6);
disp(' ')
%
disp('Hit any key to start running the simulation.')
disp(' ')
pause
disp('The simulation is now running...')
disp(' ')
%
%Value Function and Definitions
Bmax = (2 .^ bit) - 1;
m = ind;
n = bit;
%Generate the Initial Population:  "gen"
gen = rand(m,n);
for i=1:m
for j=1:n
if gen(i,j)<.5;
gen(i,j)=0;
else
gen(i,j)=1;
end
end
end
%
%Calculate the real value of each chromosome:  "BC"
m2 = 2 * ones(n,1);
for i=1:n
m2(i,1)=m2(i,1).^(n-i);
end
%
%Starting the Genetic Simulation Here!
```

```
for t=1:time
BC = ones(m,1);
for i=1:m
BC(i,1)=gen(i,:)  * m2;
end
%
%Calculate the real economic value of each chromosome:  "VC"
VC = (Umax ./ Bmax) * BC;
%
%Calculate the real value from the objective function:  "FC"
%This is the fitness function (The Most Important Function)
FC = (a - b .* VC) .* VC - d - e .* VC;
%Calcuate the relative fitness of each chromosome:  "RC"
RC = FC ./ sum(FC);
%Calculate the scaled relative fitness of each chromosome:  "SC"
A = abs(min(FC)) + epsilon;
FCA = FC + A;
SC = FCA ./ (sum(FC) + (m .* A));
genresult = [gen,VC,SC];
%
%Reproduction code
norm_fit = SC;
selected = rand(size(SC));
sum_fit = 0;
for i=1:length(SC),
sum_fit = sum_fit + norm_fit(i);
index = find(selected<sum_fit);
selected(index) = i*ones(size(index));
end
gen = gen(selected,:);
%
%This is the code for Crossover (Point & Pairwise)
%size(gen,1) = ind = number of individual
%size(gen,2) = bit = number of genes
sites = ceil(rand(size(gen,1)/2,1)*(size(gen,2)-1));
sites = sites.*(rand(size(sites))<kappa);
```

```
for i = 1:length(sites)
newgen([2*i-1 2*i],:)  = [gen([2*i-1 2*i],1:sites(i)) gen([2*i 2*i-1],
sites(i)+1:size(gen,2))];
end
gen=newgen;
%
%This is the code for mutation
mutated = find(rand(size(gen))<mu);
newgen = gen;
newgen(mutated) = 1-gen(mutated);
gen=newgen;
%
%Collecting solutions of interest
%output(t) = the mean output level in each generation
%variance(t) = the variance of output level in each generation
output(t)=mean(VC);
stddev(t)=var(VC)^.5;
optq(t) = qstar;
tt(t) = t;
end
%
%Reporting results
%mean_output = the average of the mean output level in all generations
%var_mean_output = the variance of the mean output level in all generations
%t_stat_mean_output = the t-statistics of mean_output away from the true
parameter
disp(' ')
disp(' ')
disp('Thank you for waiting.  The process of simulation is over.')
disp('Please hit any key to get the results.  ')
pause
disp(' ')
disp(' ')
mean_output=mean(output);
var_mean_output=var(output);
str_output = ['The mean of the output level in all generations is
```

```
' num2str(mean_output) '.'];
str_varout = ['The variance of the output level is ' num2str(var_mean_output)
'.'];
disp(str_output);
disp(str_varout);
%
figure(1)
subplot(2,1,1)
hold on
gop=plot(tt,output);
goptq = plot(tt,optq,'k:');
hold off
title('Output level over time')
set(gop,'Color','black','LineWidth',1.1)
xlabel('Time')
ylabel('Output level')
subplot(2,1,2)
gv=plot(stddev);
title('Standard deviation of output level over time')
set(gv,'Color','black','LineWidth',1)
xlabel('Time')
ylabel('Standard deviation of output level')
disp(' ')
disp(' ')
disp('This is the end of the simulation.')
disp(' ')
```

# Chapter 12

# An Alternative Unification Framework

An important alternative framework for methodological unification is Guillermina Jasso's *Tripartite Structure of Social Science Analysis* (Jasso 2004).[1] Like the EITM framework, she seeks to unify formal and empirical analysis. But, unlike the EITM framework, Jasso's framework can also be used for measurement purposes. The motivation for Jasso's (2004) tripartite structure:

> acknowledges the critical importance of the research activities that precede theoretical and empirical analysis — developing the framework out of which theoretical and empirical analysis emerge...it acknowledges the part played by nondeductive theories and links them to deductive theories, and it recognizes the extratheoretical empirical work...[it] represents more faithfully the varied kind of scientific work we do and their varied interrelationships. It invites to the table, so to speak, activities that in the old world of deductive theory and testing of predictions were slighted, even as they made their own fundamental contributions to the growth of knowledge (pages 401-402).

This chapter has three parts. First, we provide a brief background into Jasso's tripartite framework. Second, we apply Jasso's framework to a measurement question: the creation and use of an index for justice. Specifically, our application uses Jasso's justice evaluation function and aggregates it to justice indexes.

[1] We use the term "framework" as a substitute for Jasso's term — "structure." However, note that a component of Jasso's "structure" contains the term "framework."

Jasso's justice index sheds further light on thinking about different ways to characterize how people decide. Up to this point, the research we have highlighted, to the extent it involves decision making and decision theory, focuses on cost-benefit calculations or other "economic" motivations. But, people make decisions based on other factors. *Justice* is one such factor. In a series of papers Jasso (1999, 2002, 2004, 2008, 2010) applies her measurement of justice to a variety of social science issues. Her measure has important behavioral properties: 1) *scale-invariance* and 2) *additivity*. These properties are "thought desirable on subtantive grounds in a justice evaluation function (Jasso 2004: 408)."

Third, and not surprisingly, the use of formal and empirical analyis to constuct indexes is also done in other disciplines. The final component of this chapter is an example from economics. We show in the appendix how monetary aggregates are constructed — and like Jasso — they are based on the linkage between theory and data.

## 12.1 Jasso's Tripartite Structure: A Summary

The tools required to implement the tripartite structure are broad. Jasso incorporates a framework centering on various research design activities — ranging from the resesarch questions to the way we charactize the relations — which add to the process of linking formal and empirical analysis.

Jasso's (2004) tripartite structure is represented in Figure 12.1.1. To maintain fidelity to the concepts and definitions she describes, we use Jasso's voice, summarize the key elements, and (where possible) follow the order she describes her tripartite structure:

- Element 1: The Framework
- Element 2: Theoretical Analysis
- Element 3: Empirical Analysis.

### Element 1: The Framework

The middle column of Figure 12.1.1 lists the framework elements. In the case of justice analysis we have the following considerations.

**Social Science Analysis**

| Theoretical Analysis | Framework | Empirical Analysis |
|---|---|---|
| ***Deductive***<br>Postulates<br>Predictions<br>--------------------<br>***Hierarchical***<br>Postulates<br>Propositions | Questions<br>Actors<br>Quantities<br>Functions<br>Distributions<br>Matrices<br>Contexts | Measure/<br>estimate<br>terms/relations<br>--------------------<br>Test deduced predictions<br>--------------------<br>Test propositions |

Figure 12.1.1: Jasso's Tripartite Structure

**Fundamental Questions:** For *justice analysis* fundamental questions can include (page 405):

1. What do individuals and collectivities think is just, and why?
2. How do ideas of justice shape determination of actual situations?
3. What is the magnitude of the perceived injustice associated with given departures from perfect justice?
4. What are the behavioral and social consequences of perceived injustice?

**Fundamental Actors:** Jasso argues there are two fundamental actors in justice analysis: the observer and the rewardee. "The observer forms ideas of the just reward for particular rewardees and judges the justice or injustice of the actual rewards received by rewardees (where the observer may be among the rewardees) (page 404)."

**Fundamental Quantities:** For justice analysis, Jasso presents three fundamental quantities: the actual reward, the just reward, and the justice evaluation.

**Fundamental Functions:** Fundamental functions "become critical building blocks both for theoretical work, where they often appear as assumptions, and for empirical work, where they appear as relations to be estimated (page 408)." Each of the central questions are addressed by a function (or family of functions) that combines some of the fundamental quantities. "For justice analysis, the first central question is addressed by the just reward function, the third central question by the justice evaluation function, and so on (page 406)."

Jasso characterizes an agent's decision on what is just and not just (actual reward and just reward) with a particular functional form. The justice evaluation function's functional form reflects losses being given greater weight than gains: while the justice evaluation increases with the actual reward it does so at a decreasing marginal rate. A functional form characterizing these behavioral traits is the logarithm of the ratio of the actual reward to the just reward:

$$J = \theta ln\left(\frac{A}{C}\right), \tag{12.1.1}$$

where:

- J is the justice evaluation (the assessment by an observer that a rewardee is rewarded justly or unjustly).
- A is the rewardee's actual reward.
- C is the observer's idea of the just reward for the rewardee.
- $\theta$ is the signature constant.

Jasso calls the sign of $\theta$ the *framing coefficient*. The reason is this coefficient "embodies the observer's framing of the reward as a good or as a bad (negative for a bad, positive for a good), and the absolute value of it called the expressiveness coefficient since it transforms the observer's experience of justice into the expression thereof (page 408)."

A critical matter in linking the theory and the nature of how people decide what is just resides in the log-ratio specification. Jasso notes that this particular specification of the justice evaluation function has been shown to be the only specification that satisfies both *scale-invariance and additivity*. These two conditions are desirable on substantive grounds in a justice evaluation function.

**Fundamental Distributions:** For the case of justice many distributions are available. Jasso explains:

> In the case of cardinal goods, the actual reward distribution and the just reward distribution can assume a variety of shapes, usually modeled

> by variates specified on the positive support, such as the lognormal and Pareto. And the justice evaluation distribution, reflecting the operation of both actual reward and just reward in the production of the justice evaluation, can assume a large variety of shapes as well (page 410).

Jasso notes the distribution or distributions used will depend on how the justice evaluation is modeled. Many distributions fit, including "the negative exponential, the positive exponential, the Erlang, the normal, the logistic, the quasi-logistic, the Laplace, and the asymmetrical Laplace (page 410)."

**Fundamental Matrices:** Jasso shows the fundamental actors can be arrayed in matrix form and, therefore, applicable to applied statistical analysis. In the justice analysis example the three fundamental quantities are represented by "three fundamental matrices: the just reward matrix, the actual reward matrix (which in the absence of perception error collapses to a vector), and the justice evaluation matrix (page 409)."

In applying equation (12.1.1) to these matrices, Jasso provides the following notation (page 409). Let the observers be indexed by $i = 1, ..., N$ and rewardees by $r = 1, ...R$. Consequently, $c_{ir,} a_{ir}, j_{ir}$ represent the observer-specific and rewardee-specific just reward, actual reward, and justice evaluation, respectively. With these details we have the following matrices or vector.

The **just reward matrix**:

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & . & . & . & c_{1R} \\ c_{21} & c_{22} & c_{23} & . & . & . & c_{2R} \\ c_{31} & c_{32} & c_{33} & . & . & . & c_{3R} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ c_{N1} & c_{N2} & c_{N3} & . & . & . & c_{NR} \end{bmatrix}$$

The **actual reward matrix**:[2]

[2]Jasso indicates that "if there are no perception errors, the actual reward matrix collapses to a vector: (page 409)"

$$\mathbf{a}_{\cdot r} = \begin{bmatrix} a_{\cdot 1} & a_{\cdot 2} & a_{\cdot 3} & . & . & . & a_{\cdot R} \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & . & . & . & a_{1R} \\ a_{21} & a_{22} & a_{23} & . & . & . & a_{2R} \\ a_{31} & a_{32} & a_{33} & . & . & . & a_{3R} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ a_{N1} & a_{N2} & a_{N3} & . & . & . & a_{NR} \end{bmatrix}$$

The **justice evaluation matrix** is:

$$\mathbf{J} = \begin{bmatrix} j_{11} & j_{12} & j_{13} & . & . & . & j_{1R} \\ j_{21} & j_{22} & j_{23} & . & . & . & j_{2R} \\ j_{31} & j_{32} & j_{33} & . & . & . & j_{3R} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ j_{N1} & j_{N2} & j_{N3} & . & . & . & j_{NR} \end{bmatrix}.$$

**Fundamental Contexts:** The use of context is also a consideration. In justice analysis context-specific variation occurs in a variety of areas. Formalization of context considerations include: "$b$ for the benefit under consideration, $r$ for the type or identity of the rewardee, $o$ for the observer, $t$ for the time period, and $s$ for the society (page 410)."

## Element II: Theoretical Analysis

As shown in the left column of Figure 12.1.1, (the theoretical panel), Jasso distinguishes between two main kinds of theories — deductive theories and hierarchical (nondeductive) theories:

> both deductive and hierarchical theories have a two-part structure, the first part containing an assumption or a set of assumptions — also called postulates... the assumption set should be as short as possible, and the second part [predictions and propositions] should be as large as possible and, indeed, always growing ... (page 411).

**Deductive Theory:** Jasso argues deductive theory possesses "the starting assumption, perhaps in combination with other assumptions, is used as the starting point from which to deduce new implications. These implications (predictions) show the link of the process described by the starting assumption. They are observable, testable implications; as well. They are ceteris paribus implications, given the multifactor world in which we live (page 411)."

Deductive theories are assessed on both theoretical and empirical criteria.

> Theoretical criteria focus on the structure of the theory. A good theory has a minimum of assumptions and a maximum of predictions...Moreover, in a good theory, the predictions constitute a mix of intuitive and nonintuitive predictions, and at least some of them are novel predictions...Empirical criteria for evaluating deductive theories focus, of course, on tests of the predictions. It may happen that early in the life of a theory, the assumption set grows. It may come to be seen that the single starting assumption is not sufficient by itself to yield many predictions but that the introduction of one or two additional assumptions produces unexpected synergies and an explosion of new predictions. Often, work with a particular set of assumptions leads to codification of special methods for deriving predictions. These special methods may focus on special representations of the assumptions or special kinds of tools (page 412).

**Hierarchical Theory:** Hierarchical theory differs from deductive theory. Even though "both kinds of theories begin with an assumption, in a hierarchical theory there is no deduction; instead, propositions are constructed by linking a term from the assumption with an observable term (page 414)." For justice analysis:

> a hierarchical theory in which the justice evaluation function is an assumption might be used to construct propositions linking observables to the justice evaluation or to the proportion overrewarded or to the average underreward among the underrewarded (page 414).

## Element III: Empirical Analysis

Three forms of empirical analysis are identified in Figure 12.1.1.[3] Two involve testing the predictions deduced in deductive theories and the propositions constructed in hierarchical theories."A third kind of empirical work, and sometimes the only empirical activity — especially in the early stages of development of a particular topical subfield — consists of basic measurement and estimation operations. The quantities identified in the framework are measured, the functions and distributions estimated, and the matrices populated (page 422)."

**Testing the Predictions of Deductive Theories v. Testing Propositions Construed in Hierarchical Theories:**

Jasso argues there are similarities and differences between tests for deductive and hierarchical theories. Deductive theories test predictions based on a well defined "path," but in the case of hierachical theories tests focus on the propositions. "Testing the propositions constructed in hierarchical theories is less demanding in part because the proposition is already at least half observable, given that it was crafted by linking a term from a postulate to an observable term (page 423)."

On the other hand, when designing the specification and estimation procedures, both types of theories are equally demanding. Because many causal factors are in play, similar specification challenges exist, and both the nature of the specification and the quality of the data lead to similar estimation challenges.

A final aspect is interpreting the results. "Because in the construction of propositions in hierarchical theories no pathways have been specified, the knowledge gained from empirical tests is less informative in some sense than the results of tests of predictions, though nonetheless important (page 424)."

**Extratheoretical Measurement and Estimation:** In Figure 12.1.1, this type of work is represented in the top row of the empirical analysis column. Jasso characterizes extratheoretical research consisting "mainly of measurement and estimation of quantities and relations in the framework (page 424)." Extratheoretical research activities are quite numerous and informative in justice analysis. They include (but

[3] When it comes to determining the usefulness of a model Jasso asserts:

> There is widespread agreement that rejecting a prediction is not a sufficient condition for rejecting a theory. Moreover, rejecting a prediction is not a necessary condition for rejecting a theory; even if all of a theory's predictions survive test unrejected, one may still reject the theory—in favor of a better theory, one with "excess corroborated content" (Lakatos 1970). Indeed, the view known as sophisticated falsificationism holds that it is not possible to judge the empirical merits of a theory in isolation; falsification requires comparison of the relative merits of two theories (Lakatos 1970:116).

are not limited to):

- Measuring the true and disclosed just rewards.
- Measuring the experienced and expressed justice evaluations.
- Estimating the just reward function and the principles of microjustice.
- Assessing the extent of interindividual disagreement on the principles of justice.
- Ascertaining whether individuals frame particular things as goods or as bads.
- Estimating observer's expressiveness, comparing the just inequality with the actual inequality.
- Assessing just gender gaps and their underlying mechanisms.
- Measuring trends in overall injustice.
- Estimating the poverty and the inequality components of overall injustice.[4]

## 12.2 An Illustration of Extratheoretical Research: The Justice Index and Gender Gaps

In this illustration we apply aspects of Jasso's *tripartite stucture* to extratheoretical research, and creating a justice measurement. Jasso examines justice indexes and gender gaps in the U.S. The data are from the 1991 International Social Justice Project (ISJP) (2004: 424-427).[5] Respondents state their actual earnings and the earnings they think just for themselves (Jasso 1999).[6]

Some *fundamental questions* pertain to the experience of injustice.

1. How pervasive is the experience of unjust underreward?
2. Does the experience of injustice vary systematically by gender?

[4] This list can be found in Jasso (2004: 424).

[5] "The ISJP was the first major international effort to document the views of ordinary citizens regarding social, economic, and political justice. The project involved five Western democracies and eight formerly socialist countries (page 424)."

[6] The complete dataset of International Social Justice Project, 1991 and 1996, can be obtained from Inter-university Consortium for Political and Social Research (ICPSR) at the University of Michigan. The data of the U.S. sample are available from the following website: http://www.uh.edu/class/hcpp/EITM/EITMBook.html. Interested readers can also download the STATA do-file to compute the results in Table 12.1 (Jasso 2004: 426).

3. Is the experience of injustice driven by poverty or by inequality?

To measure the individual's justice evaluation, the justice evaluation function introduced previously is used (Equation (12.2.1)). It calculates the personal justice evaluation from the information provided on actual and just earnings. Jasso call this experienced justice evaluation. This type of evaluation omits the signature constant ($\theta$):

$$J = \ln\left(\frac{A}{C}\right), \tag{12.2.1}$$

This *fundamental function* includes the following implications:

1. In the state of perfect justice, the rewardee's actual reward ($A$) equals the perceived just reward ($C$) : such that the justice index $J = \ln(A/C) = \ln(1) = 0$.

2. The justice index is positive (*i.e.*, $J > 0$) when the actual reward ($A$) is greater than the previous just reward ($C$).

3. Implication 2 suggests that the rewardee perceives herself that she is over-benefit or under-burden in terms of her actual earnings. However, a rewardee would consider herself as under-benefit or over-burden when her actual reward ($A$) is less than her perceived just reward. It indicates that $J < 0$.

### 12.2.1 Justice Indexes

Two justice indexes are suggested by Jasso, but one is a special case of the other. The "main" justice index — **JI1** — is defined as the arithmetic mean of the experienced justice evaluation: $JI1 = E(J) = E(\ln(A/C))$. "It can assume positive, negative, and zero values. A positive value has the interpretation that the center of gravity of the distribution of justice evaluations lies in the over-reward region, and a negative value indicates that the center of gravity lies in the under-reward region (page 245)."

#### 12.2.1.1 Decomposition Methods

Along with these distributional characteristics, Jasso (1999) also presents two decompostion methods:

- *Mean-inequality decomposition*
- *Reality-ideology decomposition.*

**Mean-Inequality Decomposition**

For the method of mean-inequality decomposition, the argument is that the justice index ($JI1$) can be calculated as the sum of justice evaluation about the mean ($JI1_{Mean}$) and the justice evaluation about the inequality ($JI1_{Ineq}$).

Note that the formula of justice index can be rewritten as the log of the ratio of geometric mean of actual reward to geometric mean of just reward:

$$JI1 = E\left(\ln\left(A/C\right)\right) = \ln\left(G\left(A\right)/G\left(C\right)\right), \tag{12.2.2}$$

where $E\left(\cdot\right)$ represents the operator of computing arithmetic means (or expected values), that is, $E\left(X\right) = \left(\sum_{n=1}^{N} x_n\right)/N$. $G\left(\cdot\right)$ is the geometric mean operator where $G\left(X\right) = \left(\Pi_{n=1}^{N} x_n\right)^{1/N}$.

To calculate the component of inequality in the justice index, Jasso considers Atkinson's (1975) measure $I$, which is defined as:

$$I\left(X\right) = 1 - \left[G\left(X\right)/E\left(X\right)\right]. \tag{12.2.3}$$

According to equation (12.2.3), if the geometric mean equals the arithmetic mean in a dataset, that is, $G\left(X\right) = E\left(X\right)$, then Atkinson's measure $I\left(X\right)$ equals zero. This result indicates there is *no* inequality observed in the data.

However, the value of Atkinson's measure increases as the distribution of the observations becomes more unequal. We obtain the geometric mean from equation (12.2.3):

$$G\left(X\right) = E\left(X\right)\left[1 - I\left(X\right)\right], \tag{12.2.4}$$

where the geometric mean can be written as a function of arithmetic mean and Atkinson's measure.

Finally, expressing the geometric means for actual reward $G\left(A\right)$ and just reward $G\left(C\right)$ — according to equation (12.2.4) — we can rewrite the justice index $JI1$ as follows:

$$JI1 = \ln\left(\frac{E\left(A\right)\left[1 - I\left(A\right)\right]}{E\left(C\right)\left[1 - I\left(C\right)\right]}\right). \tag{12.2.5}$$

As a result, equation (12.2.5) can be expressed as the sum of two components:

$$\begin{aligned} JI1 &= \ln\left(\frac{E\left(A\right)}{E\left(C\right)}\right) + \ln\left(\frac{1 - I\left(A\right)}{1 - I\left(C\right)}\right) \\ &= JI1_{Mean} + JI1_{Ineq,} \end{aligned} \tag{12.2.6}$$

where $JI1_{Mean} \equiv \ln \left( E \left( A \right) / E \left( C \right) \right)$, and $JI1_{Ineq} \equiv \ln \left( \left( 1 - I \left( A \right) \right) / \left( 1 - I \left( C \right) \right) \right)$.

The justice index, about the mean ($JI1_{Mean}$), represents an (arithmetic) average value of the actual rewards relative to that of the perceived just rewards in the sample. On the other hand, the justice index about the inequality ($JI1_{Ineq}$) indicates the difference between the inequality of the actual rewards and the inequality of the just rewards in the sample.[7]

### Reality-Ideology Decomposition

While inequality measures portray factual inequality, they do not include knowledge of individual-specific ideas of justice. Jasso, therefore, suggests the justice index $JI1$ can be interpreted in an alternate way. She distinguishes between injustice due to reality ($JI1_{Reality}$)and injustice due to ideology ($JI1_{Ideology}$). Injustice due to reality is based on how a person observes their actual income in relation to the mean and the inequality of the income distribution. Injustice due to ideology is based on what a person perceives the just income in terms of the mean and the inequality distribution.

Hence, the justice index can be rewritten as follows:

$$\begin{aligned} JI1 &= \ln \left( E \left( A \right) \left( 1 - I \left( A \right) \right) \right) - \ln \left( E \left( A \right) \left( 1 - I \left( A \right) \right) \right) \\ &= JI1_{Reality} - JI1_{Ideology}, \end{aligned} \tag{12.2.7}$$

where $JI1_{Reality} \equiv \ln \left( E \left( A \right) \left( 1 - I \left( A \right) \right) \right)$ and $JI1_{Ideology} \equiv \ln \left( E \left( A \right) \left( 1 - I \left( A \right) \right) \right)$. Equation (12.2.7) shows that $JI1$ is equal to the reality component minus the ideology component (as also shown in the formula in Table 12.1).

### An Alternative Index

The second index — **JI1*** — is a special case of $JI1$ in which the just rewards (i.e., $C$) equal the mean rewards (i.e., $E \left( A \right)$). Intuitively, this measure arises when justice is equality. Hence, we can express $JI1^*$ as follows:

$$JI1^* = JI1|_{C=E(A)} = E \left[ \ln \left( \frac{A}{E \left( A \right)} \right) \right].$$

This measure Jasso (2004: 425) argues is consistent with other sociological interpretations in the literature (Blau 1960, 1964; Blalock 1967). It can be applied to small

[7] Jasso (1999) argues that the justice index about the mean ($JI1_{Mean}$) is the perceived injustice driven by scarcity or poverty, while the second component of justice is due to the inequality in the distribution of income.

homogeneous groups or utopian communities.

### 12.2.2 Results

The results, presented as five components, are in Table 12.1. The component are:

Base Data. this includes average actual earnings, average just earnings, and sample size, both for the U.S. sample as a whole and for gender-specific subsamples.

Justice Index JI1 Its Decompositions. The results indicate there is 4 percent more injustice for women (-.236) than men (–.207). Recall from equation (12.2.6) JI1 is the sum of the mean component and the inequality component: this "first decomposition of JI1 makes it possible to distinguish between two components of overall injustice, injustice due to the mean, and injustice due to inequality (page 425)."

Jasso notes these results have rival interpretations. "Depending on the context, the mean component may be interpreted as a scarcity component or a poverty component. The mean component is larger than the inequality component for both men and women; however, the relative magnitudes differ considerably (page 425)." 75 percent $\left(\frac{-0.155}{-0.207}\right)$of overall injustice for men is due to scarcity, but for women it is a higher proportion, 94 percent $\left(\frac{-0.223}{-0.236}\right)$. Note, too, nearly all of injustice for women is due to scarcity.

The reality-ideology decomposition, as defined in equation (12.2.7) show there are differences between reality and ideology. "Among both women and men, the ideology component exceeds the reality component, producing the negative $JI1$. As already known from the magnitudes of $JI1$, the discrepancy is larger among women than among men (page 425)."

Justice Index JI1 and Its Gender Decomposition. Jasso notes that "if equality was used as the standard for just earnings, experienced injustice would be greater, substantially so among men (–.207 versus –.340 among men and –.236 versus –.271 among women) (page 426)." Within-group (gender) and between-group (gender) decompositions add further information. Recall that JI1* is equal to the sum of the two components. The within-group (gender) component is "the weighted sum of the group-specific values of JI1*, where the weights represent the fraction of the population in each group (page 426)." The calculation for the between-group (gender) component is "the weighted sum of the log of the ratio of the group mean to the overall mean." The results in Table 12.1 show the within-gender component is much larger than the between-gender component, constituting 87 percent of the overall JI1*: there is more variability within genders than between genders.

| | Men | Women | All |
|---|---|---|---|
| **Base Data** | | | |
| Average actual earnings ($) | 36,950 | 20,084 | 28,847 |
| Average just earnings ($) | 43,137 | 25,106 | 34,474 |
| Number of obs | 438 | 405 | 843 |
| **Justice Index JI1 and Its Decompositions**[a] | | | |
| JI1 | -0.207 | -0.236 | -0.221 |
| Decomposition into Mean and Inequality Components | | | |
| $JI1 = JI1_{Mean} + JI1_{Ineq}$ | | | |
| *Mean component* $JI1_{Mean}$ | -0.155 | -0.223 | -0.178 |
| *Inequality component* $JI1_{Ineq}$ | -0.052 | -0.013 | -0.043 |
| Decomposition into Reality and Ideology Components | | | |
| $JI1 = JI1_{Reality} - JI1_{Ideology}$ | | | |
| *Reality component* | 10.178 | 9.637 | 9.918 |
| *Ideology component* | 10.385 | 9.873 | 10.139 |
| **Justice Index JI1* and Its Gender Decomposition**[b] | | | |
| JI1* | -0.34 | -0.271 | -0.352 |
| Decomposition into Within-Gender and Between-Gender Components | | | |
| *Within-gender component* | | | -0.307 |
| *Between-gender component* | | | -0.0453 |
| **Gender Gaps in Actual and Just Earnings** | | | |
| Actual gender gap | | | 0.544 |
| Just gender gap | | | 0.582 |
| **Special Relation Between Mean Component of JI1 and Ratio of Gender Gaps** | | | |
| $JI1_{F,Mean} - JI1_{M,Mean} = \ln(\text{Actual Gender Gap/Just Gender Gap})$ | | | -0.0684 |

*Notes:*

[a] Justice index *JI1* is defined as $JI1 = E(J) = E[\ln(A/C)]$, where $A$ = the rewardee's actual reward, and $C$ = the observer's idea of the just reward for the rewardee (Jasso, 2004).

[b] Justice index *JI1** is defined as $JI1^* = E(J^*) = E\{\ln[A/E(A)]\}$, where $A$ = the rewardee's actual reward (Jasso, 2004).

Table 12.1: Justice Indexes and Gender Gaps: U.S. Sample, ISJP 1991 (Jasso 2004: 426)

Gender Gaps in Actual and Just Earnings. On page 426 Jasso finds the "gender gaps, defined as the ratio of the women's average to the men's average, includes both actual earnings and just earnings. As shown, and as evident from the base data the gender gap is greater for actual earnings than for just earnings:" (.544 v. .582).

Special Relation Between Mean Component of JI1and Ratio of Gender Gaps. An important finding was "the usual way of measuring gender gaps is completely inattentive to within-gender inequality" (page 427). The reason for this conclusion is found in Table 12.1. This gender gap ratio "provides a numerical approximation to an exact relation between aspects of the justice index and aspects of the gender gap. The signed difference between the women's mean component of JI1 and the men's mean component of JI1 is equal to the log of the ratio of the actual gender gap to the just gender gap" (page 427). The just gender gap exceeds the actual gender gap and it is the just gender gap that captures the higher variation in within-gender inequality.

To sum up, some highlights from the justice index analysis include the following:

1. In a 1991 United States sample women experienced more injustice, on average, than men.

2. For both genders, scarity (poverty) is a more important factor in perceiving injustice than inequality. Women, however, reveal scarcity is an even greater factor than men.

3. Average "just" earnings exceed average actual earnings. Women have a greater gap than men.

### 12.2.3 Leveraging the Justice Index

As with EITM, there are several ways to build on the justice index. To begin, this index should be used in a comparative setting over time. The sample used here is only for the United States in 1991, but expanding the sample to many countries and many years is a logical next step. A second extension would to sample the same individuals over time to see if their views on justice evolve and what the context was for the changes in their justice evaluation. A third consideration is to continue with other subgroup breakdowns. Gender is important but factors such as age and education, amongst other factors, can also provide new insights on how people judge something to be just. A fourth issue would be to determine the relation between the justice index and indicators of societal and political stability and the overall legitimacy of a political, social, and economic system. A final consideration is a determination of whether a justice index based on "equality" can be extended to include a justice index based on an individual's sense of freedom and liberty.

## 12.3 Appendix

The use of theory to create valid indices can be found in other social science disciplines as well. Here we demonstrate how theory plays a role in the construction of measures of the money suppy — monetary aggregates. We relate the process and the tools involved and the challenges that are faced.

The tools introduced in this process are: 1) Aggregation Theory and 2) Index Number Theory.

### 12.3.1 Linking Instruments to Outcomes: The Case of Monetary Aggregates

Monetary aggregate measures are as straightforward as adding the components together, so-called "simple-sum" aggregates. But components can also be aggregated applying microeconomic reasoning to aggregation theory and index number theory. These "Divisia aggregates" possess important policy implications. They provide a more accurate reading on changes and the expected consequences of monetary policy than the rival simple-sum aggregates.

An example where inaccurate measurement affected the conduct of monetary policy occurred during the recession in the early 1980s. Policy, it could be argued, prolonged the recession. In reviewing the behavior of these rival monetary aggregate measures, William Barnett (1984) concluded:

> Monetary policy during the sample period induced slower and more volatile monetary growth than was indicated by the official simple sum aggregates...Because monetary policy, as indicated by Divisia monetary aggregates, was tighter than indicated by the official aggregates, our results provide an illustration of how inattention to well-established statistical theory can lead to policymaking that may be less effective than it might be (page 171).

A decoupling between instruments and the money stock has, at times, had tragic consequences, including the Great Depression.[8] One reason for the discrepancy is the public reduces its demand deposits but holds more currency. This contracts the money stock and the supply of loanable funds. At the same time, however, the shift from demand deposits to currency increases the level of the monetary base (provided reserves do not decline by a commensurate proportion, the currency increases).

#### 12.3.1.1 Measurement and Spurious Outcomes: Aggregation Error

While it is well known to students of monetary policy that the official monetary aggregates are based on simple sums of the component quantities. However, a valid simple sum aggregate requires the components be perfect substitutes. We cannot add apples to oranges to get an aggregate of oranges; it simply makes no sense to say a dollar in currency provides the same monetary services as a dollar in Series E bonds. They are not perfect substitutes.[9] The impact of aggregation error grows as the number of components increases.[10]

In sum, the effect aggregation error fostered by imperfect substitution cannot be overemphasized. Simply combining components in a monetary aggregate can contain so much aggregation error that it gives the appearance of endogeneity (responding to business cycle conditions or interest rates). A more correct aggregation index accounting for internal substitution effects reduces the potential for a spurious diagnosis on endogeneity.[11]

[8] It is possible for monetary instruments, such as the monetary base (high powered money) to exhibit behavior that is inversely related (or not related at all) to the money stock (MlB for example). Indeed, this occurred during the Great Depression (1929-1933). Friedman and Schwartz (1963: 332-333) document that deposit reserve and deposit-currency ratios fell during this period while the monetary base exhibited an increase. Up until that period, however, Friedman and Schwartz assert the relation between the money stock and the monetary base was strong.

[9] Taken to its logical extreme, a government that used simple sum monetary aggregates could monetize its entire debt holdings and have no change in the money stock as government securities and currency would be considered substitutes for each other.

[10] The problem of imperfect substitution is less severe for the narrower monetary aggregate because the respective components tend to be more closely related. In comparative analysis, one would expect that since both simple sum and Divisia aggregates are constructed differently they should exhibit different behavior. This suspicion is confirmed (See Barnett 1982; Barnett, Offenbacher, and Spindt 1984). There are exceptions, however. Barnett, Offenbacher, and Spindt (1984) have found that at lower levels of aggregation, simple sum aggregates can, under certain tests, outperform Divisia aggregates.

[11] By way of example consider an increase in interest rates (which do correspond to the business cycle). This would have an impact on user costs between various monetary assets (components of a monetary aggregate) that are either rate regulated or not.

### 12.3.2 Relating Theory to Measurement

In an effort to address the challenges above, William Barnett derived the theoretical link between monetary theory and economic aggregation theory (Barnett 1980). Barnett's aggregates, known as Divisia aggregates, have microeconomic foundations.[12] In effect, he replaced the ad-hoc component summation and weighting schemes with a monetary aggregation method that relates component quantities to their *user costs* (e.g., the rental price of holding an asset at some point in time).

The result is Divisia monetary aggregates change under certain circumstances, but the dynamics are linked to agent (public) behavior. For example, the aggregate responds when the change in the interest rate has a microeconomic effect (in this case an income effect); otherwise, all changes in interest rates will lead to pure substitution effects along the indifference curve. The implication then is the Divisia aggregate is not dependent on interest rate or business cycle fluctuations.[13] The effects are accounted for by internal substitution.[14]

#### 12.3.2.1 Defining User Costs

Recall the fundamental measurement problem resides in the simple-sum data being constructed in such a way making it vulnerable to external shocks, which undoubtedly leads to spurious inferences. To account for these distinct component values, Barnett

[12]Divisia aggregates "dominate" simple sum aggregates in comparative statistical tests (Barnett 1982). The results, to date, show that Divisia aggregates not only give different qualitative results, but they also have superior properties for statistical inference.

[13]In more technical language, the weights of components should show increasing dispersion, which would also have a commensurate affect on the Divisia quantity variance. However, the dependence of the monetary aggregate on interest rate fluctuations (or business cycle fluctuations) will be greater or less depending on the difference between the dispersion in weights (shares) of components and the user cost price variance. If increases in dispersion of the weights (shares) of components are matched by an increase (of roughly equal proportion) in the user cost price variance, then the Divisia aggregate quantity disturbance should be relatively undisturbed. These products are expenditures on the respective components. The expenditures on each component are then divided by the total expenditures on all components of the aggregate to determine the share of a given component. These shares are then averaged between the current and preceeding month. Each individual share is used in "weighting" the growth rate for the appropriate individual component, which are then summed up to determine the growth rate of the aggregate. *On the other hand, simple sum aggregates, because the components are assumed to be perfect substitutes, fail to internally substitute relative user cost changes (decisions made by agents). This weakness creates spurious findings.*

[14]In an empirical test on Divisia second moments, Barnett, Offenbacher, and Spindt (1984) found exactly this result. Divisia aggregates are not endogenous to business cycle fluctuations or interest rate fluctuations.

(1980) derived a user cost formula for monetary assets (components):

$$\Pi_{i,t} = \frac{P_t^*(R_t - r_{i,t})(1-\tau_t)}{1 + R_t(1-\tau_t)},$$

where:

$\Pi_{i,t}$= the user cost for monetary asset $i$ during period $t$.

$R_t$= expected yield on bonds during period $t$.

$r_{i,t}$ = expected nominal holding period yield on monetary asset $i$ during period $t$.

$P_t^*$ = cost of living index for period $t$.

$\tau_t$ = marginal tax rate on earnings for period $t$.

This user cost formula represents the rental price for holding a monetary asset for period $t$. User costs are crucial because as prices they are readily applicable for statistical index number theory. Statistical index number theory is important since it can be used to account for component dispersion.

Statistical index numbers are functions of component prices and quantities for a respective aggregate. Let $(q_t)$ represent a vector of quantities consumed of the component goods and $(p_t)$ represent a vector of component prices. To derive correct price and quantity aggregates we simply make the aggregate a function, $(f(p_t, q_t, p_{t-1}, q_{t-1}))$, based on values at period $t$ and $t-1$. This gives quotients for rates of change in prices and quantities during period $t$.

Since we now have respective component prices and quantities, it is appropriate to determine the approximate growth rate for the monetary aggregate. Let $m_{i,t}$ be the of monetary asset $i$ during period $t$, and $s_{i,t}$ be the user cost "weighted average" (rental price) of the component $i$ during period t.[15] Finally, let $Q_t^D$ be the monetary aggregate itself.

With these identities we now derive the Divisia index $p$ for monetary aggregate $Q_t^D$:

$$log\, Q_t^D - log\, Q_{t-1}^D = \sum_{i=1}^{N} s_{i,t}^*(log\, m_{i,t} - log\, m_{i,t-1})$$

where:

$$s_{i,t}^* = \frac{1}{2}(s_{i,t} + s_{i,t-1}).$$

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$$s_{i,t} = \frac{\Pi_{i,t} m_{i,t}}{\sum_{k=1}^{N} \Pi_{k,t} m_{k,t}}.$$

This Divisia aggregate is now the user cost weighted average of component quantities. *Its form approximates Diewert's (1976) superlative index number class*, which are extremely accurate, up to a third-order error term. Divisia aggregates, by virtue of their construction, yield findings qualitatively different, and statistically superior to, simple sum aggregates.

### 12.3.3 Aggregation Theory

Aggregation theory is a branch of economic theory that creates macroeconomic data from microeconomic data. Aggregation theory's virtue is that it ensures that aggregated (macroeconomic) data will behave exactly *as if* it were elementary, disaggregated data. From aggregation theory we can derive a well specified utility function. And from this utility function we can specify marginal utilities from each component which is vital in the construction of an economic aggregate. We can also aggregate up, given certain assumptions about utility functions. In short, aggregation theory derives aggregates from an optimization procedure that maximizes a utility function subject to a budget constraint.

To get a monetary aggregate to behave as an elementary good we derive a utility function. For ease of discussion we must introduce the following notation and conventions. Let the consumption space be represented by the nonnegative Euclidean-orthant:

$$\Lambda^n = \{(x_1, ..., x_n) = X \quad R^n;\ X\ > 0^n\},$$

where $R^n$ is Euclidean space and $0^n$ is an n-dimensional null vector. The constraint $X > 0^n$ implies at least one of the components in $X : x_i > 0$. Let $S = l, 2, ..., n$ be a set of integers and $S_1, ..., S_q$ is a partition of the set $S$ into $q$ subsets. Therefore:

$$S_1 \cup S_2 \cup ...S_q = S,$$

$$S_i \cap S_j = \varnothing \;\; for\ i \neq j \quad i, j = 1, ..., q \;\; and$$

$$S_i \neq \varnothing \;\; for\ i = 1, ..., q.$$

Now partition the consumption space $(\Lambda^n)$ as a Cartesian product with a subspace corresponding to a given partition:

$$\Lambda^n = \Lambda^n \times \Lambda^{n_2} \times ... \times \Lambda^{n_q}$$

Thus, a component vector, $X \in \Lambda^n$, and a strictly positive price vector $(P)$ can be broken down into:

$$X = \left(x_{n_1},\, x_{n_2}, ..., x_{n_q}\right),\ and$$

$$P = \left(p_{n_1}, p_{n_2}, ..., p_{n_q}\right).$$

The preference relations of the price and component vectors are represented by utility functions ($U$: $\Lambda^n \to R$). Utility functions are quadratic and as such we further assume that $U$ is continuosly twice differentiable and quasi-concave with a strictly positive marginal utility. For a utility function to exist, a change in the consumption of one component must not induce a change in consumption in another component. That is, the marginal rates of substitution (cross elasticities) of each component are independent of each other.

With these conventions in place we can now proceed to a two-stage budgetary decision. The purpose here is to establish a real-world disaggregated utility function and aggregate up to a shadow world, which behaves *as if* it were a consumer maximizing utlity subject to a budget constraint.

A two stage budgeting decision takes the following form:

$$Maximize\, U\left(x\right)$$

$$subject\, to:\ px = m$$

where x $\in \Lambda$, $p$ is is a strictly positive price vector, and $m$ is total income. With this simple optimization function we represent the consumer budget in two stages.

- Stage One:
$$Maximize\, U = F\left(U^1, U^2, ..., U^q\right)$$
$$subject\, to:\ \sum_i^q p^i x^i = m$$
- Stage Two:
$$Maximize\, U^i\left(x_{n_i}\right)$$
$$subject\, to:\ p_{n_i} X_{n_i} = p^i u^i\left(X_{n_i}\right) \quad for\, i = 1, 2, ..., q.$$

In the first stage the consumer allocates the total income ($m$) to the component groups (blocks) $\{\mathrm{U}^i(X_{n_i})\}$; this establishes a budget constraint. The second stage involves the specific expenditure on each component that make up a given block. If the results in the utility function are the same as the in the two-stage budgeting decision, then the solution is consistent or exact and we have a well-defined utility function.

Consistency is a necessary and sufficient condition for establishing an equivalent relation from the real (disaggregated) world to the shadow (aggregated) world. Green (1964) set out the conditions by which consistency (or exactness) could hold:

**Green's Theorem 4:** A two stage budgeting decision is exact and $\exists \mathrm{U}^i\left(X_{n_i}\right)$ and $p^i$ iff $U(x)$ is weakly separable in the partition $S_1, ..., S_q$ and the functions $U^i$ are linearly homogeneous w.r.t. $X^n$.

Formally, this "blockwise weak separability" condition is defined as:

**Definition 1:** $U(x)$ is weakly separable w.r.t. to the partition $S_1, ..., S_q$ if $\frac{\partial[U_i(x)/U_j(x)]}{\partial X_k} = 0 \;\; \forall i, j \in S_q$ and $K \notin S_q$.

The existence of weak separability is derived from Gorman (1953):

**Gorman's Theorem:** $U(x)$ is weakly, separable w.r.t. the partition $S_1, ..., S_q$ iff $\exists$F:$\rightarrow R^n$and $\mathrm{U}^i : \Lambda^{n_i} \rightarrow R$ such that $\mathrm{U}(x) = F\left(U^1\left(x_{n_i}\right), U^2\left(x_{n_2}\right), ..., U^q\left(x_{n_q}\right)\right)$.

From Green's Theorem 4 the weak separability condition allows for the establishment of a well defined utility function. In addition to the weak separability condition, Green's theorem imposes one more condition — linear homogeneity.[16] Linear homogeneity (homotheticity) is a necessary and sufficient condition for the existence of aggregate prices. It ensures that exact expenditure values feed into the second stage budgeting decision from the first stage.

However, linear homogeneity is a severe assumption, which cannot be justified empirically. Linear homogeneity violates Engel's law because it produces income elasticities for every good that must be unity. The implication then is expenditure shares of all goods are independent of income and only dependent on relative prices: poor people spend the same proportion of income on food as rich people. This is clearly implausible.

There are many ways to get around this problem. The key is to devise a method that will allow for aggregation across consumers, but will not also imply unitary price

[16] Linear homogeneity means when all components are increased k-fold, the aggregate itself also increases k-fold.

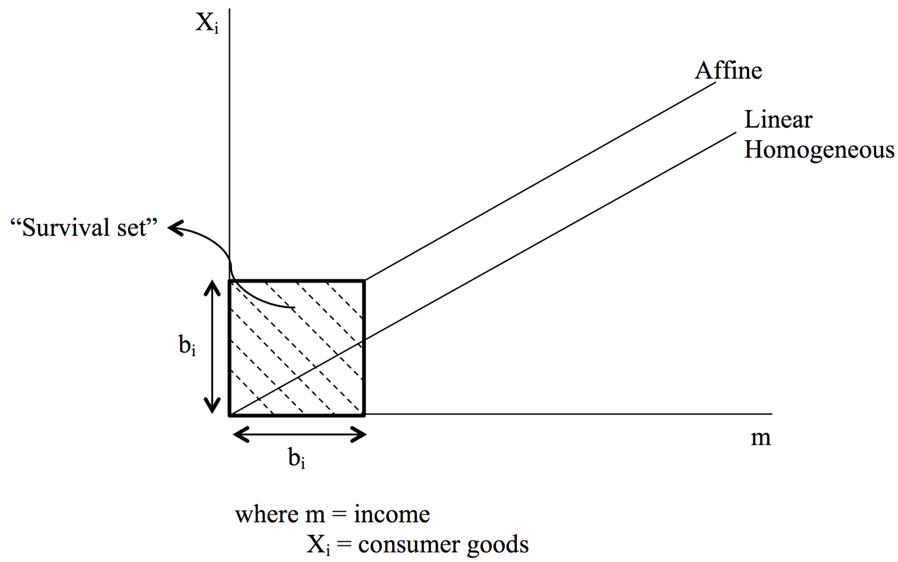


Figure 12.3.1: Affine Transformation

and income elasticities. Affine transformations present such an accomodation.[17] An Affine transformation has the simple form:

$$T(p) = A(p) + B,$$

where $A$ is a linear function and $B$ is a fixed vector. The difference between a linearly homogeneous transformation and the Affine transformation is the fixed vector B. In Figure 12.3.1 the difference is demonstrated by the fact the homothetic transformation passes through the origin. The Affine transformation, in contrast, begins at some fixed point in the northeast quadrant ($b_i$). This difference has important qualitative implications. A vector passing through the origin means consumers have zero consumption.

On the other hand, a vector beginning at some fixed point indicates consumers have a survival set, which they cannot consume less than. Yet, the Affine transformation still allows for linear homogeneity beyond $b_i$ so that we can aggregate across consumers. Of no less importance is the fixed vector $bi$. It represents a region where price and income elasticities are indeterminate; therefore, price and income elasticities are not unitary.

The Affine transformation introduces a new stage in the budgeting process. The constraint, $px = m$, must now be transformed to $\hat{m}$, where $\hat{m}= m - p'b$. The variable

[17] If the quantity aggregator function is not linearly homogeneous, (i.e., nonlinear Engel curves) then we cannot use two-stage budgeting for the purposes of finding quantity aggregator functions. Distance functions are used in this case (See Barnett 1987: 145-149).

$m$ represents supernumerary income, which the consumer uses as a budget constraint.

With the two (three) stage budget procedure and Green's Theorem 4, corrected for homotheticity, an aggregator function (or economic aggregate) is said to be "exact." Consumers treat the aggregate *as if* it were made up of elementary goods. At this stage, however, we only know how to block components comprising an aggregate function. To estimate an aggregate (or aggregator) function we must use index number theory.

### 12.3.4 Index Number Theory

Recall Barnett's Divisia aggregates are considered superlative index numbers. The specific properties they hold for accuracy require a brief discussion of index number theory. Index number theory is used to approximate aggregator functions. Index numbers come in two different forms: functional and statistical. Functional index numbers estimate aggregator functions using empirical estimates for unknown parameters. On the other hand, statistical index numbers are parameter free; they depend on component prices and quantities.[18]

For many years there was controversy as to which index numbers provided a better approximation to an economic aggregate (aggregator function). The two index numbers are necessarily competing with different strengths and weaknesses. Because functional forms are dependent on parameter specification, they are necessarily ad-hoc and data dependent approximations.

Conversely, statistical index numbers are dependent on prices and quantities to reveal a current point on an aggregator function. Yet statistical index numbers, because they are contingent on prices and quantities, they cannot provide a valid second-order approximation to track an aggregate function. They are useful in the local sense only.

However, in 1976 Diewert proved that statistical index numbers were linked to functional index numbers. To prove this Diewert utilized a modified Taylor expansion of the following form:

$$\nabla f\left(m^1\right) - \nabla f\left(m^0\right) = \frac{1}{2}\left[\nabla f\left(m^1\right) + \nabla f\left(m^0\right)\right]^T \left(M^1 - M^0\right)$$

where $\nabla f(m^n)$ is a gradient vector of function $f$ estimated at $m^n$.

[18]Functional index numbers = $F(x,\theta)$ and statistical index numbers = $F(x,p)$, where: $x$ = quantities of commodities, $\theta$ = unknown parameter(s), and $\underline{p}$ = prices. By their construction, functional index numbers require data for one period only; whereas, statistical index numbers require data for more than one period.

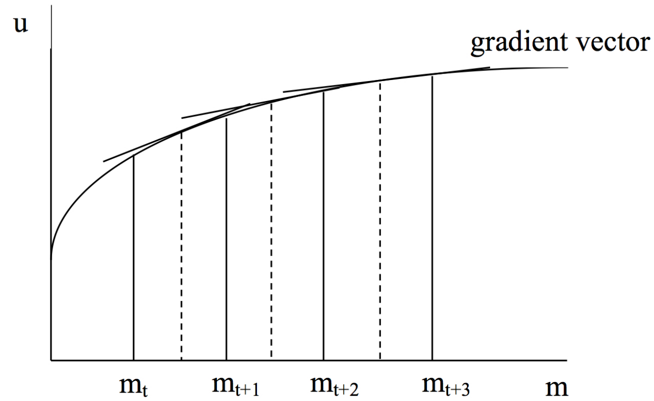


Figure 12.3.2: Gradient

Diewert's contribution was to seek many local approximations in small neighborhoods. From a graphical standpoint (Figure 12.3.2), we see how Diewert's index numbers could then track an aggregator function. Intervals were set to cover some average distance between two points, $\frac{1}{2}(m_t + m_{t-1})$, where this approximation was across a sufficiently small neighborhood, $(m_t - m_{t-1})$, yields a third-order remainder. In short, it is extremely accurate.[19]

In sum, index number theory and, in particular, superlative index numbers, are means of estimating (given prices and quantities) specific points of an aggregator function (economic aggregate). We know these estimates are exact in the Diewert sense. A flexible aggregator function can obtain first and second order derivatives. Marginal utilities can be considered as quasi-weights, which implies that these weights are theoretically and empirically driven and not ad-hoc. They must be blockwise weakly separable and Affine linearly homogeneous.

[19]Diewert's method has a residual, $(m_t - m_{t-1})$, smaller than rounding error, which speaks favorably about its accuracy. Quantity indices of the Diewert variety that are exact over a second-order approximation to a homothetic function are called superlative. Superlative index numbers, therefore, can always attain the current value of a flexible aggregator function.

# Chapter 13

# Conclusion

We believe significant scientific progress can be made by unifying formal and empirical modeling. This advancement will require ending existing barriers between formal modelers and empirical modelers. As the 2002 NSF EITM Report concluded, practices must change:

> formal modelers must subject their theories to closely related tests while, at the same time, empirical modelers must formalize their models prior to conducting any of the available statistical tests. The point is not to sacrifice logically coherent and mathematical models. Breakthroughs in theory can be enhanced with the assistance of empirical models in experimental and non-experimental settings (page 13).

This methodological unification will also lead to the use of an ever increasing set of behavioral concepts. Application of the EITM framework means new and better ways will be discovered to model human behavior. And the repeated application of competing analogues raises the possibility of conceptual proliferation in thinking how humans act, but now with a sense there is a rigor in putting these new behavioral developments to the test. The "new" developments in bounded rationality, learning, and evolutionary modeling are indeed important in EITM, but they are by no means the only ones.

## 13.1 Challenges

Recall the motivation for the 2001 EITM Workshop was the intellectual divide between formal modeling and empirical (e.g., applied statistical) modeling and the deleterious ramifications this divide presents to the social and behavioral sciences.

A consequence of this divide is research (and instruction) competent in one technical area, but lacking in another. This impaired competency contributes to a failure to identify the proximate causes explicated in a theory and, in turn, increases the difficulty of achieving a significant increase in scientific knowledge.

As we noted — at its most elementary level — EITM is a framework for unifying formal and empirical analysis. Even though methodological unification of this kind is not new in social-behavioral science, barriers exist concerning the purpose and application of EITM (e.g., "that is what we are doing anyway"). Short term barriers for adopting EITM include basic misunderstanding of what EITM means. Longer term barriers include rigid training traditions within and across social and behavioral science disciplines.

The sources of resistance are not surprising. Resistance to unifying formal and empirical modeling is due to several factors.[1] Among those factors are:

1. **The Intellectual Investment**: Scholars have to invest in different skill sets. The intellectual investment needed for formal modeling is different than the knowledge needed for empirical modeling. But, given the greater mathematical demands in formal modeling the tendency is for students and scholars not to have sufficient training in formal modeling. This deficit is compounded since there are few incentives for motivating tenured faculty to try new methods, including using formal modeling as part of their tool kit.

2. **Training Differences**: Empirical modelers devote their energies to data collection, measurement, and statistical matters, and formal modelers focus on mathematical rigor. This divide is reinforced in departments having a strong tradition in either formal or empirical analysis.

3. **Research Practice**: For empirical modelers, model failures lead to emphasis on additional statistical training or more sophisticated uses of statistics – usually to "patch over" – a model failure. Formal modelers, on the other hand, deal with model controversies by considering alternative mathematical formulations but this is usually done piecemeal. However, the one similarity between these two approaches is that both formal and empirical modelers tend to remain tied to their particular technique despite the warning signals evidenced in model breakdown. These practices are reinforced by reviewers or journal editors due to their specializaton of either formal or empirical analysis.

[1] These challenges take several forms which are sourced and many of these issues are discussed in Poteete, Janssen, and Ostrom (2010: 3-27).

These implementation challenges are deeply rooted in the academic community — fostered by career incentives — that will take years to overcome (Poteete, Ostrom, and Janssen 2010: 18-24). Consequently, "Old habits" learned in graduate school inhibit the desire to make the changes in skill development. But, the situation is worse since many things learned in graduate school tend to become out-of-date by mid-career.

If methodological instruction is subjected to these status quo forces, successive generations will only repeat the shortcomings. We now see, *and have repeatedly seen,* applied statistical practices that misuse the t-statistic and, as we have discussed earlier, are unsuitable for addressing complex issues. Valid policy prescriptions based on nonfalsifiable methodogy simply will not take place. Prediction without basic understanding of how a system under study works is of little scientific or social value.

The importance of using EITM to spur training reorientation and integration within and between disciplines cannot be overstated. Disciplines that provide incentives for this type of risk taking and re-tooling will reduce the threat of an:

> assembly-line model of research production that imperils innovative theories and methodologies and, in turn, scientific breakthroughs. One could make the argument that EITM or initiatives like it are unnecessary because the unfettered marketplace of ideas expedites best scientific practices and progress. But, it is precisely because there are significant rigidities (training and otherwise) in the current academic setting (imperfect competition) which makes EITM-type initiatives not only necessary—but imperative (EITM Report 2002: 8).

## 13.2 Instruction: A Reorientation

Establishing formal and empirical modeling competency in training in the social sciences is a must. Without that foundation any substantial progress in the use of EITM in the social sciences research is limited. Ideally, reorientation in training involves two parts:

1. Adding courses to ensure sufficient coverage of both formal and empirical tools — and appropriate competency testing.

2. Coherent sequencing of the courses so that skills increase over time.

There are a several possible choices to fulfill these requirements. A first step is requiring graduate students take one full year (usually) of mathematics for social scientists. However, what is not typically done is to continue reinforement of this training. To that end, mathematical (quantitative) competency in these graduate courses can be demonstrated not only in these foundational courses, but also in qualifying examinations in the summer after the first year of coursework. Students must clear this hurdle before being allowed to proceed with their Ph.D. It is safe to say that this latter component would be novel in most social science disciplines.

This mathematical (and quantitative) approach would also be reinforced in revised substantive courses, where examples of methodological unification are used. Not only substantive survey courses but methodological capstone courses integrating formal and empirical modeling can serve this purpose. These capstone courses could include a data gathering component in EITM. Since new technologies have an impact on theoretical implications, the curricula must reflect the fact that theory guides data collection. Further momentum would be reflected in these new syllabi, reflecting the use of EITM.

## 13.3 Supporting Science, Policy, and Society

In assessing metrics to evaluate whether EITM was making a meaningful contribution to improving social science practices, the 2002 NSF EITM Report stated the following:

> How will progress be measured? There are several performance indicators, including the number of articles that use formal and empirical analysis in the major professional journals. Another measurable indicator is the number of NSF grant proposal submissions by faculty and graduate students (doctoral dissertations) that use both approaches (page 13).

This increasing presence can be evident in the increasing number of journal articles, dissertation proposals, books, and research grants. While these metrics represent “inputs,” they still have ramifications since they represent how merging formal and empirical analysis contributes to transforming of how researchers thought about problems and undertake intellectual risks in synthesizing the model and subsequently testing it.

Ultimately, the most important metric is improving in our stock of knowledge. Measuring improvement in our quality of knowledge is not straightforward. EITM, methodological unification, is a bridge for establishing transparency and undertaking a dialogue, but a scholars ability to see and describe the patterns and puzzles in new and more accurate ways is still the driving force. To put it another way, this dialogue provides a coherent way to take every idea further, where something new is shown to be better than what has been established.

Earlier we asserted that EITM can improve current methdological practice. But, what about the policy value and social value? Consider the research in macro political economy since the 1930s. Beginning with the work of Jan Tinbergen, efforts were made to assist policymakers in devising ways to stabilize business cycles.[2] The result was the volatility of business cycles has been reduced in the past 50 years — even when we take current economic conditions into consideration. In addition, the duration of economic expansions has increased in the United States (Granato and Wong 2006) and around the world (Sheffrin 1989). In retrospect these:

> salutary economic events occurred at approximately the same time that quantitative political economic methodologies emphasized and were judged on their ability to produce identified and invariant predictions. Is this relation a coincidence? A good case can be made that the guidelines of the Cowles Commission and successor methodologies has contributed to changes in business cycle behavior (since World War II). And while they have received their share of criticism, these quantitative tools have assisted policymakers by providing useful knowledge and creating a systematic scientific justification for their actions (Granato 2005: 13).

As the preceeding chapters demonstrate, EITM applies to a variety of disciplines and also solidify a lasting change so that social and behavioral scientists consider it natural to unify formal and empirical analysis in their research designs. Or to put it another way, true change will have been achieved when social and behavioral scientists are viewed in this way by the "harder sciences": there will be no need to use the acronym EITM.

[2]The study of business cycles — a public policy and societal concern — was impetus for the development of econometrics. Economic historian Mary Morgan (1990) points out that econometrician Jan Tinbergen's first:

> ...macrodynamic model was built in response to a request from the Dutch Economic Association to present a paper in October 1936 on policies to relieve the depression...His model is a remarkable piece of work, involving not only building and estimating a model of the whole economy but also using the model to simulate the likely impact of various policies (page 102).

EITM-inspired efforts that lead to greater cooperation between the various sciences can enhance policy acumen and aid society. Prior ways of conducting policy research — where integration between the social sciences, natural sciences, and engineering is rare — can lead to misleading predictions and policy failure. In particular, downplaying or ignoring behavioral responses to various phenomena and new technologies may have negative ramifications for public policies regarding energy (i.e., the smart grid), education, health, and numerous other important policy areas where human behavior and human response is a factor.

Because it places an emphasis on modeling and testing analogues of human behavior, EITM translates at a technical level understood by the natural sciences, the physical sciences, and engineering. This potential for enhanced understanding and cooperation can be key in policy success. Among the most important broader impacts of EITM — and one with the most lasting consequence — will be simply raising awareness of the complexities and challenges involved with the linkage of models and tests to the study of social, behavioral, and economic processes.

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