

CHAPTER 13 GAME THEORY AND COMPETITIVE STRATEGY

EXERCISES

3. Two computer firms, A and B, are planning to market network systems for office information management. Each firm can develop either a fast, high-quality system (H), or a slower, low-quality system (L). Market research indicates that the resulting profits to each firm for the alternative strategies are given by the following payoff matrix:

		<i>Firm B</i>	
		<i>H</i>	<i>L</i>
<i>Firm A</i>	<i>H</i>	30, 30	50, 35
	<i>L</i>	40, 60	20, 20

a. If both firms make their decisions at the same time and follow *maximin* (low-risk) strategies, what will the outcome be?

With a maximin strategy, a firm determines the worst outcome for each option, then chooses the option that maximizes the payoff among the worst outcomes. If Firm A chooses *H*, the worst payoff would occur if Firm B chooses *H*: A's payoff would be 30. If Firm A chooses *L*, the worst payoff would occur if Firm B chooses *L*: A's payoff would be 20. With a maximin strategy, A therefore chooses *H*. If Firm B chooses *L*, the worst payoff would occur if Firm A chooses *L*: the payoff would be 20. If Firm B chooses *H*, the worst payoff, 30, would occur if Firm A chooses *L*. With a maximin strategy, B therefore chooses *H*. So under maximin, both A and B produce a high-quality system.

b. Suppose both firms try to maximize profits, but Firm A has a head start in planning, and can commit first. Now what will the outcome be? What will the outcome be if Firm B has a head start in planning and can commit first?

If Firm A can commit first, it will choose *H*, because it knows that Firm B will rationally choose *L*, since *L* gives a higher payoff to B (35 vs. 30). This gives Firm A a payoff of 50. If Firm B can commit first, it will choose *H*, because it knows that Firm A will rationally choose *L*, since *L* gives a higher payoff to A (40 vs. 30). This gives Firm B a payoff of 60.

4. Two firms are in the chocolate market. Each can choose to go for the high end of the market (high quality) or the low end (low quality). Resulting profits are given by the following payoff matrix:

		<i>Firm 2</i>	
		<i>Low</i>	<i>High</i>
<i>Firm 1</i>	<i>Low</i>	-20, -30	900, 600
	<i>High</i>	100, 800	50, 50

a. What outcomes, if any, are Nash equilibria?

If Firm 2 chooses Low and Firm 1 chooses High, neither will have an incentive to change ($100 > -20$ for Firm 1 and $800 > 50$ for Firm 2). If Firm 2 chooses High and

Firm 1 chooses Low, neither will have an incentive to change ($900 > 50$ for Firm 1 and $600 > -30$ for Firm 2). Both outcomes are Nash equilibria.

- b. If the manager of each firm is conservative and each follows a maximin strategy, what will be the outcome?**

If Firm 1 chooses Low, its worst payoff, -20, would occur if Firm 2 chooses Low. If Firm 1 chooses High, its worst payoff, 50, would occur if Firm 2 chooses High. Therefore, with a conservative maximin strategy, Firm 1 chooses High. Similarly, if Firm 2 chooses Low, its worst payoff, -30, would occur if Firm 1 chooses Low. If Firm 2 chooses High, its worst payoff, 50, would occur if Firm 1 chooses High. Therefore, with a maximin strategy, Firm 2 chooses High. Thus, both firms choose High, yielding a payoff of 50 for both.

- c. What is the cooperative outcome?**

The cooperative outcome would maximize *joint* payoffs. This would occur if Firm 1 goes for the low end of the market and Firm 2 goes for the high end of the market. The joint payoff is 1,500 (Firm 1 gets 900 and Firm 2 gets 600).

- d. Which firm benefits most from the cooperative outcome? How much would that firm need to offer the other to persuade it to collude?**

Firm 1 benefits most from cooperation. The difference between its best payoff under cooperation and the next best payoff is $900 - 100 = 800$. To persuade Firm 2 to choose Firm 1's best option, Firm 1 must offer at least the difference between Firm 2's payoff under cooperation, 600, and its best payoff, 800, i.e., 200. However, Firm 2 realizes that Firm 1 benefits much more from cooperation and should try to extract as much as it can from Firm 1 (up to 800).

- 5. Two major networks are competing for viewer ratings in the 8:00-9:00 P.M. and 9:00-10:00 P.M. slots on a given weeknight. Each has two shows to fill this time period and is juggling its lineup. Each can choose to put its "bigger" show first or to place it second in the 9:00-10:00 P.M. slot. The combination of decisions leads to the following "ratings points" results:**

		Network 2	
		First	Second
Network 1	First	18, 18	23, 20
	Second	4, 23	16, 16

- a. Find the Nash equilibria for this game, assuming that both networks make their decisions at the same time.**

A Nash equilibrium exists when neither party has an incentive to alter its strategy, taking the other's strategy as given. By inspecting each of the four combinations, we find that (First, Second) is the only Nash equilibrium, yielding a payoff of (23, 20). There is no incentive for either party to change from this outcome.

- b. If each network is risk averse and uses a maximin strategy, what will be the resulting equilibrium?**

This conservative strategy of minimizing the maximum loss focuses on limiting the extent of the worst possible outcome, to the exclusion of possible good outcomes. If Network 1 plays First, the worst payoff is 18. If Network 1 plays Second, the worst payoff is 4. Under maximin, Network 1 plays First. (Here, playing First is a dominant strategy.) If Network 2 plays First, the worst payoff is 18. If Network 2 plays Second, the worst payoff is 16. Under maximin, Network 2 plays First. The maximin equilibrium is (First, First) with a payoff of (18,18).

- c. **What will be the equilibrium if Network 1 can make its selection first? If Network 2 goes first?**

If Network 1 plays First, Network 2 will play Second, yielding 23 for Network 1. If Network 1 plays Second, Network 2 will play First, yielding 4 for Network 1. Therefore, if it has the first move, Network 1 will play First, and the resulting equilibrium will be (First, Second). If Network 2 plays First, Network 1 will play First, yielding 18 for Network 2. If Network 2 plays Second, Network 1 will play First, yielding 20 for Network 2. If it has the first move, Network 2 will play Second, and the equilibrium will again be (First, Second).

- d. **Suppose the network managers meet to coordinate schedules, and Network 1 promises to schedule its big show first. Is this promise credible, and what would be the likely outcome?**

A move is credible if, once declared, *there is no incentive to change*. Network 1 has a dominant strategy: play the bigger show First. In this case, the promise to schedule the bigger show first is credible. Knowing this, Network 2 will schedule its bigger show Second. The coordinated outcome is likely to be (First, Second).

6. **Two competing firms are each planning to introduce a new product. Each firm will decide whether to produce Product A, Product B, or Product C. They will make their choices at the same time. The resulting payoffs are shown below.**

We are given the following payoff matrix, which describes a product introduction game:

		<i>Firm 2</i>		
		A	B	C
<i>Firm 1</i>	A	-10,-10	0,10	10,20
	B	10,0	-20,-20	-5,15
	C	20,10	15,-5	-30,-30

- a. **Are there any Nash equilibria in pure strategies? If so, what are they?**

There are two Nash equilibria in pure strategies. Each one involves one firm introducing Product A and the other firm introducing Product C. We can write these two strategy pairs as (A, C) and (C, A), where the first strategy is for player 1. The payoff for these two strategies is, respectively, (10,20) and (20,10).

- b. **If both firms use *maximin* strategies, what outcome will result?**

Recall that maximin strategies maximize the minimum payoff for both players. For each of the players the strategy that maximizes their minimum payoff is A. Thus (A,A) will result, and payoffs will be (-10,-10). Each player is much worse off than at either of the pure strategy Nash equilibrium.

- c. **If Firm 1 uses a maximin strategy, and Firm 2 knows, what will Firm 2 do?**

If Firm 1 plays its maximin strategy of A, and Firm 2 knows this then Firm 2 would get the highest payoff by playing C. Notice that when Firm 1 plays conservatively, the Nash equilibrium that results gives Firm 2 the highest payoff of the two Nash equilibria.

7. **We can think of the U.S. and Japanese trade policies as a Prisoners' Dilemma. The two countries are considering policies to open or close their import markets. Suppose the payoff matrix is:**

Japan

		Open	Close
U.S.	Open	10, 10	5, 5
	Close	-100, 5	1, 1

- a. **Assume that each country knows the payoff matrix and believes that the other country will act in its own interest. Does either country have a dominant strategy? What will be the equilibrium policies if each country acts rationally to maximize its welfare?**

Choosing Open is a dominant strategy for both countries. If Japan chooses Open, the U.S. does best by choosing Open. If Japan chooses Close, the U.S. does best by choosing Open. Therefore, the U.S. should choose Open, no matter what Japan does. If the U.S. chooses Open, Japan does best by choosing Open. If the U.S. chooses Close, Japan does best by choosing Open. Therefore, both countries will choose to have Open policies in equilibrium.

- b. **Now assume that Japan is not certain that the U.S. will behave rationally. In particular, Japan is concerned that U.S. politicians may want to penalize Japan even if that does not maximize U.S. welfare. How might this affect Japan's choice of strategy? How might this change the equilibrium?**

The irrationality of U.S. politicians could change the equilibrium from (Close, Open). If the U.S. wants to penalize Japan they will choose Close, but Japan's strategy will not be affected since choosing Open is still Japan's dominant strategy.

The question #8 below is for graduate students only.

8. **You are a duopolist producer of a homogeneous good. Both you and your competitor have zero marginal costs. The market demand curve is**

$$P = 30 - Q$$

where $Q = Q_1 + Q_2$. Q_1 is your output and Q_2 is your competitor's output. Your competitor has also read this book.

- a. **Suppose you are to play this game only once. If you and your competitor must announce your output at the same time, how much will you choose to produce? What do you expect your profit to be? Explain.**

These are some of the cells in the payoff matrix:

		Firm 2's Output						
Firm 1's Output		0	5	10	15	20	25	30
	0	0,0	0,125	0,200	0,225	0,200	0,125	0,0
	5	125,0	100,100	75,150	50,150	25,100	0,0	0,0
	10	200,0	150,75	100,100	50,75	0,0	0,0	0,0
	15	225,0	100,50	75,50	0,0	0,0	0,0	0,0
	20	200,0	100,25	0,0	0,0	0,0	0,0	0,0
	25	125,0	0,0	0,0	0,0	0,0	0,0	0,0
	30	0,0	0,0	0,0	0,0	0,0	0,0	0,0

If both firms must announce output at the same time, both firms believe that the other firm is behaving rationally, and each firm treats the output of the other firm as a fixed number, a Cournot equilibrium will result.

For Firm 1, total revenue will be

$$TR_1 = (30 - (Q_1 + Q_2))Q_1, \text{ or } TR_1 = 30Q_1 - Q_1^2 - Q_1Q_2.$$

Marginal revenue for Firm 1 will be the derivative of total revenue with respect to Q_1 ,

$$\frac{\partial TR}{\partial Q_1} = 30 - 2Q_1 - Q_2.$$

Because the firms share identical demand curves, the solution for Firm 2 will be symmetric to that of Firm 1:

$$\frac{\partial TR}{\partial Q_2} = 30 - 2Q_2 - Q_1.$$

To find the profit-maximizing level of output for both firms, set marginal revenue equal to marginal cost, which is zero:

$$Q_1 = 15 - \frac{Q_2}{2} \quad \text{and}$$

$$Q_2 = 15 - \frac{Q_1}{2}.$$

With two equations and two unknowns, we may solve for Q_1 and Q_2 :

$$Q_1 = 15 - (0.5)\left(15 - \frac{Q_1}{2}\right), \text{ or } Q_1 = 10.$$

By symmetry, $Q_2 = 10$.

Substitute Q_1 and Q_2 into the demand equation to determine price:

$$P = 30 - (10 + 10), \text{ or } P = \$10.$$

Since no costs are given, profits for each firm will be equal to total revenue:

$$\pi_1 = TR_1 = (10)(10) = \$100 \quad \text{and}$$

$$\pi_2 = TR_2 = (10)(10) = \$100.$$

Thus, the equilibrium occurs when both firms produce 10 units of output and both firms earn \$100. Looking back at the payoff matrix, note that the outcome (100, 100) is indeed a Nash equilibrium: neither firm will have an incentive to deviate, given the other firm's choice.

- b. Suppose you are told that you must announce your output before your competitor does. How much will you produce in this case, and how much do you think your competitor will produce? What do you expect your profit to be? Is announcing first an advantage or disadvantage? Explain briefly. How much would you pay to be given the option of announcing either first or second?**

If you must announce first, you would announce an output of 15, knowing that your competitor would announce an output of 7.5. (Note: This is the Stackelberg equilibrium.)

$$TR_1 = (30 - (Q_1 + Q_2))Q_1 = 30Q_1 - Q_1^2 - Q_1\left(15 - \frac{Q_1}{2}\right) = 15Q_1 - \frac{Q_1^2}{2}.$$

Therefore, setting $MR = MC = 0$ implies:

$$15 - Q_1 = 0, \text{ or } Q_1 = 15 \quad \text{and} \\ Q_2 = 7.5.$$

At that output, your competitor is maximizing profits, given that you are producing 15. At these outputs, price is equal to

$$30 - 15 - 7.5 = \$7.5.$$

Your profit would be

$$(15)(7.5) = \$112.5.$$

Your competitor's profit would be

$$(7.5)(7.5) = \$56.25.$$

Announcing first is an advantage in this game. The difference in profits between announcing first and announcing second is \$56.25. You would be willing to pay up to this difference for the option of announcing first.

- c. Suppose instead that you are to play the first round of a series of 10 rounds (with the same competitor). In each round, you and your competitor announce your outputs at the same time. You want to maximize the sum of your profits over the 10 rounds. How much will you produce in the first round? How much would you expect to produce in the tenth round? The ninth round? Explain briefly.**

Given that your competitor has also read this book, you can assume that he or she will be acting rationally. You should begin with the Cournot output and continue with the Cournot output in each round, including the ninth and tenth rounds. Any deviation from this output will reduce the sum of your profits over the ten rounds.

- d. Once again you will play a series of 10 rounds. This time, however, in each round your competitor will announce its output before you announce yours. How will your answers to (c) change in this case?**

If your competitor always announces first, it might be more profitable to behave by reacting “irrationally” in a single period. For example, in the first round your competitor will announce an output of 15, as in Exercise (7.b). Rationally, you would respond with an output of 7.5. If you behave this way in every round, your total profits for all ten rounds will be \$562.50. Your competitor’s profits will be \$1,125. However, if you respond with an output of 15 every time your competitor announces an output of 15, profits will be reduced to zero for both of you in that period. If your competitor fears, or learns, that you will respond in this way, he or she will be better off by choosing the Cournot output of 10, and your profits after that point will be \$75 per period. Whether this strategy is profitable depends on your opponent’s expectations about your behavior, as well as how you value future profits relative to current profits.

(Note: A problem could develop in the last period, however, because your competitor will know that you realize that there are no more long-term gains to be had from behaving strategically. Thus, your competitor will announce an output of 15, knowing that you will respond with an output of 7.5. Furthermore, knowing that you will not respond strategically in the last period, there are also no long-term gains to be made in the ninth period from behaving strategically. Therefore, in the ninth period, your competitor will announce an output of 15, and you should respond rationally with an output of 7.5, and so on.)