Inferences from Small Samples

1. The Central Limit Theorem, which describes
   the mean, standard deviation and shape of the
   sampling distribution, depends on large samples
   \((n \geq 30)\).

2. What can be said about the mean, standard
   deviation and shape of the sampling
   distribution when \(n\) is small?

3. If the sample is drawn from approximately
   normal populations, the mean of the sampling
   distribution is the same for the sampling
   distribution of the means for large sample.

4. The shape is symmetric.

5. The standard deviation is not 1, it varies
   because the shape varies for different sample
   sizes.

“Student’s” \(T\) scores/distribution

1. Developed by W.S. Gosset in 1908.

2. Based on a ratio of the standard normal
   distribution function and the square root of the
   \(\chi^2\) distribution.
3. Given a sample of \( n \) observations, the statistics

\[
    t = \frac{\bar{x} - \mu}{s/\sqrt{n}}
\]

has a \( t \) distribution with \( n-1 \) degrees of freedom.

1. There are many different \( t \) distributions.

2. Every distribution resembles a standard normal distribution but each have “fatter” tails.

3. Small samples have more variability.

4. As the sample size approaches 30, the distribution becomes normal.

**Degrees of Freedom**

1. Number of independent units of observations in the sample relevant to estimation of a particular point estimate.

2. There are \( n \) observations as the initial units of information.

3. Only one observation is used to determine the point estimate.

4. The remaining observations are allowed to be any value, they are allowed to be “free”.
Mathematica Examples

**How do T scores compare to Z scores?**

Here are the two-tailed values at $p=0.05$

<table>
<thead>
<tr>
<th>Z Scores</th>
<th>1.96</th>
</tr>
</thead>
<tbody>
<tr>
<td>T Scores</td>
<td></td>
</tr>
<tr>
<td>df=29</td>
<td>2.045</td>
</tr>
<tr>
<td>df=20</td>
<td>2.086</td>
</tr>
<tr>
<td>df=15</td>
<td>2.131</td>
</tr>
<tr>
<td>df=10</td>
<td>2.228</td>
</tr>
<tr>
<td>df=  5</td>
<td>2.571</td>
</tr>
</tbody>
</table>