Miscible density-stable displacement flows in inclined tube

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We study density-stable laminar miscible displacement flow of two iso-viscous Newtonian fluids in an inclined pipe (diameter $\hat{D}$). We present a wide range of novel experimental results. We illustrate the non-monotone relation in displacement efficiency at the density difference moves from positive (density unstable) to negative (density stable), the efficiency being minimal for iso-dense fluids. The density stable configuration has been found to produce highly efficient displacements, with the bulk of the interface moving steadily at the mean velocity. The streamwise length of the stretched interface, or stretch length $\hat{L}$, is measured over a wide range of parameters. The stretch length increases with the mean flow velocity, increases with inclination $\beta$ from vertical, decreases with density difference, and increases with viscosity. Our data are well represented by the scaled expression $\hat{L} = -\tan \beta = -3680/\chi$, where $\chi$ is the ratio of buoyancy and viscous stresses. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.4766197]

I. INTRODUCTION

We study the effect of pipe inclination and flow rate on the density stable displacement of one miscible fluid by another of the same viscosity. By density stable we mean that the less dense fluid displaces the heavier fluid in the downwards direction. Our displacement flow experiments are performed in a long pipe, but in regimes where the Péclet number is significantly larger than the ratio of length to diameter. Thus, we are far from the laminar dispersive regimes of Taylor1 and Aris.2

Miscible displacements in this regime have been studied experimentally by a number of authors, considering both density differences and viscosity differences between the fluids, and in both pipes3–6 and Hele-Shaw cells.7–9 In general for upwards displacements, a more effective displacement is achieved by using a displacing fluid that is both heavier and more viscous. Where density differences are present there is a large body of work targeted at viscosity effects in vertical geometries, and much of the published work is focused at onset of fingering/spike type instabilities. There is also a significant body of computational work. Much of the earlier work has been focused at symmetric pipe or channel displacements.10, 11 More recently, some authors have considered two-dimensional buoyant inclined channel flow displacements12–14 and also three-dimensional displacements in channels.15

The significant novelty of our study is as follows. First, we know of no other experimental or numerical study of miscible displacement flow that covers a broad range of pipe inclinations in the density stable configuration. In our previous work,13, 14, 16 we have presented the results of an extensive experimental and computational study of density unstable displacements in pipes and channels that are only inclined close to horizontal. Second, we illustrate the very interesting transition that occurs as the density difference (dimensionlessly represented by the Atwood number $At$) passes through zero. For identical fluids ($At = 0$) the velocity adopts a Poiseuille profile, the centre of which

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moves at twice the mean speed. For density unstable $At > 0$ a buoyancy driven slumping interface is found in Refs. 13, 14, and 16, with front speed around $26 \pm 5\%$ higher than the mean speed. Intuition suggests that the density stable displacement flow ($At < 0$) should displace at approximately the mean speed, as we indeed find. In a long pipe, the displacement efficiency is given by the ratio of mean speed to front velocity and it is curious to observe non-monotone variation of the displacement efficiency with $At$. The third novel contribution of the study is in finding a simple dimensionless relation that captures the spreading of the interface in the stable configuration.

II. EXPERIMENTAL SETUP AND DESIGN CONSIDERATIONS

Our experiments are conducted in an inclined acrylic pipe (4 m long, $D = 19$ mm) capable of being tilted to any angle $\beta$; see Fig. 1. A gate valve separates the upper 80 cm of the pipe from the lower 320 cm. Two fish tanks surround the pipes to reduce refraction effects and obtain high quality images. The gate valve is initially closed, separating displacing and displaced fluid. The fluids are pumped from two transparent tanks that are pressurized (to $\sim 70$ Pa gauge) by compressed air. By adjusting a needle valve located before the drain we can control the flow rate accurately, for low-to-moderate Reynolds numbers.

We cover a range of pipe inclinations at different fixed density differences and viscosities. Both fluids used are water. Salt (NaCl) is used as the weighting agent, added to the displaced fluid. We add black dye (ink) to the displaced heavier fluid to allow for optical absorption measurements. The low concentration of the dye added has a negligible effect on the fluid properties. The optical measurement method consists of acquiring images of the pipe using digital cameras. We use two high speed cameras. Each camera covers a 160 cm portion of the pipe below the gate valve and records images at given frame rates. The pipes are back-lit using light-emitting diode (LED) strips with a diffusive layer placed between LED strips and fish tank wall to improve light homogeneity. Based on the images of the experiments we can estimate the concentration of the fluid at each pixel following standard calibration methods for light absorption. In the case of two stratified fluids with no interfacial mixing, the concentration of each point is either 0 or 1. On averaging the concentration field across the pipe cross section we get a gray scale between 0 and 1, which can be plotted in the form of a spatiotemporal plot showing the evolution of the mean concentration as a function of the streamwise location from the gate valve, $\hat{x}$ and time, $\hat{t}$, measured from the start of the experiment. This same average may also be interpreted as an interface height in the case that there is little mixing over the experimental time scale.

The dimensional parameter ranges we have considered for our experiments are given in Table I. We also carried a series of experiment for $\beta = 85^\circ$, over similar ranges of parameters. The data for this set are not included here as the stretch length (to be defined later in Sec. III B) was mostly

![Fig. 1. Schematic of the experimental set-up (the shape of the interface is illustrative only).](image-url)
not fully developed during the time of the experiment and within the length of the apparatus. As $\beta \to 90^\circ$ we approach a singular case in which the interface between the two fluids in the pipe extends to infinity, given a sufficiently long pipe/experiment.

A dimensional analysis of this type of flow is given in Ref. 14. We denote the density of the displacing fluid by $\hat{\rho}_L$ and that of the displaced fluid by $\hat{\rho}_H$. The Atwood number is defined as $At = (\hat{\rho}_L - \hat{\rho}_H)/(\hat{\rho}_L + \hat{\rho}_H)$, representing a dimensionless density difference. Note that for this study $At < 0$ since $\hat{\rho}_L < \hat{\rho}_H$. Our experiments are all performed at small $|At|$, the significance of which is that a Boussinesq approximation is valid. This means that density differences affect the buoyancy force significantly through the densimetric Froude number, $Fr = \hat{V}_0/\sqrt{At \hat{g} \hat{D}}$, but not the acceleration of individual fluids. Here $\hat{V}_0$ is the mean flow velocity and $\hat{g}$ the gravitational acceleration. Since we also study a high Péclet number range (as is most relevant industrially), the flow is governed only by 3 dimensionless groups: $\beta$, $Fr$, and the Reynolds number $Re = \hat{V}_0 \hat{D}/\hat{\nu}$, where $\hat{\nu}$ is defined using the mean density $\hat{\rho} = (\hat{\rho}_L + \hat{\rho}_H)/2$ and the common viscosity of the two fluids, $\hat{\mu}$. The range of dimensionless parameters is also shown in Table I, from which we can see that significant buoyancy forces and inertial effects are present in our experimental range, as well as a broad set of pipe inclinations.

### III. RESULTS

We present our results in three main sections below. First we examine the interesting transition from density unstable to density stable flows as $At$ decreases through zero (Sec. III A). The remainder of the results concern only density stable flows. In Sec. III B we characterise the main features of the density stable displacements, describing the evolution to a steady stretch length. Parametric variations in stretch length are described in Sec. III C.

#### A. The transition from density unstable to density stable

In density unstable displacement the heavier displacing layer slumps beneath the lighter displaced layer. In the absence of significant inertial mixing, we are able to identify two displacement fronts: the leading front advances more rapidly along the bottom of the pipe, whereas the trailing front lags behind so that the interface is progressively extended between the two fronts. If buoyancy forces are strong, the trailing front may propagate backwards against the mean imposed flow. After an initial transient the speed of the leading front settles to a steady value, $\hat{V}_{f0}$ (behind the leading front) is occupied by displaced fluid. The displacement efficiency is therefore approximately equal to $\hat{V}_{f0}/\hat{V}_{f} < 1$. This ratio is very important in the design of fluid displacements. Our previous studies of density unstable displacement flows in near-horizontal pipes yielded front speeds typically in the range $26 \pm 5\%$ faster than $\hat{V}_{f0}$.

Figure 2 compares a sequence of images for density-unstable and density-stable displacement flows at the same $|At|$, for $\beta = 70^\circ$, $\hat{\nu} = 1$ mm$^2$/s and with very similar $\hat{V}_0$. The slumping interface in the density unstable case (Fig. 2(a)) is qualitatively similar to our previous work (at larger $\beta$).
This particular displacement appears to be in inertial regime. The density stable displacement (Fig. 2(b)) also shows two fronts advancing. Two key differences are that (i) after an initial phase in which the interface is extended, the trailing front moves steadily at approximately the same speed as the leading front; (ii) the lighter displacing fluid layer now advances on top of the heavier displaced fluid.

FIG. 2. Density-unstable and density-stable displacements at \( \beta = 70^\circ \) (see Figure 1 for definition) and \( \theta = 1 \text{ mm}^2/\text{s} \). (a) \( At = 0.0035 \) (dark fluid is less dense than the lighter coloured fluid) and mean velocity \( \bar{V}_0 = 20.3 \text{ mm/s} \); the sequence starts 16.25 s after opening the gate valve and the time interval between images is 2.5 s. (b) \( At = -0.0035 \) (dark fluid is more dense than the lighter coloured fluid) and \( \bar{V}_0 = 20.9 \text{ mm/s} \); the sequence starts 21.5 s after opening the gate valve and the time interval between images is 3.25 s. The field of view is 430 × 19 mm², taken 800 mm below the gate valve. The last image at the bottom of (a) shows the gray scale for the concentration values, which can also be used for (b). The arrows indicate the flow direction and the left end of the tube is higher than the right (the fluid flows downhill).

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Figures 3(a) and 3(b) show the spatiotemporal diagram of the same experiments as in Fig. 2. From these plots we are able to compute the velocity of the leading displacement front, \( \bar{V}_f \). For the density unstable displacement, \( \bar{V}_f = 38.9 \text{ mm/s} > \bar{V}_0 = 20.3 \text{ mm/s} \), so that the interface elongates as the displacement evolves, whereas for Fig. 2(b) we have \( \bar{V}_f \approx \bar{V}_0 \), meaning a significantly more efficient displacement. This transition in displacement efficiency (\( \bar{V}_0/\bar{V}_f \) apparently increases) as we change \( At \) from positive to negative appears intuitive. However, it is also remarkable what happens at the transition.

First of all, we comment that the density differences involved in our experiments are relatively small (\( At = \pm 0.0035 \) corresponds to a density difference of \( \approx 0.7% \)), which suggests that the transition occurs very close to \( At = 0 \). Second, the change in \( \bar{V}_0/\bar{V}_f \) is not monotone! For \( At = 0 \) the iso-viscous displacement flow is simply that of a passive tracer advected by a Poiseuille flow. The centerline advects at twice the mean flow and the entire front elongates progressively as the
displacement continues. The spatiotemporal plot for an experiment at \( At = 0 \) is illustrated in Fig. 3(c). We have marked on this plot the line \( \tilde{V}_f = 2 \tilde{V}_0 \) (as a guide to the eye). This leading front is much less sharp than either of those with a density difference. We observe significant dispersion, but note that we are still far from the diffusive Taylor regime.

The leading front speed at \( At = 0 \) is significantly faster than those for small positive or negative \( At \). There is no monotone increase in efficiency as \( At \) decreases through zero. Indeed, it seems that as \( At \rightarrow 0 \) from above, \( \tilde{V}_0/\tilde{V}_f \) decreases towards a value of 0.5, but then increases rapidly to 1 for \( At < 0 \). Experimentally, we cannot determine if the transition in efficiency is discontinuous or simply rapid at \( At = 0 \). This feature is at least noteworthy.

### B. Characteristics of density stable displacements

We turn now to exploring density stable displacements in more detail. The experiment of Figs. 2(b) and 3(b) is quite typical in that, after an initial transient the leading and trailing fronts move together. In order to capture this development we plot the averaged concentration profiles when the gate valve is opened at the start of the experiment. During this period the mean flow itself evolves. To obtain the stretch length in a robust and consistent way from our experiments, we neglect that we are still far from the diffusive Taylor regime.

Where either \( \tilde{h}/\tilde{D} < 0.15 \) or \( \tilde{h}/\tilde{D} > 0.85 \) (approximately) the layers of displacing or displaced fluid become thin and visualisation from the side is through the top or bottom of the pipe where curvature is maximal. These wall regions are consequently those most affected by noise in measurement of the averaged concentrations. Even though the behaviour in these wall regions is hard to visualize, it is clear that the bulk of the interface is unaffected by the wall regions. Indeed the slope of the evolution profiles in the center of the pipe remains constant as they advance. Interestingly, the evolution profiles change curvature at an inflection point around \( \tilde{h}/\tilde{D} = 0.5 \), which is the pipe center. The self-similar evolution profiles shown in Fig. 4(a) suggest that these profiles might collapse onto one curve with an appropriate scaling. Figure 4(b) shows the evolution profiles \( \tilde{h}/\tilde{D} \) plotted against \( \tilde{x}/\tilde{t} \). We can see that the profiles converge to a value \( \approx 22.8 \) (mm/s) at large times, (the very left curves in Figure 4(b)). This value is very close to the mean flow speed, \( \tilde{V}_0 = 20.9 \) mm/s, suggesting that the interface between displacing and displaced fluids moves at a speed very close to that of the mean flow. Later on we will discuss this and its effect on displacement efficiency in more detail.

We call the streamwise distance between the leading and trailing fronts the *stretch length*, \( \tilde{L} \). In order to obtain the stretch length in a robust and consistent way from our experiments, we neglect the part of the interface below 15\% and above 85\% of the pipe’s diameter. Figure 4(c) shows the evolution of \( \tilde{L} \) with time for the same experiment. We can distinguish an initial elongation phase, when the gate valve is opened at the start of the experiment. During this period the mean flow itself

![FIG. 4.](image)

(a) Experimental profiles of normalized \( h(\tilde{x}, \tilde{t}) \) for \( \tilde{t} = 68, 74.5, \ldots, 126.5, 133 \) s for the same experiment as Figures 2(b) and 3(b). The dotted lines show \( \tilde{h}/\tilde{D} = 0.85 \) and 0.15. The horizontal solid line indicates \( \tilde{h}/\tilde{D} = 0.5 \) and the inclined bold dashed line (eye guide) shows the slope of the collapsed curves which stays the same for different profiles. (b) Collapse of the normalized profiles \( \tilde{h}/\tilde{D} \) with \( \tilde{x}/\tilde{t} \) for the same experiment as part (a). (c) The evolution of stretch length value, \( \tilde{L} \), with time for the same experiment as part (a). The fully developed stretch length is attained for approximately \( \tilde{t} > 40 \) s. The inset in (c) shows the dependency of the scaled stretch length for different choices of lower and upper bounds of \( \tilde{h}/\tilde{D} \) (0.15–0.85, 0.1–0.9, and 0.2–0.8).
displacing and displaced fluids measured at any time after opening the gate valve, and will refer to (Fig. 4(a)). This in turn confirms that the scaled stretch length measurements are not greatly sensitive to the choice of diameter filter percentage at short times, they converge into a constant value over longer times when the case is naturally used. We see that although the stretch length values are dependent on the choice of filter percentage, and we stay with 15% and 85% as filter values. From that for scaling each stretch length, \( \hat{L} \) which is the mean travel time for one diameter.

accelerates from zero to the set flow rate and we can also expect strong buoyancy effects as the initial interface is perpendicular to the pipe. In the second phase we see a slower relaxation towards a constant length, in this case after about 40 s. A dimensionless stretch length is defined by scaling \( \hat{L} \) with \((0.85 \sim 0.15) \hat{D} \). The inset in Figure 4(c) shows the dependency of the scaled stretch length for different choices of lower and upper bounds of \( \hat{h}/\hat{D} \) (0.15–0.85, 0.1–0.9, and 0.2–0.8). Note that for scaling each stretch length, \( \hat{L} \), the corresponding percentage of the diameter, \( \hat{D} \), in each case is naturally used. We see that although the stretch length values are dependent on the choice of diameter filter percentage at short times, they converge into a constant value over longer times when the flow is developed. Basically this is due to the constant slope over the central part of the pipe (Fig. 4(a)). This in turn confirms that the scaled stretch length measurements are not greatly sensitive to the choice of diameter filter percentage, and we stay with 15% and 85% as filter values. From hereon when we use the expression stretch length, we mean the longitudinal distance \( \hat{L} \) between displacing and displaced fluids measured at any time after opening the gate valve, and will refer to the constant value obtained at long times as the fully developed stretch length.

Figure 5(a) plots the range of development times, \( t_{FD} \), required for the stretch length to become fully developed. We have scaled \( t_{FD} \) with \( \hat{D}/\hat{V}_0 \) which is the mean travel time for one diameter. Thus, the vertical axis measures the number of pipe diameters from the gate valve until the flow is fully developed. The length of pipe below the gate valve is roughly 170 diameters. The horizontal axis in Fig. 5(a) is the parameter,

\[
\chi = \frac{Re \cos \beta}{Fr^2} = \frac{2\beta \hat{g} \cos \beta \hat{D}^2}{\hat{V}_0},
\]

which captures the ratio of buoyant forces to viscous forces. We can see that as the buoyancy force decreases with respect to the viscous forces, development times increase. Also note that when the ratio of inertial to buoyant forces increases the dimensional time for a fully developed stretch length increases as well; see Fig. 5(b).

In all cases when the stretch length is fully developed we have a leading front velocity that is close to the imposed velocity. Figure 6 shows the range of \( \hat{V}_f \) vs \( \hat{V}_0 \) from our experiments. The broken line is the linear fit \( \hat{V}_f = 1.04 \hat{V}_0 \). In this case an average displacement efficiency for all our density stable experiments is 1/1.04 \( \approx 96\% \). The discrepancy from \( \hat{V}_f = \hat{V}_0 \) is quite small. Partly this may be attributed to measurement errors (e.g., \( \hat{V}_f \) comes from flow visualisation and \( \hat{V}_0 \) from the flow meter). Partly there may be imperfect displacement in the upper and lower wall regions (thin film effects) and partly we must consider that there is some dispersion.

With regard to dispersion, it is worth reflecting on the consequences of steadily advancing leading and trailing fronts with \( \hat{V}_f \approx \hat{V}_0 \) and steady stretch length. Both far upstream and downstream of the front, the flows adopts a Poiseuille profile but the frontal region advances uniformly. Globally speaking, this means that the centreline fluid behind the front must decelerate and move towards the walls. In advance of the front, the fluid close to the walls is pushed towards the centreline.
and accelerates forward. Thus, there is a net counter-current flow in the radial direction for any steadily moving front. For large Péclet number miscible displacements, significant diffusion only occurs close to sharp gradients in fluid concentration. However, now we have significant secondary flows on both sides of the front that can advect diffused mixture away from the frontal region. One consequence is that although the averaged concentration profiles can propagate apparently steadily (e.g., Fig. 4(a)) the fluid concentration in the frontal region can become diffuse (e.g., Fig. 2(b)).

A second consequence of the secondary flows described above is that there must exist recirculatory zones in the streamlines, as considered in a moving frame of reference. Thus, although the flow may become parallel far from the frontal region, close to the front it is multi-dimensional and the secondary flow velocity scale is evidently $\hat{V}_0$. This means that inertial effects are always present in the frontal region, whenever $Re > 1$, which is for all our experiments.

**C. Parametric variation in the fully developed stretch length**

We now explore the sensitivity of the fully developed stretch length to our experimental variables: $At$, $\beta$, $\hat{V}_0$, and $\hat{\nu}$. Figure 7(a) shows the profiles of normalized $h/D$ for density-stable displacements at different $\hat{V}_0$, with fixed $\beta = 60^\circ$, $\hat{\nu} = 1 \text{ mm}^2/\text{s}$, and $At = -0.0035$. An increase in density difference decreases the fully developed stretch length. Figure 7(b) shows the effect of the density difference ($At$) on the length of the interface for two otherwise similar cases ($\hat{V}_0$ values are also very close together). While the second of these is perhaps intuitive, the effect of increasing mean velocity is less obvious. One explanation, that we reinforce later below, is that the stretch length is controlled by

![Graph](image_url)

**FIG. 6.** Front velocity values, $\hat{V}_f$, plotted against the mean flow velocity, $\hat{V}_0$ for different experiments as in Figure 5. The dashed-dotted line is the linear fit $\hat{V}_f = 1.04\hat{V}_0$. In this case an average displacement efficiency for all density stable experiments is $1/1.04 \sim 96\%$. The solid line represents the ideal displacement case with $\hat{V}_f = \hat{V}_0$.

![Graph](image_url)

**FIG. 7.** Experimental profiles of $\hat{h}(\hat{x}, \hat{t})/\hat{D}$ at different times but at around the same position in the pipe for $\beta = 60^\circ$ and $\hat{\nu} = 1 \text{ mm}^2/\text{s}$; (a) $At = -0.0035$ different $\hat{V}_0$; (b) similar $\hat{V}_0$ and different $At < 0$. All experiments are density stable.
the ratio of axial buoyancy forces to viscous stresses, $|\chi|$. As we increase the flow rate ($V_i$) this ratio is reduced allowing the interface to spread more. Note too that we have seen that the development times $t_{FP}$ also increase as $|\chi|$ decreases.

We note in Fig. 7 that the profiles $h/h_0$ are measured when the stretch length is fully developed. There is a degree of asymmetry that appears to increase with both $V_i$ and $|\chi|$. Partly this may be due to asymmetry in the wall regions at top and bottom, i.e., the top is an advancing film and the bottom is a draining film. This may also be related to the pattern of secondary flows and dispersion being non-uniform across the pipe depth. It is hard to develop simplistic models that will give an interface shape prediction (e.g., by assuming a Poiseuille flow), since in the interfacial region the velocity field is far from a Poiseuille flow.

Figures 8(a) and 8(b) show the effects of inclination angle, mean flow speed, density difference, and fluid viscosities on the fully developed stretch length. Figure 8(a) shows results for $At = -0.0035$ and $At = -0.01$. As we approach a horizontal configuration the stretch length increases. The trend shown in this figure suggests that as we move towards strictly vertical pipe the stretch length should become very small, which is also observed. Figure 9 shows snapshots of the flows for a series of strictly vertical experiments. The parameters used are $\beta = 0^\circ$, $\hat{\nu} = 1$ mm$^2$/s, $At = -0.0035$, and $V_i = 18.2, 48.1$ and 70.6 mm/s. Due to the symmetric Poiseuille profile, advective velocity effects act to disperse the light displacing fluid ahead of the mean flow in the pipe centre. This dispersive effect is balanced by the stabilizing density difference between the fluids, preventing the interface from spreading excessively.

![Figure 8](image1.png)

**FIG. 8.** (a) Variations in dimensional stretch length $\hat{L}$, for different $\beta$ and $V_i$. The solid symbols are for $At = -0.0035$ and the open symbols for $At = -0.01$. Note that $\hat{\nu} = 1$ mm$^2$/s. (b) Dimensional stretch length $\hat{L}$ versus $V_i$ for two different dynamic viscosities at fixed $At = -0.0035$ and $\beta = 20^\circ$.

![Figure 9](image2.png)

**FIG. 9.** Snapshots of the vertical displacement flow experiment, $\beta = 0^\circ$, carried for $\hat{\nu} = 1$ mm$^2$/s, $At = -0.0035$ and (a) $V_i = 18.2$ mm/s, (b) $V_i = 48.1$ mm/s, and (c) $V_i = 70.6$ mm/s. The sequence for part a starts 30.75 s after opening the gate valve and the time interval between images is 5 s. For parts (b) and (c) the sequences start 80.75 s and 19.5 s after opening the gate valve and the time intervals between images are 2 s and 1.25 s, respectively. The arrow indicates the flow direction and the left end of the tube is higher than the right (the fluid flows vertically downhill). The field of view is 700 x 19 mm$^2$, taken 1550 mm below the gate valve. The last image at the bottom of (a) is the colourbar of the concentration values and can be used for (b) and (c) too. The white dashed lines in (a)-(c) are eye guide and indicate the boundaries of the mixed region moving upfront.
from elongating noticeably: the stretch length remains very small (close to zero). We see that the interface between the fluids flattens out with a small cap region advancing in front of the pure displacing fluid. This cap region could either be due to fluid mixing or could be a visual effect due to a film of displaced fluid remaining on the wall. In any case the length of cap region reduces as the mean flow is increased, suggesting that the residual film is more likely. Ahead of the cap and quite faint in these figures is dispersed spike-like region of mixed fluid (mostly displaced fluid), which we have outlined for clarity. These regions are formed during the initial phase of mixing after opening the gate valve, but they may also be “fed” by a small amount of diffusion/dispersion, from the tip of the front as the flow progresses. This diffuse mixed region disperses further ahead of the cap region with increasing mean flow, as is expected. These effects are interesting in their own right, but this has not been the focus of our study. The main practical conclusion is that the fully developed stretch length is small for vertical pipes. More details on vertical displacement flows in ducts can be found in Refs. 3, 4, 7, and 10. Where experiments have been performed they have typically been performed with more viscous fluids and at lower Reynolds numbers than here.

To investigate the effect of viscosity (note we are still interested in iso-viscous displacements) we have run comparison experiments at higher viscosity, obtained with a 54 weight-percent glycerine-water solution with \( \nu \sim 5.6 \, \text{mm}^2/\text{s} \). Figure 8(b) shows that when the fluids become more viscous the stretch length also increases. A few experiments were run with an even higher glycerine concentration (72% (wt./wt.)) with \( \nu \sim 18.6 \, \text{mm}^2/\text{s} \). It was observed that the stretch length was much longer than the length of our apparatus (results are not presented here). In these experiments the trailing front did not appear to move at all from the gate valve over the duration of the experiment.

The increase in viscosity exposed a difference in qualitative behaviour of the displacement front near the top and bottom of the pipe. The leading front displaces immediately and keeps moving, stretching the interface. We postulate that the trailing front requires a critical stretching \( \hat{L} \) before it begins to move. Presumably, the angle of the interface contributes to a force balance in which buoyancy forces are opposed by viscous forces (hence the elongation with increased viscosity). Once the trailing front is moving it does so at approximately the same speed as the imposed flow, and we may measure \( \hat{V}_f \approx \hat{V}_0 \) from spatiotemporal plots such as Fig. 3(b). In this case the displacement is very efficient (close to 100%); see Fig. 6. Note that although this suggests a viscous:buoyancy balance, if we were to develop a lubrication model such as in Ref. 17, this would predict that the fully developed stretch length is governed primarily by a balance between axial buoyancy forces and viscous stresses (originating from the mean flow). This balance is captured in the parameter \( \chi = 2At \hat{g} \cos \beta \hat{D}^2/2 \hat{V}_0 \). Since our stretch length values are calculated between \( \hat{h}/\hat{D} = 0.15 \) and \( \hat{h}/\hat{D} = 0.85 \) we subtract off the hydrostatic stretch length \( (0.7\hat{D}\tan\beta) \) from \( \hat{L} \) and normalize with \( 0.7\hat{D} \). Figure 10 plots the normalized fully developed stretch length

![Figure 10](http://scitation.aip.org/termsconditions. Downloaded to IP: 137.82.113.109 On: Fri, 15 Nov 2013 21:38:50)
against $\chi$ for our data, in linear and log scales. It appears that the data collapse well onto a single curve regardless of $\beta$, $At$, and $\tilde{v}$. Horizontal error bars for $\chi$ are shown. Vertical error bars are negligible. A model curve is fitted to the experimental data of the form

$$L - \tan \beta = -3680/\chi. \tag{2}$$

This in itself is interesting since our flows are performed at a range of non-trivial $Re$ and the occurrence of secondary flows about a steadily advancing frontal region means that inertial effects are always present close to the advancing front. Note that only the data for $\beta \geq 20^\circ$ has been used to fit (2). As discussed above, the stretch length in a vertical pipe is observed to be of the order of the diameter and consequently negligible. The expression (2) at $\beta = 0^\circ$ is not singular, but indicates a stretch length larger than that observed and one that is strongly dependent on $\chi$. This represents an extrapolation of (2) outside the range of the data used to make the curve fit. It is unclear how far below $\beta = 20^\circ$ this expression will be valid.

By choosing $0.7\bar{D}$ as the scale for dimensional stretch length values, $\bar{L}$, one can now re-scale $L - \tan \beta = -3680/\chi$ with the full diameter value, $\bar{D}$, and calculate a predicted stretch length value over the whole pipe (not only over 70% of the pipe). In rescaling (2) in this way we effectively assume that the interface evolution profiles (e.g., Fig. 4) extend linearly up to the point where they intersect with the pipe walls. This gives a closer prediction of the stretch length in the pipe. It is also worth noting that for the 72% (wt/wt) glycerine-water experiments in which the stretch length could not be measured within our apparatus, $\chi \approx -14$, for which the fitted model predicts a stretch length on the order of 5 m, exceeding the length of our apparatus. This is precisely what we observed.

IV. SUMMARY

Laminar displacement flow of two miscible iso-viscous Newtonian fluids in an inclined pipe has been investigated experimentally in the case where the displacing fluid is less dense than the displaced fluid (i.e., density stable). Our experiments have covered a broad range of the governing dimensionless parameter space ($\beta$, $Re$, $Fr$).

The practical importance of our study is in predicting the displacement efficiency, which is quantified by the ratio $\tilde{V}_0/\tilde{V}_f$. We have demonstrated that there is an interesting transition in $\tilde{V}_0/\tilde{V}_f$ as the $At$ changes from positive (density unstable) to negative (density stable). For $At > 0$ our previous work shows that efficiencies are significantly $< 1$. As $At$ decreases towards zero $\tilde{V}_0/\tilde{V}_f$ reduces further, bounded below by the theoretical $\tilde{V}_0/\tilde{V}_f = 0.5$ of passive scalar advection. However, for $At < 0$ the efficiency $\tilde{V}_0/\tilde{V}_f$ increases sharply to a value close to 1. Remarkably, this transition shows order 1 changes in $\tilde{V}_0/\tilde{V}_f$ over a very small range of $At$ (corresponding to 0.7% density difference).

In the density stable range of our experiments ($At < 0$), our data is approximated well by the linear fit: $\tilde{V}_f = 1.04\tilde{V}_0$, suggesting an efficiency of around 96%. We expect that the displacement is never 100% efficient due to a very thin residual lower layer of the displaced fluid, and possible dispersion of diffused mixture in the frontal region.

At the start of a typical density stable experiment the trailing front remains stationary while the leading front advances, stretching the interface. On attaining a critical length the trailing front also moves, at approximately the same speed as the mean flow. The length of the interface along the pipe has been called the stretch length, $L$. We have characterised the time taken for the stretch length to become constant in time, and portrayed this in terms of a development length. This development time increases with the mean imposed velocity and decreases with the axial buoyancy force.

The fully developed stretch length increases with the mean flow velocity, decreases with the density difference, increases with inclination from vertical and increases with fluid viscosity. Our data are well represented by the scaled expression $L - \tan \beta = -3680/\chi$, where $\chi = 2At\bar{g}\bar{D}^2 \cos \beta/(\tilde{v}\tilde{V}_0)$. This implies that stretch length is primarily due to a balance between axial buoyancy forces and viscous stresses from the mean flow. This in itself is interesting since our flows are performed at a range of non-trivial $Re$ and the occurrence of secondary flows about a steadily advancing frontal region means that inertial effects are always present close to the advancing front.
Future experimental work on density stable displacement flows will focus on the effects of a viscosity ratio between two Newtonian fluids and on examining shear-thinning effects in these flows. It would also be of interest to derive a simple predictive model that produces the observed dominant dependence on $\chi$, but apparent insensitivity to inertial effects.

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