

## CHAPTER 10 MARKET POWER: MONOPOLY AND MONOPSONY

### EXERCISES

**3. A monopolist firm faces a demand with constant elasticity of -2.0. It has a constant marginal cost of \$20 per unit and sets a price to maximize profit. If marginal cost should increase by 25 percent, would the price charged also rise by 25 percent?**

Yes. The monopolist's pricing rule as a function of the elasticity of demand for its product is:

$$\frac{(P - MC)}{P} = - \frac{1}{E_d}$$

or alternatively,

$$P = \frac{MC}{\left(1 + \left(\frac{1}{E_d}\right)\right)}$$

In this example  $E_d = -2.0$ , so  $1/E_d = -1/2$ ; price should then be set so that:

$$P = \frac{MC}{\left(\frac{1}{2}\right)} = 2MC$$

Therefore, if MC rises by 25 percent price, then price will also rise by 25 percent. When  $MC = \$20$ ,  $P = \$40$ . When MC rises to  $\$20(1.25) = \$25$ , the price rises to  $\$50$ , a 25% increase.

**4. A firm faces the following average revenue (demand) curve:**

$$P = 100 - 0.01Q$$

**where Q is weekly production and P is price, measured in cents per unit. The firm's cost function is given by  $C = 50Q + 30,000$ . Assuming the firm maximizes profits,**

**a. What is the level of production, price, and total profit per week?**

The profit-maximizing output is found by setting marginal revenue equal to marginal cost. Given a linear demand curve in inverse form,  $P = 100 - 0.01Q$ , we know that the marginal revenue curve will have twice the slope of the demand curve. Thus, the marginal revenue curve for the firm is  $MR = 100 - 0.02Q$ . Marginal cost is simply the slope of the total cost curve. The slope of  $TC = 30,000 + 50Q$  is 50. So  $MC$  equals 50. Setting  $MR = MC$  to determine the profit-maximizing quantity:

$$100 - 0.02Q = 50, \text{ or}$$

$$Q = 2,500.$$

Substituting the profit-maximizing quantity into the inverse demand function to determine the price:

$$P = 100 - (0.01)(2,500) = 75 \text{ cents.}$$

Profit equals total revenue minus total cost:

$$\pi = (75)(2,500) - (30,000 + (50)(2,500)), \text{ or}$$

$$\pi = \$325 \text{ per week.}$$

**b. If the government decides to levy a tax of 10 cents per unit on this product, what will be the new level of production, price, and profit?**

Suppose initially that the consumers must pay the tax to the government. Since the total price (including the tax) consumers would be willing to pay remains unchanged, we know that the demand function is

$$P^* + T = 100 - 0.01Q, \text{ or}$$

$$P^* = 100 - 0.01Q - T,$$

where  $P^*$  is the price received by the suppliers. Because the tax increases the price of each unit, total revenue for the monopolist decreases by  $TQ$ , and marginal revenue, the revenue on each additional unit, decreases by  $T$ :

$$MR = 100 - 0.02Q - T$$

where  $T = 10$  cents. To determine the profit-maximizing level of output with the tax, equate marginal revenue with marginal cost:

$$100 - 0.02Q - 10 = 50, \text{ or}$$

$$Q = 2,000 \text{ units.}$$

Substituting  $Q$  into the demand function to determine price:

$$P^* = 100 - (0.01)(2,000) - 10 = 70 \text{ cents.}$$

Profit is total revenue minus total cost:

$$p = (70)(2,000) - ((50)(2,000) + 30,000) = 10,000 \text{ cents, or}$$

\$100 per week.

Note: The price facing the consumer after the imposition of the tax is 80 cents. The monopolist receives 70 cents. Therefore, the consumer and the monopolist each pay 5 cents of the tax.

If the monopolist had to pay the tax instead of the consumer, we would arrive at the same result. The monopolist's cost function would then be

$$TC = 50Q + 30,000 + TQ = (50 + T)Q + 30,000.$$

The slope of the cost function is  $(50 + T)$ , so  $MC = 50 + T$ . We set this  $MC$  to the marginal revenue function from part (a):

$$100 - 0.02Q = 50 + 10, \text{ or}$$

$$Q = 2,000.$$

Thus, it does not matter who sends the tax payment to the government. The burden of the tax is reflected in the price of the good.

**5. The following table shows the demand curve facing a monopolist who produces at a constant marginal cost of \$10.**

Price	Quantity
27	0
24	2
21	4
18	6
15	8
12	10
9	12
6	14
3	16
0	18

**a. Calculate the firm's marginal revenue curve.**

To find the marginal revenue curve, we first derive the inverse demand curve. The intercept of the inverse demand curve on the price axis is 27. The slope of the inverse demand curve is the change in price divided by the change in quantity. For example, a decrease in price from 27 to 24 yields an increase in quantity from 0 to 2. Therefore, the slope is  $-\frac{3}{2}$  and the demand curve is

$$P = 27 - 1.5Q.$$

The marginal revenue curve corresponding to a linear demand curve is a line with the same intercept as the inverse demand curve and a slope that is twice as steep. Therefore, the marginal revenue curve is

$$MR = 27 - 3Q.$$

**b. What are the firm's profit-maximizing output and price? What is its profit?**

The monopolist's maximizing output occurs where marginal revenue equals marginal cost. Marginal cost is a constant \$10. Setting  $MR$  equal to  $MC$  to determine the profit-maximizing quantity:

$$27 - 3Q = 10, \text{ or } Q = 5.67.$$

To find the profit-maximizing price, substitute this quantity into the demand equation:

$$P = 27 - (1.5)(5.67) = \$18.5.$$

Total revenue is price times quantity:

$$TR = (18.5)(5.67) = \$104.83.$$

The profit of the firm is total revenue minus total cost, and total cost is equal to average cost times the level of output produced. Since marginal cost is constant, average variable cost is equal to marginal cost. Ignoring any fixed costs, total cost is  $10Q$  or  $56.67$ , and profit is

$$104.83 - 56.67 = \$48.17.$$

**c. What would the equilibrium price and quantity be in a competitive industry?**

For a competitive industry, price would equal marginal cost at equilibrium. Setting the expression for price equal to a marginal cost of 10:

$$27 - 1.5Q = 10 \Rightarrow Q = 11.3 \Rightarrow P = 10.$$

Note the increase in the equilibrium quantity compared to the monopoly solution.

**d. What would the social gain be if this monopolist were forced to produce and price at the competitive equilibrium? Who would gain and lose as a result?**

The social gain arises from the elimination of deadweight loss. Deadweight loss in this case is equal to the triangle above the constant marginal cost curve, below the demand curve, and between the quantities 5.67 and 11.3, or numerically

$$(18.5 - 10)(11.3 - 5.67)(.5) = \$24.10.$$

Consumers gain this deadweight loss plus the monopolist's profit of \$48.17. The monopolist's profits are reduced to zero, and the consumer surplus increases by \$72.27.

**6. A firm has two factories for which costs are given by:**

$$\text{Factory \# 1: } C_1(Q_1) = 10Q_1^2$$

$$\text{Factory \# 2: } C_2(Q_2) = 20Q_2^2$$

**The firm faces the following demand curve:**

$$P = 700 - 5Q$$

where  $Q$  is total output, i.e.  $Q = Q_1 + Q_2$ .

- a. On a diagram, draw the marginal cost curves for the two factories, the average and marginal revenue curves, and the total marginal cost curve (i.e., the marginal cost of producing  $Q = Q_1 + Q_2$ ). Indicate the profit-maximizing output for each factory, total output, and price.

The average revenue curve is the demand curve,

$$P = 700 - 5Q.$$

For a linear demand curve, the marginal revenue curve has the same intercept as the demand curve and a slope that is twice as steep:

$$MR = 700 - 10Q.$$

Next, determine the marginal cost of producing  $Q$ . To find the marginal cost of production in Factory 1, take the first derivative of the cost function with respect to  $Q$ :

$$\frac{dC_1(Q_1)}{dQ} = 20 Q_1.$$

Similarly, the marginal cost in Factory 2 is

$$\frac{dC_2(Q_2)}{dQ} = 40 Q_2.$$

Rearranging the marginal cost equations in inverse form and horizontally summing them, we obtain total marginal cost,  $MC_T$ :

$$Q = Q_1 + Q_2 = \frac{MC_1}{20} + \frac{MC_2}{40} = \frac{3MC_T}{40}, \text{ or}$$

$$MC_T = \frac{40Q}{3}.$$

Profit maximization occurs where  $MC_T = MR$ . See Figure 10.6.a for the profit-maximizing output for each factory, total output, and price.

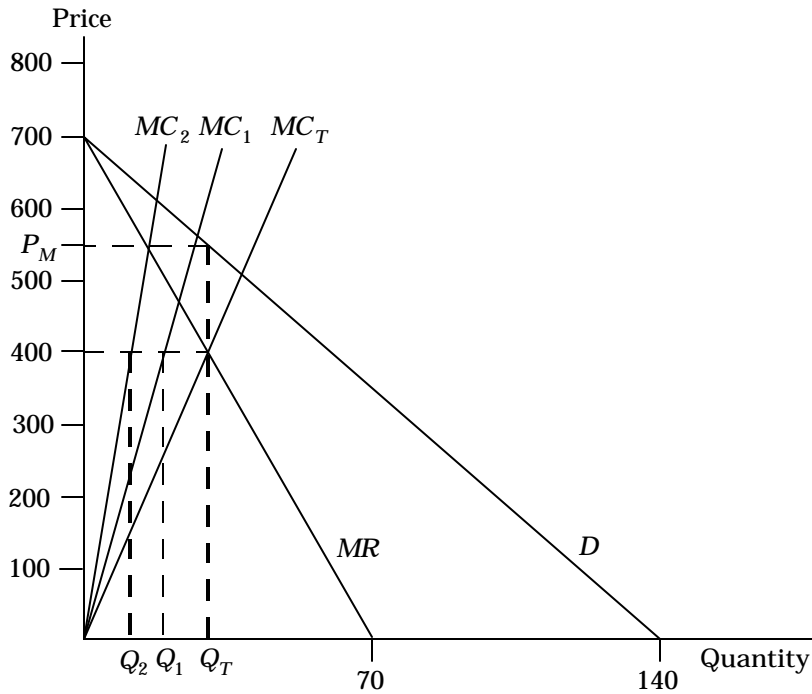


Figure 10.6.a

**b. Calculate the values of  $Q_1$ ,  $Q_2$ ,  $Q$ , and  $P$  that maximize profit.**

Calculate the total output that maximizes profit, i.e.,  $Q$  such that  $MC_T = MR$ :

$$\frac{40Q}{3} = 700 - 10Q, \text{ or } Q = 30.$$

Next, observe the relationship between  $MC$  and  $MR$  for multiplant monopolies:

$$MR = MC_T = MC_1 = MC_2.$$

We know that at  $Q = 30$ ,  $MR = 700 - (10)(30) = 400$ .

Therefore,

$$MC_1 = 400 = 20Q_1, \text{ or } Q_1 = 20 \text{ and}$$

$$MC_2 = 400 = 40Q_2, \text{ or } Q_2 = 10.$$

To find the monopoly price,  $P_M$ , substitute for  $Q$  in the demand equation:

$$P_M = 700 - (5)(30), \text{ or}$$

$$P_M = 550.$$

**c. Suppose labor costs increase in Factory 1 but not in Factory 2. How should the firm adjust the following (i.e., raise, lower, or leave unchanged): Output in Factory 1? Output in Factory 2? Total output? Price?**

An increase in labor costs will lead to a horizontal shift to the left in  $MC_1$ , causing  $MC_T$  to shift to the left as well (since it is the horizontal sum of  $MC_1$  and  $MC_2$ ). The new  $MC_T$  curve intersects the  $MR$  curve at a lower quantity and higher marginal revenue. At a higher level of marginal revenue,  $Q_2$  is greater than at the original level for  $MR$ . Since  $Q_T$  falls and  $Q_2$  rises,  $Q_1$  must fall. Since  $Q_T$  falls, price must rise.

**7. A drug company has a monopoly on a new patented medicine. The product can be made in either of two plants. The costs of production for the two plants are**

**$MC_1 = 20 + 2Q_1$ , and  $MC_2 = 10 + 5Q_2$ . The firm's estimate of the demand for the product is**

**$P = 20 - 3(Q_1 + Q_2)$ . How much should the firm plan to produce in each plant, and at what price should it plan to sell the product?**

First, notice that *only*  $MC_2$  is relevant because the marginal cost curve of the first plant lies above the demand curve.

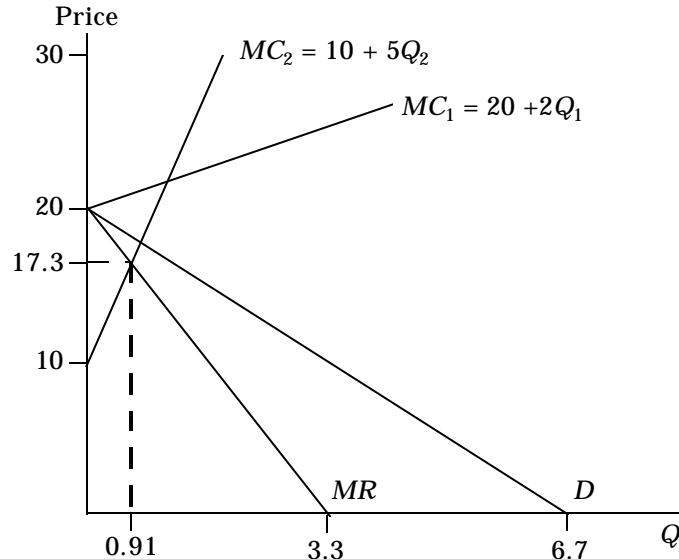


Figure 10.7

This means that the demand curve becomes  $P = 20 - 3Q_2$ . With an inverse linear demand curve, we know that the marginal revenue curve has the same vertical intercept but twice the slope, or  $MR = 20 - 6Q_2$ . To determine the profit-maximizing level of output, equate  $MR$  and  $MC_2$ :

$$20 - 6Q_2 = 10 + 5Q_2, \text{ or}$$

$$Q = Q_2 = 0.91.$$

Price is determined by substituting the profit-maximizing quantity into the demand equation:

$$P = 20 - 3(0.91) = 17.3.$$

**9. A monopolist faces the demand curve  $P = 11 - Q$ , where  $P$  is measured in dollars per unit and  $Q$  in thousands of units. The monopolist has a constant average cost of \$6 per unit.**

**a. Draw the average and marginal revenue curves and the average and marginal cost curves. What are the monopolist's profit-maximizing price and quantity? What is the resulting profit? Calculate the firm's degree of monopoly power using the Lerner index.**

Because demand (average revenue) may be described as  $P = 11 - Q$ , we know that the marginal revenue function is  $MR = 11 - 2Q$ . We also know that if average cost is constant, then marginal cost is constant and equal to average cost:  $MC = 6$ .

To find the profit-maximizing level of output, set marginal revenue equal to marginal cost:

$$11 - 2Q = 6, \text{ or } Q = 2.5.$$

That is, the profit-maximizing quantity equals 2,500 units. Substitute the profit-maximizing quantity into the demand equation to determine the price:

$$P = 11 - 2.5 = \$8.50.$$

Profits are equal to total revenue minus total cost,

$$\begin{aligned}\pi &= TR - TC = (AR)(Q) - (AC)(Q), \text{ or} \\ \pi &= (8.5)(2.5) - (6)(2.5) = 6.25, \text{ or } \$6,250.\end{aligned}$$

The degree of monopoly power is given by the Lerner Index:

$$\frac{P - MC}{P} = \frac{8.5 - 6}{8.5} = 0.294.$$

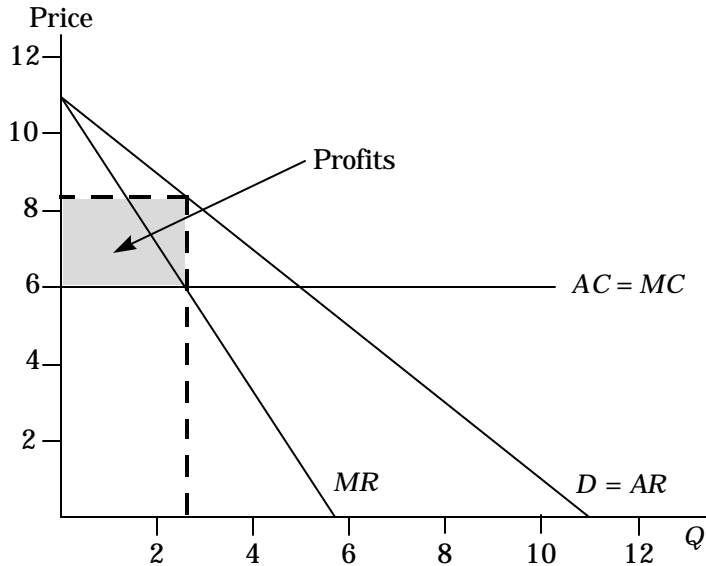


Figure 10.9.a

- b. A government regulatory agency sets a price ceiling of \$7 per unit. What quantity will be produced, and what will the firm's profit be? What happens to the degree of monopoly power?**

To determine the effect of the price ceiling on the quantity produced, substitute the ceiling price into the demand equation.

$$\begin{aligned}7 &= 11 - Q, \text{ or} \\ Q &= 4,000.\end{aligned}$$

The monopolist will pick the price of \$7 because it is the highest price that it can charge, and this price is still greater than the constant marginal cost of \$6, resulting in positive monopoly profit.

Profits are equal to total revenue minus total cost:

$$\pi = (7)(4,000) - (6)(4,000) = \$4,000.$$

The degree of monopoly power is:

$$\frac{P - MC}{P} = \frac{7 - 6}{7} = 0.143.$$

- c. What price ceiling yields the largest level of output? What is that level of output? What is the firm's degree of monopoly power at this price?**

If the regulatory authority sets a price below \$6, the monopolist would prefer to go out of business instead of produce because it cannot cover its average costs. At any price above \$6, the monopolist would produce less than the 5,000 units that would be produced in a

competitive industry. Therefore, the regulatory agency should set a price ceiling of \$6, thus making the monopolist face a horizontal effective demand curve up to  $Q = 5,000$ . To ensure a positive output (so that the monopolist is not indifferent between producing 5,000 units and shutting down), the price ceiling should be set at  $\$6 + \delta$  where  $\delta$  is small.

Thus, 5,000 is the maximum output that the regulatory agency can extract from the monopolist by using a price ceiling. The degree of monopoly power is

$$\frac{P - MC}{P} = \frac{6 + d - 6}{6} = \frac{d}{6} \rightarrow 0 \text{ as } \delta \rightarrow 0.$$

**10. Michelle's Monopoly Mutant Turtles (MMMT) has the exclusive right to sell Mutant Turtle t-shirts in the United States. The demand for these t-shirts is  $Q = 10,000/P^2$ . The firm's short-run cost is  $SRTC = 2,000 + 5Q$ , and its long-run cost is  $LRTC = 6Q$ .**

**a. What price should MMMT charge to maximize profit in the short run? What quantity does it sell, and how much profit does it make? Would it be better off shutting down in the short run?**

MMMT should offer enough t-shirts such that  $MR = MC$ . In the short run, marginal cost is the change in  $SRTC$  as the result of the production of another tshirt, i.e.,  $SRMC = 5$ , the slope of the  $SRTC$  curve. Demand is:

$$Q = \frac{10,000}{P^2},$$

or, in inverse form,

$$P = 100Q^{-1/2}.$$

Total revenue ( $PQ$ ) is  $100Q^{1/2}$ . Taking the derivative of  $TR$  with respect to  $Q$ ,  $MR = 50Q^{-1/2}$ . Equating  $MR$  and  $MC$  to determine the profit-maximizing quantity:

$$5 = 50Q^{-1/2}, \text{ or } Q = 100.$$

Substituting  $Q = 100$  into the demand function to determine price:

$$P = (100)(100^{-1/2}) = 10.$$

The profit at this price and quantity is equal to total revenue minus total cost:

$$\pi = (10)(100) - (2000 + (5)(100)) = -\$1,500.$$

Although profit is negative, price is above the average variable cost of 5 and therefore, the firm should not shut down in the short run. Since most of the firm's costs are fixed, the firm loses \$2,000 if nothing is produced. If the profit-maximizing quantity is produced, the firm loses only \$1,500.

**b. What price should MMMT charge in the long run? What quantity does it sell and how much profit does it make? Would it be better off shutting down in the long run?**

In the long run, marginal cost is equal to the slope of the  $LRTC$  curve, which is 6.

Equating marginal revenue and long run marginal cost to determine the profit-maximizing quantity:

$$50Q^{-1/2} = 6 \text{ or } Q = 69.44$$

Substituting  $Q = 69.44$  into the demand equation to determine price:

$$P = (100)[(50/6)^2]^{-1/2} = (100)(6/50) = 12$$

Therefore, total revenue is \$833.33 and total cost is \$416.67. Profit is \$416.67. The firm should remain in business.



- c. **Can we expect MMT to have lower marginal cost in the short run than in the long run? Explain why.**

In the long run, MMT must replace all fixed factors. Therefore, we can expect *LRMC* to be higher than *SRMC*.

12. **The employment of teaching assistants (TAs) by major universities can be characterized as a monopsony. Suppose the demand for TAs is  $W = 30,000 - 125n$ , where  $W$  is the wage (as an annual salary), and  $n$  is the number of TAs hired. The supply of TAs is given by  $W = 1,000 + 75n$ .**

- a. **If the university takes advantage of its monopsonist position, how many TAs will it hire? What wage will it pay?**

The supply curve is equivalent to the average expenditure curve. With a supply curve of  $W = 1,000 + 75n$ , the total expenditure is  $Wn = 1,000n + 75n^2$ . Taking the derivative of the total expenditure function with respect to the number of TAs, the marginal expenditure curve is  $1,000 + 150n$ . As a monopsonist, the university would equate marginal value (demand) with marginal expenditure to determine the number of TAs to hire:

$$30,000 - 125n = 1,000 + 150n, \text{ or} \\ n = 105.5.$$

Substituting  $n = 105.5$  into the supply curve to determine the wage:

$$1,000 + (75)(105.5) = \$8,909 \text{ annually.}$$

- b. **If, instead, the university faced an infinite supply of TAs at the annual wage level of \$10,000, how many TAs would it hire?**

With an infinite number of TAs at \$10,000, the supply curve is horizontal at \$10,000. Total expenditure is  $(10,000)(n)$ , and marginal expenditure is 10,000. Equating marginal value and marginal expenditure:

$$30,000 - 125n = 10,000, \text{ or} \\ n = 160.$$

13. **Dayna's Doorstops, Inc. (DD), is a monopolist in the doorstep industry. Its cost is  $C = 100 - 5Q + Q^2$ , and demand is  $P = 55 - 2Q$ .**

- a. **What price should DD set to maximize profit? What output does the firm produce? How much profit and consumer surplus does DD generate?**

To maximize profits, DD should equate marginal revenue and marginal cost. Given a demand of  $P = 55 - 2Q$ , we know that total revenue,  $PQ$ , is  $55Q - 2Q^2$ . Marginal revenue is found by taking the first derivative of total revenue with respect to  $Q$  or:

$$MR = \frac{dTR}{dQ} = 55 - 4Q.$$

Similarly, marginal cost is determined by taking the first derivative of the total cost function with respect to  $Q$  or:

$$MC = \frac{dTC}{dQ} = 2Q - 5.$$

Equating  $MC$  and  $MR$  to determine the profit-maximizing quantity,

$$55 - 4Q = 2Q - 5, \text{ or}$$

$$Q = 10.$$

Substituting  $Q = 10$  into the demand equation to determine the profit-maximizing price:

$$P = 55 - (2)(10) = \$35.$$

Profits are equal to total revenue minus total cost:

$$\pi = (35)(10) - (100 - (5)(10) + 10^2) = \$200.$$

Consumer surplus is equal to one-half times the profit-maximizing quantity, 10, times the difference between the demand intercept (the maximum price anyone is willing to pay) and the monopoly price:

$$CS = (0.5)(10)(55 - 35) = \$100.$$

**b. What would output be if DD acted like a perfect competitor and set  $MC = P$ ? What profit and consumer surplus would then be generated?**

In competition, profits are maximized at the point where price equals marginal cost, where price is given by the demand curve:

$$55 - 2Q = -5 + 2Q, \text{ or}$$

$$Q = 15.$$

Substituting  $Q = 15$  into the demand equation to determine the price:

$$P = 55 - (2)(15) = \$25.$$

Profits are total revenue minus total cost or:

$$\pi = (25)(15) - (100 - (5)(15) + 15^2) = \$125.$$

Consumer surplus is

$$CS = (0.5)(55 - 25)(15) = \$225.$$

**c. What is the deadweight loss from monopoly power in part (a)?**

The deadweight loss is equal to the area below the demand curve, above the marginal cost curve, and between the quantities of 10 and 15, or numerically

$$DWL = (0.5)(35 - 15)(15 - 10) = \$50.$$

**d. Suppose the government, concerned about the high price of doorstops, sets a maximum price at \$27. How does this affect price, quantity, consumer surplus, and DD's profit? What is the resulting deadweight loss?**

With the imposition of a price ceiling, the maximum price that DD may charge is \$27.00. Note that when a ceiling price is set above the competitive price the ceiling price is equal to marginal revenue for all levels of output sold up to the competitive level of output.

Substitute the ceiling price of \$27.00 into the demand equation to determine the effect on the equilibrium quantity sold:

$$27 = 55 - 2Q, \text{ or } Q = 14.$$

Consumer surplus is

$$CS = (0.5)(55 - 27)(14) = \$196.$$

Profits are

$$\pi = (27)(14) - (100 - (5)(14) + 14^2) = \$152.$$

The deadweight loss is \$2.00 This is equivalent to a triangle of

$$(0.5)(15 - 14)(27 - 23) = \$2$$

**e. Now suppose the government sets the maximum price at \$23. How does this affect price, quantity, consumer surplus, DD's profit, and deadweight loss?**

With a ceiling price set below the competitive price, DD will decrease its output. Equate marginal revenue and marginal cost to determine the profit-maximizing level of output:

$$23 = -5 + 2Q, \text{ or } Q = 14.$$

With the government-imposed maximum price of \$23, profits are

$$\pi = (23)(14) - (100 - (5)(14) + 14^2) = \$96.$$

Consumer surplus is realized on only 14 doorsteps. Therefore, it is equal to the consumer surplus in part d., i.e. \$196, plus the savings on each doorstep, i.e.,

$$CS = (27 - 23)(14) = \$56.$$

Therefore, consumer surplus is \$252. Deadweight loss is the same as before, \$2.00.

**f. Finally, consider a maximum price of \$12. What will this do to quantity, consumer surplus, profit, and deadweight loss?**

With a maximum price of only \$12, output decreases even further:

$$12 = -5 + 2Q, \text{ or } Q = 8.5.$$

Profits are

$$\pi = (12)(8.5) - (100 - (5)(8.5) + 8.5^2) = -\$27.75.$$

Consumer surplus is realized on only 8.5 units, which is equivalent to the consumer surplus associated with a price of \$38 ( $38 = 55 - 2(8.5)$ ), i.e.,

$$(0.5)(55 - 38)(8.5) = \$72.25$$

plus the savings on each doorstep, i.e.,

$$(38 - 12)(8.5) = \$221.$$

Therefore, consumer surplus is \$293.25. Total surplus is \$265.50, and deadweight loss is \$84.50.

**15. A monopolist faces the following demand curve:**

$$Q = 144/P^2$$

**where Q is the quantity demanded and P is price. Its average variable cost is**

$$AVC = Q^{1/2},$$

**and its fixed cost is 5.**

**a. What are its profit-maximizing price and quantity? What is the resulting profit?**

The monopolist wants to choose the level of output to maximize its profits, and it does this by setting marginal revenue equal to marginal cost. To find marginal revenue, first rewrite the demand function as a function of Q so that you can then express total revenue as a function of Q, and calculate marginal revenue:

$$Q = \frac{144}{P^2} \Rightarrow P^2 = \frac{144}{Q} \Rightarrow P = \sqrt{\frac{144}{Q}} = \frac{12}{\sqrt{Q}}$$

$$R = P * Q = \frac{12}{\sqrt{Q}} * Q = 12 \cdot \sqrt{Q}$$

$$MR = \frac{\Delta R}{\Delta Q} = 0.5 * \frac{12}{\sqrt{Q}} = \frac{6}{\sqrt{Q}}.$$

To find marginal cost, first find total cost, which is equal to fixed cost plus variable cost. You are given fixed cost of 5. Variable cost is equal to average variable cost times Q so that total cost and marginal cost are:

$$TC = 5 + Q * Q^{\frac{1}{2}} = 5 + Q^{\frac{3}{2}}$$

$$MC = \frac{\Delta TC}{\Delta Q} = \frac{3\sqrt{Q}}{2}$$

To find the profit-maximizing level of output, we set marginal revenue equal to marginal cost:

$$\frac{6}{\sqrt{Q}} = \frac{3\sqrt{Q}}{2} \Rightarrow Q = 4.$$

You can now find price and profit:

$$P = \frac{12}{\sqrt{Q}} = \frac{12}{\sqrt{4}} = \$6$$

$$\Pi = PQ - TC = 6 * 4 - (5 + 4^{\frac{3}{2}}) = \$11.$$

**b. Suppose the government regulates the price to be no greater than \$4 per unit. How much will the monopolist produce? What will its profit be?**

The price ceiling truncates the demand curve that the monopolist faces at  $P=4$  or  $Q = \frac{144}{16} = 9$ . Therefore, if the monopolist produces 9 units or less, the price must be

\$4. Because of the regulation, the demand curve now has two parts:

$$P = \begin{cases} \$4, & \text{if } Q \leq 9 \\ 12 Q^{-1/2}, & \text{if } Q > 9. \end{cases}$$

Thus, total revenue and marginal revenue also should be considered in two parts

$$TR = \begin{cases} 4Q, & \text{if } Q \leq 9 \\ 12 Q^{1/2}, & \text{if } Q > 9 \end{cases} \text{ and}$$

$$MR = \begin{cases} \$4, & \text{if } Q \leq 9 \\ 6Q^{-1/2}, & \text{if } Q > 9. \end{cases}$$

To find the profit-maximizing level of output, set marginal revenue equal to marginal cost, so that for  $P = 4$ ,

$$4 = \frac{3}{2}\sqrt{Q}, \text{ or } \sqrt{Q} = \frac{8}{3}, \text{ or } Q = 7.11.$$

If the monopolist produces an integer number of units, the profit-maximizing production level is 7 units, price is \$4, revenue is \$28, total cost is \$23.52, and profit is \$4.48. There is a shortage of two units, since the quantity demanded at the price of \$4 is 9 units.

**c. Suppose the government wants to set a ceiling price that induces the monopolist to produce the largest possible output. What price will do this?**

To maximize output, the regulated price should be set so that demand equals marginal cost, which implies;

$$\frac{12}{\sqrt{Q}} = \frac{3\sqrt{Q}}{2} \Rightarrow Q = 8 \text{ and } P = \$4.24.$$

The regulated price becomes the monopolist's marginal revenue curve, which is a horizontal line with an intercept at the regulated price. To maximize profit, the firm produces where marginal cost is equal to marginal revenue, which results in a quantity of 8 units.