

CHAPTER 7 THE COST OF PRODUCTION

EXERCISES

1. Assume a computer firm's marginal costs of production are constant at \$1,000 per computer. However, the fixed costs of production are equal to \$10,000.

a. Calculate the firm's average variable cost and average total cost curves.

The variable cost of producing an additional unit, marginal cost, is constant at \$1,000, so $VC = \$1000Q$, and $AVC = \frac{VC}{Q} = \frac{\$1,000Q}{Q} = \$1,000$. Average fixed cost is

$\frac{\$10,000}{Q}$. Average total cost is the sum of average variable cost and average fixed cost:

$$ATC = \$1,000 + \frac{\$10,000}{Q}.$$

b. If the firm wanted to minimize the average total cost of production, would it choose to be very large or very small? Explain.

The firm should choose a very large output because average total cost decreases with increase in Q . As Q becomes infinitely large, ATC will equal \$1,000.

5. A chair manufacturer hires its assembly-line labor for \$22 an hour and calculates that the rental cost of its machinery is \$110 per hour. Suppose that a chair can be produced using 4 hours of labor or machinery in any combination. If the firm is currently using 3 hours of labor for each hour of machine time, is it minimizing its costs of production? If so, why? If not, how can it improve the situation?

If the firm can produce one chair with either four hours of labor or four hours of capital, machinery, or any combination, then the isoquant is a straight line with a slope of -1 and intercept at $K = 4$ and $L = 4$, as depicted in Figure 7.5.

The isocost line, $TC = 22L + 110K$ has a slope of $-\frac{22}{110} = -0.2$ when plotted with capital on the vertical axis and has intercepts at $K = \frac{TC}{110}$ and $L = \frac{TC}{22}$. The cost minimizing point is a corner solution, where $L = 4$ and $K = 0$. At that point, total cost is \$88.

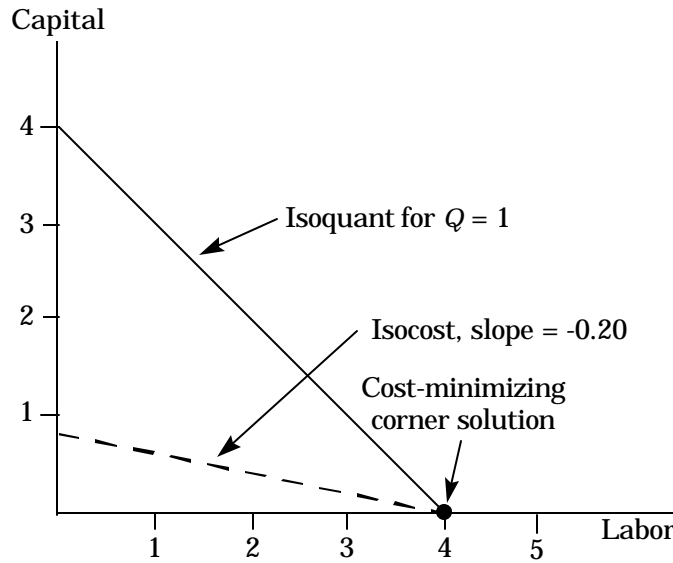


Figure 7.5

6. Suppose the economy takes a downturn, and that labor costs fall by 50 percent and are expected to stay at that level for a long time. Show graphically how this change in the relative price of labor and capital affects the firm's expansion path.

Figure 7.6 shows a family of isoquants and two isocost curves. Units of capital are on the vertical axis and units of labor are on the horizontal axis. (Note: In drawing this figure we have assumed that the production function underlying the isoquants exhibits constant returns to scale, resulting in linear expansion paths. However, the results do not depend on this assumption.)

If the price of labor decreases while the price of capital is constant, the isocost curve pivots outward around its intersection with the capital axis. Because the expansion path is the set of points where the *MRTS* is equal to the ratio of prices, as the isocost curves pivot outward, the expansion path pivots toward the labor axis. As the price of labor falls relative to capital, the firm uses more labor as output increases.

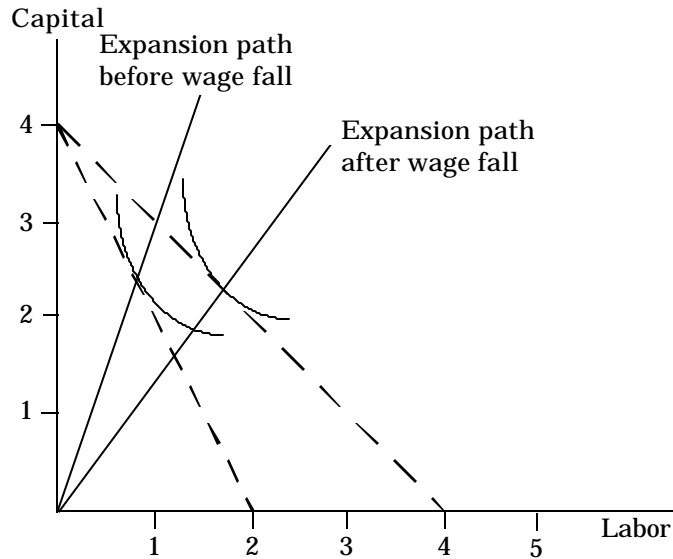


Figure 7.6

11. The short-run cost function of a company is given by the equation $C=190+53Q$, where C is the total cost and Q is the total quantity of output, both measured in tens of thousands.

a. What is the company's fixed cost?

When $Q = 0$, $C = 190$, so fixed cost is equal to 190 (or \$1,900,000).

b. If the company produced 100,000 units of goods, what is its average variable cost?

With 100,000 units, $Q = 10$. Variable cost is $53Q = (53)(10) = 530$ (or \$5,300,000).

Average variable cost is $\frac{TVC}{Q} = \frac{\$530}{10} = \53 . or \$530,000.

c. What is its marginal cost per unit produced?

With constant average variable cost, marginal cost is equal to average variable cost, \$53 (or \$530,000).

d. What is its average fixed cost?

At $Q = 10$, average fixed cost is $\frac{TFC}{Q} = \frac{\$190}{10} = \19 or (\$190,000).

CHAPTER 7

PRODUCTION AND COST THEORY— A MATHEMATICAL TREATMENT

EXERCISES

1. Of the following production functions, which exhibit increasing, constant, or decreasing returns to scale?

- a. $F(K, L) = K^2 L$
 b. $F(K, L) = 10K + 5L$
 c. $F(K, L) = (KL)^{0.5}$

Returns to scale refer to the relationship between output and proportional increases in all inputs. This is represented in the following manner:

$F(\lambda K, \lambda L) > \lambda F(K, L)$ implies increasing returns to scale;

$F(\lambda K, \lambda L) = \lambda F(K, L)$ implies constant returns to scale; and

$F(\lambda K, \lambda L) < \lambda F(K, L)$ implies decreasing returns to scale.

a. Applying this to $F(K, L) = K^2 L$,

$$F(\lambda K, \lambda L) = (\lambda K)^2 (\lambda L) = \lambda^3 K^2 L = \lambda^3 F(K, L).$$

This is greater than $\lambda F(K, L)$; therefore, this production function exhibits increasing returns to scale.

b. Applying the same technique to $F(K, L) = 10K + 5L$,

$$F(\lambda K, \lambda L) = 10\lambda K + 5\lambda L = \lambda F(K, L).$$

This production function exhibits constant returns to scale.

c. Applying the same technique to $F(K, L) = (KL)^{0.5}$,

$$F(\lambda K, \lambda L) = (\lambda K \lambda L)^{0.5} = (\lambda^2)^{0.5} (KL)^{0.5} = \lambda (KL)^{0.5} = \lambda F(K, L).$$

This production function exhibits constant returns to scale.

2. The production function for a product is given by $Q = 100KL$. If the price of capital is \$120 per day and the price of labor \$30 per day, what is the minimum cost of producing 1000 units of output?

The cost-minimizing combination of capital and labor is the one where

$$MRTS = \frac{MP_L}{MP_K} = \frac{w}{r}.$$

The marginal product of labor is $\frac{dQ}{dL} = 100K$. The marginal product of capital is

$\frac{dQ}{dK} = 100L$. Therefore, the marginal rate of technical substitution is

$$\frac{100K}{100L} = \frac{K}{L}.$$

To determine the optimal capital-labor ratio set the marginal rate of technical substitution equal to the ratio of the wage rate to the rental rate of capital:

$$\frac{K}{L} = \frac{30}{120}, \text{ or } L = 4K.$$

Substitute for L in the production function and solve where K yields an output of 1,000 units:

$$1,000 = (100)(K)(4K), \text{ or } K = 1.58.$$

Because L equals $4K$ this means L equals 6.32.

With these levels of the two inputs, total cost is:

$$TC = wL + rK, \text{ or}$$

$$TC = (30)(6.32) + (120)(1.58) = \$379.20.$$

To see if $K = 1.58$ and $L = 6.32$ are the cost minimizing levels of inputs, consider small changes in K and L . around 1.58 and 6.32. At $K = 1.6$ and $L = 6.32$, total cost is \$381.60, and at $K = 1.58$ and $L = 6.4$, total cost is \$381.6, both greater than \$379.20. We have found the cost-minimizing levels of K and L .

3. Suppose a production function is given by $F(K, L) = KL^2$, the price of capital is \$10, and the price of labor \$15. What combination of labor and capital minimizes the cost of producing any given output?

The cost-minimizing combination of capital and labor is the one where

$$MRTS = \frac{MP_L}{MP_K} = \frac{w}{r}.$$

The marginal product of labor is $\frac{dQ}{dL} = 2KL$. The marginal product of capital is

$$\frac{dQ}{dK} = L^2.$$

Set the marginal rate of technical substitution equal to the input price ratio to determine the optimal capital-labor ratio:

$$\frac{2KL}{L^2} = \frac{15}{10}, \text{ or } K = 0.75L.$$

Therefore, the capital-labor ratio should be 0.75 to minimize the cost of producing any given output.