Miscible density-unstable displacement flows in inclined tube

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We study the displacement flow of two Newtonian fluids in an inclined pipe. The fluids have the same viscosity but different densities. The displacing fluid is denser than the displaced fluid and is placed above the displaced fluid (i.e., a density-unstable configuration). Three dimensionless groups describe these flows: a densimetric Froude number \( F_r \), a Reynolds number \( R_e \), and the pipe inclination \( \beta \). Our experiments cover fairly broad ranges of these parameters: \( 0 \leq F_r \leq 9 \); \( 0 \leq R_e \leq 2400 \); \( 0 \leq \beta \leq 85^\circ \). Phenomenologically, our experimental flow observations vary from well mixed fully diffusive regimes, through buoyancy-dominated inertial exchange regimes, to laminar viscous flows, all with varying degrees of stability. We characterize the different flow regimes observed in terms of the three dimensionless groups and provide leading order approximations to the velocity of the displacement front and the macroscopic diffusion in each regime.

I. INTRODUCTION

We study the effect of inclination angle and flow rate on the density-unstable pipe displacement flow of one miscible fluid by another of the same viscosity. By density-unstable we mean that the denser fluid displaces the lighter fluid as the fluids are pumped in the downwards direction. Our experiments are performed in a long pipe (approximately 200 diameters), but at flow rates for which the Péclet number is significantly larger than the ratio of length to diameter. In this regime, although the fluids are miscible, they do not have time to mix over experimental timescales in the absence of hydrodynamic effects (which are very present here). Our study represents a continuation of our previous work on density unstable displacement flows; see Refs. 1–4, which has been primarily focused at flows in ducts that are close to horizontal (\( \beta \approx 90^\circ \)). At large inclinations stratified viscous regimes are a dominant flow feature, but at lower inclinations inertial effects become increasingly significant, eventually resulting in instability and effective transverse mixing. Thus, inertial transitional and fully diffusive regimes occur.

As well as an extension of our previous work to progressively vertical inclinations, our study may be interpreted as an extension of exchange flow studies into the range of displacement flows, i.e., by adding a mean imposed flow in the downwards direction. For the exchange flow the only driving force is buoyancy. Debaq et al.\(^{5,6}\) and later Seon et al.\(^{7-11}\) experimentally investigated the exchange flow of two miscible fluids in vertical and inclined pipes, respectively; see also Ref. 12. We highlight these studies as they have been carried out with similar fluids and in a similar scale of apparatus. Depending on the flow parameters, fully diffusive, transitional, inertial, and viscous flows

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can appear. The transition between diffusive/non-diffusive regimes in a vertical pipe ($\beta = 0$) was found to be a function of density difference; see Ref. 5. Debacq et al. showed that diffusive flows could be characterized by a macroscopic diffusion coefficient, $D_M$, which could be up to $10^5$ times bigger than the molecular diffusivity. They then characterized the macroscopic diffusion coefficient and the velocity of the inter-penetrating exchange flow fronts ($\dot{V}_f$) as a function of density contrast, fluid viscosities, and tube diameter; see Ref. 6. This approach was extended to inclined pipes ($\beta > 0$) by Seon et al. who found the boundary between diffusive/non-diffusive flows and investigated the dependency of $D_M$ on the dimensionless governing parameters. They showed that the transition between diffusive/non-diffusive flows is a function of $\beta$ that increases with density contrast and decreases with fluid viscosity. Increasing the fluid viscosities coarsens the mixing zones within the flow. This, in turn, increases local density contrast and buoyant forces which finally yields to higher front velocity, $\dot{V}_f$, and less mixing. In the current study we also look for the boundary between diffusive/non-diffusive flows, but now in the presence of an imposed flow, added to the exchange flows. We will find that this boundary is not only a function of density difference and inclination angle but also a function of mean flow speed, through the relevant dimensionless numbers.

Later on, Seon et al. focused on the front dynamics of the exchange flows in tilted geometries by analyzing front velocity measurements ($\dot{V}_f$). They observed three different regimes: starting from vertical, $\dot{V}_f$ first increased with $\beta$, reached a plateau and then decreased again. In the first regime, the interplay between fluid layer segregation and mixing dictates $\dot{V}_f$. The segregation effect increases with $\beta$, while the degree of mixing decreases with $\beta$. In the plateau regime, $\dot{V}_f$ is not dependent on the fluid viscosity and is proportional to $\dot{V}_t$, a velocity scale obtained by the balance between inertia and buoyancy ($\dot{V}_t = \sqrt{At\dot{g}D}$, where $At$ is the Atwood number and $\dot{g}$ and $D$ are gravitational acceleration and tube diameter, respectively. In the third regime, the inclination angle is close to horizontal, the fluid layers separate into two counter current streams which are almost parallel to one another. This regime is controlled by viscosity and the relevant velocity scale is $\dot{V}_v = (At\dot{g}D^3)/\nu$: obtained from the balance between buoyant and viscous forces. Here $\nu$ is the kinematic viscosity obtained from the common viscosity and average density of the fluids.

Seon et al. studied the plateau regime further using a Laser Induced Florescence (LIF) technique, finding that the front velocity in the plateau regime is a function of the local fluid concentration at the front ($C_f \in [0, 1]$), in the form $\dot{V}_f \propto C_f^0.2$. Later, focusing only on diffusive exchange flows, Seon et al. found that mixing can have two different mechanisms depending on the value of $Re_t$ ($= \dot{V}_t D/\nu$), a Reynolds number defined based on the inertial velocity, $\dot{V}_t$. For $Re_t \lesssim 1000$, the scaled diffusivity $D_M/\dot{V}_f D$ varies like $Re_t^{-3/2}$, whereas for $Re_t \gtrsim 1000$, $D_M/\dot{V}_f D$ decreases as $Re_t^{-1/2}$. This difference in behaviours was attributed to changes in the quality of the transverse mixing in different ranges of $Re_t$. The third regime, for nearly horizontal pipes, was studied in Ref. 10. It was found that the front velocity is initially limited by inertia but later controlled by viscous effects. This results in a decay of $\dot{V}_f$ to a steady state, over a time proportional to $\cos \beta$. These long-time flows can be either viscous or inertial, with the transition occurring at $Re_t \cos \beta \approx 50$. In pipe exchange flow simulations, non-zero radial and azimuthal components of velocity were found to govern the mixing mechanism. Although studying different mechanisms of mixing in the diffusive flow regime is not in the scope of the current work, we do characterize the boundaries of viscous/inertial regimes through the important parameter $Re_t \cos \beta$. Here we have imposed mean flow added to the buoyant exchange flows, however the underlying mechanism for the diffusive regimes are likely to be similar to those in exchange flows.

Because of the imposed mean flow and a natural comparison for dispersive spreading within the diffusive regime flows is with turbulent Taylor dispersion; see Ref. 14. In such flows the total axial diffusivity is the combination of the Taylor dispersion and the eddy diffusivity for turbulent flow. Tichacek et al. modified Taylor’s analysis to include the effect of both Schmidt and Reynolds numbers. They showed that the degree of axial spreading increases rapidly as the flow approaches the laminar regime. This lower range of Reynolds numbers is relevant to our experiments. At low $Re$ viscous sub-layers become relatively thick and can significantly affect axial dispersion. This is also a Reynolds number range in which calculated dispersivities will be
sensitive to the velocity close to matching layer, and in which the flow can have a high degree of intermittency.

Finally, considering viscous and inertial regimes at nearly horizontal inclinations, the effect of mean flow on the exchange flow of two miscible fluids has been studied extensively by Taghavi et al. in both 2D channels and pipes; see Refs. 1–4. The first study was based on an analytical model of thin-film/lubrication type; see Ref. 1. This gave simple predictions of front velocity, interface shape and displacement efficiency for channel flows of both Newtonian and non-Newtonian fluids. In general at long times, two displacement fronts resulted. The leading front displaced with speed larger than the mean velocity and the second (trailing) front moves slower. For sufficiently strong buoyancy forces, the trailing front moves back against the mean flow (a "backflow"). Although the current study is focused on pipe inclinations away from nearly horizontal, when the flow is viscous the same lubrication model is used to predict the front velocity as a comparison with experimental data. Experimental pipe displacement flows were studied in Ref. 2, identifying three main regimes as the mean flow velocity \( (V_0) \) was increased from zero. At low \( V_0 \) the flow resembles the exchange flows of Ref. 8. As mean flow is increased the front velocity \( \hat{V}_f \) was found to vary linearly with \( V_0 \) (with proportionality \( > 1 \)). Finally when the mean speed is further increased we enter the turbulent regime where \( \hat{V}_f = \hat{V}_0 \). They also found that the mean flow can counter-intuitively stabilize the flow through increasing the local gradient Richardson number, \( Ri \). In this case the backflow is reduced and the flow becomes stabilized. In the current study we will investigate whether or not the stabilizing effect of the mean flow is valid for other inclination angles. Taghavi et al.3 studied the behaviour of the trailing displacement front. They showed that above a critical ratio of \( \chi = 2Re \cos \beta / Fr^2 \) (representing the competition between buoyancy stresses in the axial direction and viscous stresses due to the mean flow), the secondary trailing front would advance backwards against the mean flow under the action of buoyancy. Here \( Fr = V_0 / \hat{V}_t \) is the densimetric Froude number. In the current study we have also studied the movement of the trailing front, in a sense of determining whether there exists a backflow or not. A unified perspective on iso-viscous nearly horizontal displacement flows is presented by Taghavi et al.,4 based on experimental, numerical and analytical results. This has resulted in a complete categorisation of flow types and front velocities, in terms of \( Fr \) and \( Re \cos \beta / Fr \), for large inclination angles \( \beta \approx 90^\circ \). One of the main contributions of the current study is to extend this flow type categorization over a wider range of inclination angles and thus \( Re \cos \beta / Fr \). The displacement flow experimental studies have been extended recently into density-stable configurations by Alba et al.,16 i.e., the light fluid is on top of the heavy fluid. They showed that the flow structure is much simpler than the density-unstable case and the displacement efficiency is very close to 100%, i.e., a complete removal of the displaced fluid. Using a fairly simple balance between buoyant and viscous stresses, they gave a dimensionless formula to predict the longitudinal extent of the interface between the two fluids based on the experimental data obtained over a wide rage of inclination angles, density differences, and fluid viscosities.

Our review above has been selective in covering those experimental (and associated modelling) studies where the fluids used are not particularly viscous. There are also a number of displacement flow studies in vertical pipes for which the flow is more structured, due to a combination of either large viscosity, stable density differences, or slow flows. For example, high Péclet number miscible displacements have been studied computationally by Chen and Meiburg,17 and experimentally by Petitjeans and Maxworthy.18 Downward displacement flow in a vertical Hele-Shaw cell was studied experimentally and theoretically by Lajunesse et al.19 Fluids were chosen to be miscible and Newtonian. Neglecting the diffusive effects, the dependence of the flow stability on viscosity ratio and flow rate value was investigated. Scoffioni et al.20 studied interfacial instabilities in a vertical pipe during displacements of two miscible fluids. Three flow types were found depending on the flow rate and viscosity ratio value: namely, a stable finger, axisymmetric, and corkscrew modes. Balasubramaniam et al.21 focused on interfacial instabilities in the experimental study of both upward and downward displacement of Newtonian fluids in pipe flow, with the displaced fluid being less viscous. Kuang et al.22 investigated displacement flows in vertical tubes when displaced fluid is denser and more viscous than the displacing fluid. Both upward and downward configurations were considered and Particle Image Velocimetry (PIV) measurement was used to get the velocity distribution and finger tip shape. Although the flows considered in the above studies are generally
far more structured than those here and/or exhibit symmetries not present here, the underlying mechanisms and dynamics are useful to understand the current displacement flow results better.

The significant novelty of our study is as follows. First, we know of no other experimental or numerical study of miscible displacement flow in pipes (non-zero imposed mean flow velocity, $\tilde{V}_0$) that covers a broad range of pipe inclinations in the density-unstable configuration. Second, we classify the different flow regimes observed in terms of measurements of the advancing front velocity and the degree of diffusive mixing. Third, we develop simple dimensionless relations that capture two important characteristics: namely, the front velocity and the macroscopic diffusion coefficient.

Below in Sec. II we introduce the experimental setup. In presenting our results we first benchmark against existing exchange flow results for $\tilde{V}_0 = 0$ and then (Sec. III A) discuss the main qualitative features of the displacement flows; i.e., $\tilde{V}_0 > 0$. Front velocities and macroscopic diffusion are studied in Secs. III B and III C, respectively. We give a quantitative characterisation of the different flow regimes in Sec. IV and the paper closes with a brief summary (Sec. V).

II. EXPERIMENTAL SETUP AND SCOPE OF STUDY

Our experiments have been carried out in a 4-m two-fluid flow loop capable of being tilted at any angle between horizontal and vertical via ball-screw jack; see Figure 1(a). The flow loop consists of a transparent acrylic pipe (diameter $\bar{D} = 19.05$ mm) mounted within two fish tanks to reduce the refraction errors and improve image quality. The pipe is divided into two parts, separated initially by an automated gate valve: 80 cm in the upper part and 320 cm in the lower part.

In an experiment the two fluids (both water) are initially filled above and below the gate valve. We add black dye (ink) to the displaced fluid in order to measure concentration via optical absorption. The dye does not change the fluid properties. For the experiments presented in this paper the less dense fluid is always the lower displaced fluid. Salt (NaCl) is added to the upper displacing fluid as a weighting agent. At the start of the experiment the gate valve is opened and the flow is driven by an over-pressure ($\approx 70$ kPa gauge pressure) applied by compressed air to a pressurized tank containing the displacing fluid. This ensures a smooth steady inflow. The flow rate, $\tilde{Q} = \pi \bar{D}^2 \tilde{V}_0 / 4$, is regulated by adjusting a needle valve located before the drain.

The pipes are back-lit using Light-Emitting Diode (LED) strips. A diffusive layer is placed between LED strips and the fish tank wall to improve light homogeneity. The optical measurement method consists of acquiring images of the pipe using digital cameras, for which we use two digital cameras with 4096 gray-scale levels. This allows us to analyze a reasonably wide range of concentrations. Each camera covers 150 cm of the lower part of the pipe and records images at a rate of 2 or 4 Hz. Further details of the image acquisition method and its calibration are given in Ref. 23.
A dimensional analysis of the flow suggests that five dimensional parameters may govern the flow; see Ref. 4. We denote the density of the displacing fluid by \( \hat{\rho}_H \) and that of the displaced fluid by \( \hat{\rho}_L \). The Atwood number is defined as \( At = (\hat{\rho}_H - \hat{\rho}_L)/(\hat{\rho}_H + \hat{\rho}_L) \), representing a dimensionless density difference. For this study \( At > 0 \) since \( \hat{\rho}_H > \hat{\rho}_L \). Our experiments are all performed for small \( At \), the significance of which is that a Boussinesq approximation is valid. Briefly, this means that density differences can significantly affect buoyancy forces, captured by the densimetric Froude number, \( Fr = \hat{V}_0/\sqrt{At \hat{g} \hat{D}} \), but not the acceleration of individual fluids. Here \( \hat{g} \) is the gravitational acceleration and \( \hat{V}_0 \) is the mean imposed velocity. A third dimensionless parameter is the pipe inclination, \( \beta \), measured from vertical. A fourth dimensionless parameter is the Reynolds number \( Re = \hat{V}_0 \hat{D}/\hat{v} \), where \( \hat{v} \) is defined using the mean density \( \hat{\rho} = (\hat{\rho}_L + \hat{\rho}_H)/2 \) and the common viscosity of the two fluids, \( \hat{\mu} \). Finally, since our fluids are miscible there is the potential for the Péclet number \( Pe = \hat{V}_0 \hat{D}/\hat{D}_m \) to influence the flow (here \( \hat{D}_m \) is the molecular diffusivity). Typically, for the range of flow rates in our pipe we have \( Pe \gg 1 \). This suggests that on the time scale of interest, molecular diffusivity does not play a major role in the flows studied. In a typical experimental sequence we would fix the pipe inclination and Atwood number and then run a number of experiments at increasing fixed flow rates. The range of dimensionless and dimensional parameters is shown in Table I. The three most relevant dimensionless parameters are \( \beta \), \( Re \), and \( Fr \) (providing \( At \ll 1 \) and \( Pe \gg 1 \)). We see that we are able to cover a wide range of \( \beta \), \( Re \), and \( Fr \) with our experiments.

### III. RESULTS

We now present our experimental results. We first give a broad phenomenological description of the main features we have observed in our displacement flow experiments (\( \hat{V}_0 > 0 \)); see Sec. III A. We also benchmark against existing exchange flow results (\( \hat{V}_0 = 0 \)). The variations in measured front velocity is studied in Sec. III B. For those experiments dominated by strong transverse mixing we analyse the effective bulk axial diffusivity (or dispersivity), from measured concentration profiles (Sec. III C). In Sec. IV we classify the flow regimes observed, approximating the boundaries in terms of \((Re, Fr, \beta)\). We also give leading order approximations to the front velocity and bulk axial diffusivity.

#### A. Displacement flows: Main qualitative features

We first address the most basic question: namely is the global qualitative behaviour observed for exchange flows in Ref. 9 (\( \hat{V}_0 = 0 \)) significantly affected by introducing the mean flow (\( \hat{V}_0 > 0 \)). Figure 1(b) shows typical results from our experiments, primarily illustrating the effect of inclination angle \( \beta \) on the flow pattern. The snapshots of the experiments shown in Figure 1(b) are calibrated so that the color varies between 0 (displaced fluid) and 1 (displacing fluid), thus illustrating the concentration of displaced fluid. The data shown are obtained for \( At = 0.0035 \) and kinematic viscosity \( \hat{v} = 1 \) mm\(^2\)/s, all with a mean imposed velocity in the range \( \hat{V}_0 \in [29, 47] \) mm/s. This figure suggests that the same overall behaviour is observed as for the exchange flows.9 For a given mean flow (\( \hat{V}_0 \)) the degree of mixing and disorder in the system increases as we move towards the

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<th>Parameter</th>
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<td>( \hat{V}_0 )</td>
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<tr>
<td>( \hat{v} )</td>
<td>1 (mm(^2)/s)</td>
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<td>( At )</td>
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<tr>
<td>( \beta )</td>
<td>0, 10, 20, 30, 45, 60, 70, 85(deg)</td>
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vertical ($\beta \to 0^\circ$). The flow pattern for more horizontal inclinations than shown remains qualitatively similar to that for $\beta = 85^\circ$, i.e., viscous flow of two separated layers; see Ref. 4.

Seon et al.\textsuperscript{9} used a LIF technique to visualize exchange flow patterns. They found the fluid layers well stratified with some slight mixing at the interface due to Kelvin-Helmholtz instabilities for near-horizontal angles. For near-vertical pipes the strong buoyancy component in the axial direction induces a backflow which in turn yields to a mix of Rayleigh-Taylor and Kelvin-Helmholtz type instabilities, resulting in effective transverse mixing. These two regimes are separated at intermediate angles (and $At$) by flow patterns somewhere between strongly segregated and fully mixed, in which there is significant interfacial mixing, but also unbroken streams of pure fluid remain (see Refs. 9 or 24, for examples of the observed flow patterns). In Ref. 8, Seon et al. characterized their results and analysis in terms of two dimensional velocities,

$$\hat{V}_v = \frac{At \hat{g} D^2}{\nu}, \quad \hat{V}_t = \sqrt{At \hat{g} D}, \quad (1)$$

representing velocity scales for which viscous and inertial stresses balance buoyancy, respectively. Seon et al.\textsuperscript{8} measured the front velocity $\hat{V}_f$ in the different regimes and found that for an intermediate range of inclination angles the dimensionless ratio $\hat{V}_f/\hat{V}_t$ attained a constant plateau, $\hat{V}_f/\hat{V}_t \approx 0.7$. The front velocity deviates from this line at high inclinations, where viscous effects dominate, and at small inclinations where the flow becomes fully diffusive.

Figure 2 shows the dimensionless experimental front velocity measured in our experiments (with $\hat{V}_0 = 0$) scaled with $\hat{V}_v \cos \beta$, plotted against $\hat{V}_v \cos \beta/\hat{V}_t$; cf. Fig. 4 in Ref. 8. The solid line in the figure is $\hat{V}_f/\hat{V}_t = 0.7$. We have also plotted data from Ref. 8 for the same $At$. The agreement between the two studies is clear. The data coincide both in terms of attaining the plateau values and in terms of the point of deviation from the plateau (transition to fully mixed regime). This occurs at $\hat{V}_v \cos \beta/\hat{V}_t \approx 230$ for $At = 0.001$ and at $\hat{V}_v \cos \beta/\hat{V}_t \approx 500$ for $At = 0.01$.

Figure 3 shows the spatiotemporal diagrams of the depth-averaged concentration fields for the same experiments as shown in Fig. 1(b). The depth-averaged concentration value $C(x, t)$ is simply the mean value of the fluid concentration averaged across the pipe at location $\hat{x}$ and time $\hat{t}$ and can be used to estimate the degree of mixing inside the pipe, or the height of the fluid when there is no mixing. As can be seen in Figs. 3(a)–3(f) the flow stability increases as we move towards nearly horizontal inclinations. In principle we could extract statistical data on the wavelengths and growth rates of the instabilities from this data, but this is not the aim of the present study. When the degree of mixing is high it is difficult to distinguish the front between two fluids (Figs. 3(a) and 3(b)). However, when the fluids become more separated the location of advancing displacement front can be recognized clearly (Figs. 3(c)–3(f)). The inverse slope of the line bounding the black region in the...
FIG. 3. Spatio-temporal diagrams of depth-averaged concentration field, for the same experiments as shown in Figure 1(b): (a) $\beta = 0^\circ$, $\hat{V}_0 = 32 \text{ mm/s}$; (b) $\beta = 30^\circ$, $\hat{V}_0 = 29 \text{ mm/s}$; (c) $\beta = 45^\circ$, $\hat{V}_0 = 32 \text{ mm/s}$; (d) $\beta = 60^\circ$, $\hat{V}_0 = 47 \text{ mm/s}$; (e) $\beta = 70^\circ$, $\hat{V}_0 = 43 \text{ mm/s}$; (f) $\beta = 85^\circ$, $\hat{V}_0 = 43 \text{ mm/s}$. The dashed line in (f) indicates the position of the displacing front and its slope is $-1/\hat{V}_f$. The spatiotemporal diagram gives a front speed $\hat{V}_f = 64 \text{ mm/s}$.

The spatiotemporal diagram is minus the front velocity (see the dashed line in Fig. 3(f)). This boundary can be obtained through standard image processing methods when the boundary is clear (e.g., Fig. 3(f)), but when the boundary between displacing and displaced fluid concentrations is unclear in the spatiotemporal diagram, the front velocity needs to be measured more carefully (as we discuss later in Sec. III B).

To gain additional insight into the flow dynamics we measured the velocity profile 150 cm downstream of the gate valve using an ultrasonic Doppler velocimeter DOP2000 (model 2125, Signal Processing SA). For the tracer, we used polyamid seeding particles with a mean particle diameter of 50 $\mu$m with volumetric concentration equal to 0.2 g/l in the both fluids. The Ultrasound
B. Front velocity measurement and characteristics

Doppler Velocimetry (UDV) technique suits our experimental needs well since it does not require a transparent medium. The measuring volume has a cylindrical shape, with axial resolution in our fluids of about 0.375 mm and the lateral resolution is equal to the transducer diameter (5 mm), slightly varying with depth. The UDV probe was mounted at an angle approximately 76° relative to the axis of the pipe, selected to balance a good signal to noise ratio with a small ultrasonic signal reflections; see Ref. 25. The method is non-intrusive as the probe is mounted outside the pipe, with the ultrasonic beam entering the fluid by passing through a 3.175 mm-thick plexiglass pipe wall. The method measures the flow velocity projection on the ultrasound beam: essentially giving the axial velocity. Reflection effects at the lower wall of the pipe make it hard to measure a zero velocity at the lower wall. Figure 4 shows characteristic velocities for the three different flow patterns we have observed. First, Fig. 4(a) shows the velocity profile across the channel at different times for a flow that mixes fully across the pipe. After mixing initially, the measured velocity profile appears to be close to a Poiseuille profile, which may be because the two fluids become a mixture with a density very close to the average density of the two fluids. The second regime is where there is significant mixing at the interface but it is not strong enough to completely break/mix the pure fluid layers. Some of the characteristic of such flows are puffs and waves appearing at the interface that get convected towards downstream/upstream depending on how strong the mean flow force is with respect to the buoyancy.

Figure 4(b) shows the UDV profile of such a flow obtained at β = 60°. Positive high speed velocity regions adjacent to the lower wall are accompanied by strong negative velocity regions towards the upper layer (due to incompressibility). The third regime is where interfacial instabilities are weak in the system and the flow layers are basically separated at the interface by buoyancy forces. A typical velocity profile for this regime is given in Figure 4(c) for β = 85°. It is common to observe negative values of the velocity close to the top wall (backflow) where the lighter fluid exists and positive values close to the lower part of the tube where the heavier fluid flows. Note that the current UDV results only depict the streamwise velocity components. Similar qualitative results were reported for exchange flows at varying inclination angles by Znaien et al. However the Atwood number in the results given in Ref. 12 was different in different experiments thus making robust comparison a bit difficult. In Figs. 4(a)–4(c) we have a mean imposed flow added (with similar values) and the Atwood number is kept the same. The only varying parameter is inclination angle, making a comparison easier between the different cases. Note that although the effects shown in Figure 4 are qualitatively similar to those in Ref. 12, the mean flow has subtle influences on the flow by stabilizing and/or destabilizing it. The effects of the mean flow speed on the exchange flows are in depth explained later in Sec. IV.

B. Front velocity measurement and characteristics

In a long pipe, the ratio of the mean flow speed to the leading front velocity, $\frac{\dot{V}_0}{\dot{V}_f}$, indicates the proportion of the pipe displaced in an experiment, i.e., the displacement efficiency. Consequently, it becomes important to measure $\dot{V}_f$ in a consistent and repeatable way, regardless of the degree of mixing. Whereas edge detection and calculation of the slope from the spatiotemporal diagram has proven effective in our previous work, at pipe inclinations close to horizontal (see Ref. 4),
FIG. 5. (a) Evolution of the depth-averaged concentration field, \( C \), with time, \( \hat{t} = [2, 7, 12, \ldots, 32] \) s, and streamwise location, \( \hat{x} \), measured from the gate valve for the same experiment as in Figures 1(b) (\( \beta = 30^\circ \)) and 3(b). The dashed line shows \( C = 0.1 \) which is used for measuring the displacing front velocity, \( \hat{V}_f \), consistently. (b) Evolution of the front velocity value, \( \hat{V}_f \), with time for the same experiment. The inset shows the constant value of the front velocity when the flow is fully developed (\( \hat{V}_f = 31.3 \) mm/s in this case with 2% standard deviation).

Strong transverse mixing such as in Fig. 1(b) (Figs. 3(a) and 3(b)) makes this method less reliable. Figure 5(a) shows the concentration profiles at instants of time along a section of the pipe for the same experiment as shown in Fig. 3(b). Although the data presented in Figs. 5 and 3(b) belong to the same experiment, they are not at exactly the same instants of time. To avoid very diffuse concentrations and noise in the data close to the lower wall of the pipe, we estimate the speed of the displacement front by the velocity of the concentration level \( C = 0.1 \) (see the dashed line in Figure 5(a)). Evidently, selection of a threshold value is a trade-off between robustness and proximity to \( C = 0 \). Figure 5(b) shows the variation of the front velocity, \( \hat{V}_f \), with time, which is quite typical of most of our experiments. In those cases that instability and mixing causes slow oscillation and noise in the profile of front velocity with time, an average value is adopted over long times so that the flow is well developed.

Initially the front velocity accelerates as the gate valve is opened and the flow initiates. There is some development timescale during which instability and mixing initiates, slowing the front propagation. As the flow becomes fully developed the front velocity reaches an almost constant value (Fig. 5(b), inset). We use this thresholded front velocity, taken at long times in the experiment, as our measured \( \hat{V}_f \). The transient and short-time behaviour of the front in the limit of exchange flows in nearly horizontal pipes was well studied by Seon et al.\(^{10}\) It was also found that the front is initially controlled by inertia but later limited by viscous effects. This yields a high front velocity value that decays to a steady state with time. A similar effect is observed in almost all our displacement flow experiments. Due to the fluid separation by gate valve the buoyancy force at the very beginning of the experiment is dominant over all other forces. This results in a fast acceleration of the displacing fluid along the pipe. Similar results are reported for gravity currents forming from the release of a heavy fluid into a light fluid.\(^{26}\) In the case of density-stable displacements (light fluid displacing heavy fluid), we also found a similar accelerating behaviour at the start of the experiment.\(^{16}\) Although studying transient/short-time effects in displacement flows is not studied much in the literature, and can be interesting, it is beyond the scope of our study.

We can compare our measured \( \hat{V}_f \) with that obtained directly from the spatiotemporal plot via edge detection. For near-horizontal displacement flows, where the latter method is effective, typical relative errors in front velocity measurement are less than 2%. For example, for the experiment in Fig. 3(f) the threshold method gives \( \hat{V}_f = 63 \) mm/s and the edge detection method from the spatiotemporal diagram gives \( \hat{V}_f = 64 \) mm/s. By its nature, the threshold method tends to underpredict front velocity. The threshold method proves quite repeatable. Also it was checked that the general qualitative and quantitative features of displacement flows are reproducible for repeated experiments (the comparisons are not shown here though for brevity).
FIG. 6. Change in displacing front velocity, \( \hat{V}_f \) with tilt angle, \( \beta \), and imposed velocity, \( \hat{V}_0 \) for \( \beta = 1 \) mm\(^2\)/s and (a) \( At = 0.001 \), (b) \( At = 0.0035 \), and (c) \( At = 0.01 \). Different markers represent \( \hat{V}_0 = 0 \) mm/s (○), 10 (■), 20 (▲), 40 (▼), 60 (●), 80 (◇), and 100 mm/s (●). The dashed lines are guide to the eyes plotted at \( \hat{V}_f = 10, 20, 30, \ldots \) mm/s.

We now explore the main characteristics of front velocity measurements across our experimental range, where we have varied inclination angles, \( \beta \), density differences, \( At \), and mean flow speeds \( \hat{V}_0 \). Figures 6(a)–6(c) show \( \hat{V}_f \) for experiments conducted with \( At = 0.001 \), \( At = 0.0035 \), and \( At = 0.01 \), respectively. In each experimental sequence, at fixed \( At \) and \( \beta \), the experiment is repeated at successively higher \( \hat{V}_0 \), starting from exchange flows (\( \hat{V}_0 = 0 \)). The data for nearly horizontal experiments from Ref. 4 are also added for completeness. The first thing to note is that the values of the front velocity naturally increase with mean flow speed \( \hat{V}_0 \). The second observation is that the front velocity seems to be highest for \( \beta \) between 60° and 70° roughly. This might be due to the fact that in this range the counter-current flow is strong enough to increase \( \hat{V}_f \) but not strong enough to promote Kelvin-Helmholtz instabilities (which in turn tend to decrease \( \hat{V}_f \)). Note that the flow patterns for a sequence of experiments at different inclination angles and approximately the same imposed flow velocity value are shown in Figure 1(b). Qualitatively, at each \( \hat{V}_0 \), the front velocity variation with \( \beta \) is similar to that reported by Seon et al.\(^{8,9} \) for \( \hat{V}_0 = 0 \), although a constant plateau is not clearly distinguished in all our experiments. In fact the plateau region observed in Refs. 8 and 9 is limited by the viscous regime on one side (low values of \( \hat{V}_0 \cos \beta/\hat{V}_f \)) and by the buoyancy-dominated regime on the other side (high values of \( \hat{V}_0 \cos \beta/\hat{V}_f \)). It seems as if adding the imposed mean velocity to the exchange flow can extend both limits. The stabilizing effect of the mean flow was previously observed in Ref. 2, for nearly horizontal displacement flow experiments. In that study the mean flow extended the viscous flow limit by stabilizing inertial effects, thus narrowing the plateau regime. For the other limit where buoyancy destabilizes the flow, adding the mean flow seems to have amplified the instabilities even more by enlarging the buoyancy-driven zone. We will discuss the stabilizing/de-stabilizing effect of mean flow in details later on in Sec. IV.

In order to better see the effect of the mean flow, relative to the underlying buoyancy driven exchange flow, we also subtracted the mean flow (\( \hat{V}_0 \)) and exchange flow front velocity (\( \hat{V}_{\text{Exchange}} \)) from the measured front velocity values \( \hat{V}_f \). We found that most values of \( \hat{V}_f - \hat{V}_{\text{Exchange}} - \hat{V}_0 \) were still positive and especially for smaller values of \( At \). This suggests that imposing a displacement velocity on these exchange flows does not change the flow structure. The flow remained mostly stratified and the front moved with a relatively high velocity, augmented by the imposed flow. Note that the increase in \( \hat{V}_f \) over \( \hat{V}_{\text{Exchange}} - \hat{V}_0 \) means that some fluid (especially within the displaced layer) is moving at a much lower value (due to the buoyancy), i.e., the total flow is fixed. For higher Atwood numbers we observed more negative values of \( \hat{V}_f - \hat{V}_{\text{Exchange}} - \hat{V}_0 \) indicating that simple superposition of physical effects becomes meaningless. For these flows significant instability and transverse mixing is induced by the combination of mean imposed flow and strong buoyancy, modifying the flow structure. The front velocity is significantly lower than that which would be predicted by methods such as the lubrication/thin-film approach we have developed in Ref. 1, as the physical assumptions underlying these models are not valid. A new characterization is needed.

We now proceed with a dimensionless analysis of the results, focusing on the normalized front velocity \( \hat{V}_f = \hat{V}_f/\hat{V}_0 \). In our previous study of near-horizontal displacements,\(^3 \) we were able to classify all flows in the (\( Fr, Re \cos \beta/Fr \)) plane, and therefore start with this description. Figure 7(a) shows the normalized front velocity \( \hat{V}_f = \hat{V}_f/\hat{V}_0 \), plotted against \( Fr \) and \( Re \cos \beta/Fr \), for
all experiments conducted in this study. The parameter regime studied by Taghavi et al.\textsuperscript{4} is marked by the rectangle: $0 < \Re \cos \beta / \Fr < 120$ and $0 < \Fr < 6$. It can be seen that the present study covers a much wider parameter range (primarily due to variations in $\beta$).

Our first observation is that over nearly the entire range of $\Re \cos \beta / \Fr$ studied $V_f$ increases as $\Fr$ decreases. The parameter $\Re \cos \beta / \Fr$ is independent of $\tilde{V}_0$, so that as $\Fr \rightarrow 0$ we approach the exchange flow limit. For small $\Fr$ we observe a number of flows for which $V_f > 2$. We refer to these as exchange flow dominated and note that since we have scaled with the mean flow velocity, large $V_f > 2$ strongly suggests that some part of the velocity field is moving backwards against the mean flow, driven by buoyancy. The data are replotted in Fig. 7(b) with scale adjusted to emphasize those experiments for which $V_f > 2$. Backflows also can occur for lower values of $V_f > 2$. Figure 7(b) identifies those parameters for which backflows are observed and those that displace instantaneously. Exchange dominated flows generally lie within the limits $50 < \Re \cos \beta / \Fr < 650$ and $0 < \Fr < 2$, although this is not precise. Similar flows were identified in Taghavi et al.\textsuperscript{4} in the range $50 < \Re \cos \beta / \Fr < 120$ and $0 < \Fr < 0.9$, but the upper limit on $\Re \cos \beta / \Fr$ was simply due to the extent of experiments performed. The lower limit is an extension of the exchange flow studies of Seon et al.\textsuperscript{8} into the range of positive imposed flow: Seon et al.\textsuperscript{8} identified the transition from viscous to inertial exchange flows at $\Re \cos \beta / \Fr \approx 50$. Thus, the lower range of our results coincides with those of Refs. 4 and 8. The main finding is therefore that the upper limit for exchange dominated inertial flows extends up to approximately $\Re \cos \beta / \Fr = 650$.

In considering practical aspects of displacement, one important parameter is the displacement efficiency $1 / V_f$, based on the behaviour of the leading displacement front. A second important consideration is knowing whether the trailing front advances along the pipe in the direction of flow or upstream against the flow, driven by buoyancy (a backflow). In the former case, if it is possible to wait long enough the pipe will be fully displaced, but in the latter case residual fluid will remain. In Ref. 3, the criterion $\Re \cos \beta / \Fr \lesssim 58 \Fr$ was developed to predict the onset of backflows. The methodology used involved a thin-film/lubrication model of viscous regime displacements. In such models a single parameter, $\chi = 2 \Re \cos \beta / \Fr^2$ governs the dynamics at long times: $\chi$ represents the competition between buoyancy stresses in the axial direction and viscous stresses due to the mean flow. The critical value $\chi_c = 116.32$ was found to be the limit above which there is always a backflow. It is interesting to see how this value compares with our experiments, where mixing and instability are inevitably present. Figure 7(b) shows the experimental data in the ($\Fr, \Re \cos \beta / \Fr$) plane. We have marked those displacements that proceed instantaneously, i.e., with no evidence of a backflow, and have included the prediction of the lubrication model ($\chi_c = 116.32$) for comparison.
Interestingly we observe that all the data points falling below $\chi = \chi_c$ line are found to be displacing instantaneously after opening the gate valve, i.e., $\chi < \chi_c$ appears to be a sufficient condition to avoid backflows. However, we see that many flows in the regime $\chi > \chi_c$ also displace instantaneously. Phenomenologically, these flows are inertial and well-mixed, and thus not well represented by the assumptions underlying the model behind $\chi = \chi_c$.

On examining Fig. 7 closely we observe that within the exchange dominated regime, there are interesting variations in $V_f$ with $Re \cos \beta/\mathcal{F}r$. Let us consider a fixed $\mathcal{F}r$ in the range $0 < \mathcal{F}r < 0.9$ and start with a low value of $Re \cos \beta/\mathcal{F}r$. First, for $Re \cos \beta/\mathcal{F}r \lesssim 58\mathcal{F}r$ ($\chi < \chi_c$) the flows are viscous dominated at long times and do not have a sufficiently strong buoyancy component to promote a backflow. For $58\mathcal{F}r \lesssim Re \cos \beta/\mathcal{F}r \lesssim 50$ the flows are exchange dominated but still viscous, i.e., with counter-current buoyancy driven well-defined stable laminar layers advancing downstream and upstream back against the flow. These regimes are studied at length in Refs. 3 and 4. Increasing $Re \cos \beta/\mathcal{F}r > 50$ we enter the inertial regime. Although counter-current streams persist we begin to observe instability and mixing close to the interface. On further increasing $Re \cos \beta/\mathcal{F}r$ buoyancy forces increase, driving both lighter fluid upwards and denser fluid downwards, hence increasing $V_f$. However, the degree of instability and mixing also increases progressively as $Re \cos \beta/\mathcal{F}r$ is increased. As transverse mixing comes to dominate, the streams of pure fluid are no longer able to remain intact while flowing counter-currently and a fully diffusive regime is entered as $V_f$ decreases towards unity.

The competition between buoyancy-driven advection and transverse mixing leads to a maximum in $V_f$ at an intermediate value of $Re \cos \beta/\mathcal{F}r$. This competition is captured physically in the parameter $Re \cos \beta/\mathcal{F}r$, which is defined as

$$\frac{Re \cos \beta}{\mathcal{F}r} = \frac{\hat{V}_0 \hat{D}}{\hat{v}} \cos \beta \sqrt{At\hat{g}} \hat{D} = \frac{\cos \beta \sqrt{2}}{\sqrt{2}} \left[ \frac{(\hat{\rho}_H - \hat{\rho}_L)(\hat{\rho}_H - \hat{\rho}_L)}{2\mu^2} \hat{D}^3 \right]^{1/2}. \tag{2}$$

The parameter in the square brackets above is the Archimedes number. An interpretation of this expression is as the relative strengths of buoyancy stresses $(\hat{\rho}_H - \hat{\rho}_L)\hat{g}\hat{D}$, and viscous stresses, $\mu \hat{V}_t/\hat{D}$. Note that here the velocity scale used for the viscous stresses is $\hat{V}_t$, which is itself driven by buoyancy. It is this feature of self-reinforcement that tends to promote the onset of (buoyancy driven) instability, i.e., if the viscous stresses induced by the buoyant motion are unable to balance the buoyant stresses.

### C. Macroscopic diffusion

In this section we focus on those experiments where the degree of transverse mixing is high. In such cases we expect that advective transport due to the mean flow will be supplemented by diffusive spreading along the pipe. Debacq et al.\textsuperscript{5,6} used a similarity scaling for exchange flows in vertical pipes to collapse profiles of the cross-sectionally averaged concentration $C$, measured at each location $\hat{x}$ and time $\hat{t}$, onto a master curve defined with respect to $\hat{x}/\sqrt{\hat{t}}$. On fitting the master curve to an error function form, estimates were derived for the macroscopic diffusion of the mean concentration along the pipe. Seon et al.\textsuperscript{11} followed the same approach for inclined pipes and we adopt an analogous approach here.

In the presence of a mean flow ($\hat{V}_0$), when the flows fully mix transversely it is logical to assume a core of the mixture traveling with the speed $\hat{V}_0$ and therefore use $(\hat{x} - \hat{V}_0\hat{t})/\sqrt{\mathcal{F}t}$ as a similarity scaling. Figure 8(a) shows an example of this for $\beta = 45^\circ$, $At = 0.08$, $\hat{v} = 1$ mm$^2$/s. We can see that all the concentration evolution profiles have collapsed onto one curve. The solid line shows a curve fit of the form $C = 0.5erfc((\hat{x} - \hat{V}_0\hat{t})/\sqrt{\mathcal{F}t})$, motivated by the analytical solution to the linear diffusion equation.

In Fig. 8(a) we find $\hat{D}_M = 0.006$ m$^2$/s. Here $\hat{D}_M$ is the macroscopic diffusion coefficient. Although for most experiments with a high degree of mixing the similarity scaling provides an excellent fit to the data (e.g., Fig. 8(a)), as we examine a wider set of our experimental data this scaling fails in variety of different ways. First, this error-function fit assumes that the area behind and ahead of the mixed core (moving with speed $\hat{V}_0$) should be symmetric. This is not always true since the mixing
FIG. 8. Collapse of depth-averaged concentration profiles with $(\hat{x} - \hat{V}_0\hat{t})/\sqrt{\hat{t}}$ for $\hat{\nu} = 1$ mm$^2$/s and (a) $\beta = 45^\circ$, $\hat{V}_0 = 113$ mm/s, (b) $\beta = 20^\circ$, $\hat{V}_0 = 44$ mm/s, (c) $\beta = 45^\circ$, $\hat{V}_0 = 32$ mm/s, and (d) $\beta = 85^\circ$, $\hat{V}_0 = 43$ mm/s. In (a) $At = 0.08$ and in (b)–(d) $At = 0.0035$. The solid lines in (a) and (b) show the error-function fits with $\hat{D}_M = 0.006, 0.002$ m$^2$/s, respectively. The upper right insets show the qualitative flow pattern in each case. In (d) the lower left inset shows the collapse of the concentration profiles when $\hat{x}/\hat{t}$ is used instead of $(\hat{x} - \hat{V}_0\hat{t})/\sqrt{\hat{t}}$.

quality can be slightly different behind and ahead of the mixing core. Unlike the exchange flow, adding a mean flow can break the symmetry. Figure 8(b) shows one example of this: for negative $(\hat{x} - \hat{V}_0\hat{t})/\sqrt{\hat{t}}$ the measured profiles fall slightly below the curve which suggest that the mixing is happening over a slower timescale behind the mixing core. Assuming a symmetric error function fit to the collapsed concentration profiles is still a reasonable leading order approximation, but some aspects of the physical phenomena present are not captured. Second, we have experiments which are highly inertial, exhibiting instability and mixing, but for which transverse mixing is incomplete and which are not fully diffusive. For example, see Fig. 8(c) at $\beta = 45^\circ$. The figure confirms that the concentration evolution should not be categorized as fully diffusive. The fluctuations in the profiles shown relate to the large scale instabilities that appear intermittently at the interface. A final limit where we do not have diffusive flows is where the layers remain well segregated at the interface, i.e., viscous flows. Figure 8(d) shows again that the averaged concentration profiles do not always collapse onto a master curve using the similarity scaling $(\hat{x} - \hat{V}_0\hat{t})/\sqrt{\hat{t}}$. These flows are essentially advective and the profiles collapse instead under a similarity scaling $\hat{x}/\hat{t}$ (see Ref. 1) as is illustrated in the inset of Fig. 8(d).

Note that to determine $\hat{D}_M$ we only analyze the concentration in the lower section of the experimental apparatus and at later times in the experiment, so that the flow is as fully developed as our experiment allows. The initial stages of every experiment are complex, as there is an initial acceleration phase, followed by an inertia-buoyancy balance, then development of either instabilities and mixing, or advective transport (as considered earlier). While physically interesting, these initial phenomena are not the focus of our study.
Seon et al.\textsuperscript{11} fitted their measured $\hat{D}_M$, for the case $\hat{V}_0 = 0$, to an expression of form,

$$\hat{D}_{M,\text{Exchange}} = 5 \times 10^3 (\hat{V}_t \hat{D})(1 + 3.6 \tan \beta)^{3/2} \left(\frac{\hat{V}_t}{\hat{V}_v}\right)^3. \quad (3)$$

Note that the range of applicability of (3) in Ref. 11 is for $Re/Fr \lesssim 1000$, which covers the range of our exchange flow experiments. In order to check the consistency of our macroscopic diffusion coefficient measurements, the data are compared with (3) in Fig. 9, for those of our experiments run at $\hat{V}_0 = 0$ that were fully mixed. The agreement is good, with a similar deviation as for the data in Ref. 11.

We now focus only on those experiments for which we have been able to reliably collapse the data at long times, using the similarity scaling $(\hat{x} - \hat{V}_0 \hat{t})/\sqrt{\hat{t}}$, and have been able to estimate a macroscopic diffusion coefficient from the master curve. We call these flows fully diffusive. Figure 10 plots the measured macroscopic diffusion coefficients for displacement flows at different inclination angles and two Atwood numbers: $At = 0.0035$ and $At = 0.01$. Two clear conclusions are that, within the range of fully diffusive experiments, $\hat{D}_M$ increases with both inclination angle, $\beta$, and mean flow speed, $\hat{V}_0$.

None of the above trends are surprising, but they are interesting in suggesting two independent factors contributing to $\hat{D}_M$. The relatively large values of $\hat{D}_M$ confirm that the diffusive transport...
cannot be molecular in origin, but is due to advection, i.e., this is largely dispersion. The flows are disordered and well-mixed transversely, to all appearances locally turbulent. In such cases we may expect a scaling of the dispersivity with length and velocity scales that are relevant to the eddy structure. This is evidently complex in general, but we might guess that the relevant velocity will scale approximately with either the inertial velocity or the mean flow, depending on the relative strengths of buoyancy or the mean flow in driving the mixing. To explore this, we plot $\hat{D}_M/(\hat{V}_t \hat{D})$ and $\hat{D}_M/(\hat{V}_0 \hat{D})$ in the $(Fr, Re \cos \beta/Fr)$-plane (see Fig. 11), the axes reflecting the competition between inertia from the mean flow and buoyancy ($Fr$) and between buoyancy driven motion and viscous dissipation of that motion ($Re \cos \beta/Fr$), respectively.

A first observation is that our experimental data for fully diffusive experiments closely corresponds to the region of the $(Fr, Re \cos \beta/Fr)$-plane in which the front velocities $\hat{V}_f$ are very close to the mean flow $\hat{V}_0$, e.g., compare with the region in Fig. 7(b) where $\hat{V}_f \approx 1$. Second, we can see that scaling of $\hat{D}_M$ suggest that both mechanisms are responsible for the mixing in different limits. For small $Fr$, at large $(Re \cos \beta/Fr)$ as we transition out of the exchange dominated regime see that $\hat{D}_M/(\hat{V}_t \hat{D})$ appears to become independent of $Fr$, suggesting buoyancy driven mixing. At the other extreme, for modest $(Re \cos \beta/Fr)$ and as we increase $Fr$ we find $\hat{D}_M/(\hat{V}_0 \hat{D}) \approx$ constant, suggesting mixing driven by the mean flow. The pattern of $\hat{D}_M/(\hat{V}_0 \hat{D})$ in Fig. 11(b) is suggestive of lines of constant $Re$, and note that in turbulent/mixed shear flows the turbulent dispersivity (when scaled with $\hat{V}_0 \hat{D}$) has only slow variation with $Re$. Figure 12 shows all our fully diffusive experiments and plots with the color-scale the $Re$ corresponding to each experiment. Lines of constant $Re$ (keeping all other parameters fixed) would appear as hyperbolae in this plane, and we have included the line $Re = 2100$ for different inclination angles, as a nominal transitional value in the absence of buoyancy effects. Those data points that exceed $Re = 2100$ are highlighted by the superposed symbols. We
observe that although some of our data satisfy this criterion, many of the experiments have $Re$ far below 2100, although the mixing behaviour appears turbulent. This suggests that buoyancy has a significant role in at least instigating this regime, even for those regimes where the mean flow dominates. It is worth noting that our study has not explored very high Reynolds numbers.

In order to compare the macroscopic diffusion coefficients, $\hat{D}_M$, against predictions from the Taylor dispersion coefficient, $\hat{D}_T$, for turbulent pipe flow, we used the well-established formula $\hat{D}_T = 1.785 V_0 D_f \sqrt{f_f}$. For simplicity the Nikuradze formula was used for the friction factor, $f_f = 0.266/Re^{1/4}$. In general we observed that $\hat{D}_M$ exceeds $\hat{D}_T$ by up to an order of magnitude, over the full range of our experiments. This is purely comparative of course and no exact agreement is to be expected, due to the relatively low $Re$. It is well known that the actual dispersion increases rapidly as the turbulence weakens and the viscous wall layers grow thicker; see, e.g., Refs. 15, 27, and 28. Here, although apparently turbulent even at lower $Re$, the driving force is at least partly buoyancy and we cannot expect a classical turbulent velocity profile to drive dispersion. Although we have some velocity data, the UDV system is not ideal for capturing turbulent characteristics of the flow and we feel that making further inference could be beyond the range of reliability of our data. At very high Reynolds numbers, say $Re > 10^4$, since all our experiments are at low $Ar$ we might expect to recover the classical results for turbulent dispersion. However, this is beyond the scope of this study and would be difficult to study in our present apparatus.

IV. REGIME CLASSIFICATION AND LEADING ORDER APPROXIMATIONS

In Sec. III we presented our results in terms of the velocity of the leading displacement front, and in terms of bulk axial diffusivity in the case of fully diffusive flows. We now give a leading order quantitative description of the different flow regimes observed at long times in our experiments, in terms of the three dimensionless parameters: $(Fr, Re, \beta)$. Much of the parametric variation appears to be captured in the $(Fr, Re \cos \beta/Fr)$-plane, as was the case for nearly horizontal displacements studied in Ref. 4. Figure 13 shows the experimental points from our study, characterized in terms of flow type. We distinguish flows according to the following criteria: (i) instantaneous displacement (if there is no displaced fluid observed above the gate valve); (ii) fully diffusive (if we are able to collapse the data at long times via similarity solution of form, $C = 0.5\text{erfc}(\frac{\hat{x} - \hat{V}_0 \hat{t}}{2\sqrt{\hat{D}_M}})$); (iii) for non-diffusive flows we classify as either, viscous flows if there is no instability evident in the spatiotemporal image behind the leading displacement front, and inertial otherwise. It can be seen that this classification, although not perfect, does appear to separate the data within the $(Fr, Re \cos \beta/Fr)$-plane.

Two lines are plotted in Fig. 13. First, the solid line is the prediction of instantaneous displacements by the lubrication model in Ref. 3, $(\chi = \chi_c = 116.32)$, as discussed earlier. Below this line

![Figure 13](image-url)
only instantaneous displacements may occur, in the viscous limit. The broken line in Fig. 13 is given by

\[ \frac{Re \cos \beta}{Fr} = 500 - 50Fr. \] (4)

For \( \chi > \chi_c \), this line represents the approximate position of the boundary between fully diffusive and non-diffusive flows. The two lines intersect at \( Fr \approx 4.62 \) and \( Re \cos \beta/Fr \approx 270 \).

For \( \chi < \chi_c \) and sufficiently small flow rates we find viscous dominated flows, characteristically laminar with well defined interfaces separating distinct fluid streams. These flows are an extension of the viscous instantaneous displacements studied in Refs. 3 and 4, but here we have a higher range of \( Fr \) and \( Re \cos \beta/Fr \); see Fig. 7(a). In this regime the viscous stresses generated by the mean flow appear to dominate any destabilizing effects of buoyancy. Using the lubrication/thin-film displacement model in Refs. 3 and 4 we can compute an approximation to the front velocity, that depends only on the parameter \( \chi = 2Re \cos \beta/Fr^2 \). Figure 14(a) compares the scaled front velocity values obtained from the current experiments with the predictions \( V_f(\chi) \), of the lubrication model. Data from Ref. 4, belonging to viscous flows in nearly horizontal experiments, are also included. The lubrication model appears to be effective in predicting the front velocity for our data, i.e., at significantly larger \( Re \cos \beta/Fr \) than was previously studied.

On increasing the flow rate (here \( Fr \)) the flow transitions from viscous laminar layers to fully diffusive turbulent flow. As we have seen earlier, the fully diffusive regime is driven by a combination of buoyancy and mean flow effects. For \( \chi < \chi_c \), we expect that the mean flow effects will be dominant as buoyancy decreases. Although (4) appears to capture the transition reasonably well close to the point of interception of the two curves in Fig. 13, this relationship cannot be valid as \( Re \cos \beta/Fr \rightarrow 0 \) (e.g., this gives \( Fr = 10 \) for a horizontal pipe). Instead, we expect a more classical criterion for transition to be valid, primarily based on a \( Re \) threshold. We have seen in Fig. 12 that the fully diffusive flows observed in this range do have \( Re > 1500 \), which tends to support our viewpoint. Although not high enough for fully shear driven turbulence, flows at these \( Re \) are highly inertial and it is quite conceivable that the transition could be triggered. First, it could be that in the initial stages of the experiment when buoyancy effects are large, these trigger instability. Second, we must admit that our apparatus was not designed for a classical study of shear flow transition, but for observing displacement flows. At high \( Re \), geometric imperfections (e.g., near the gate valve) or entry conditions may play a role in triggering instability. Furthermore, if we consider a long smooth pipe, the viscous flows observed at lower flow rates tend to have an elongating interfacial region that essentially stratifies. The effect of density on the base flow and on the (linear) stability problem is

\[ \frac{2V_f}{\chi} \] (a) Comparison of the front velocity values obtained from experiments classified as viscous with the predictions \( V_f(\chi) \) from the lubrication model in Refs. 3 and 4 (thick solid line). Color values indicate \( Re \cos \beta/Fr \) for each experiment. The data with the solid boundary are for near-horizontal pipe, the viscous flows observed at lower flow rates tend to have an elongating interfacial region that essentially stratifies. The effect of density on the base flow and on the (linear) stability problem is

\[ \frac{\hat{V}_f}{\hat{V}_t} \] (b) comparison of the experimental and predicted values of \( \hat{V}_f/\hat{V}_t \), for intermittent flows with the predictions following (6). The dashed line in Figure 14(b) indicates \( \hat{V}_f/\hat{V}_t(\text{experiment}) = \hat{V}_f/\hat{V}_t(\text{curve fit}) \). The contour values in both figures show the corresponding \( Re \cos \beta/Fr \) to each experiment.
felt only through $\chi$, which decreases like $1/\hat{V}_0$ (i.e., as $Re^{-1}$ increases with flow rate). Thus, the limit of large $Re$ and small $\chi$ recovers the single fluid flow. From these perspectives (and in the absence of viscosity differences), we feel that in the regime $\chi < \chi_c$, a better criterion to use as a sufficient condition for transition to the fully diffusive regime is $Re \gtrsim Re_r = 2100$. Finally, we might reflect on why (4) appears to capture this transition so well in the range studied. The main point here is that our experiments are performed using a single pipe diameter and hence $Fr$ simply reflects an increase in $\hat{V}_0$, which is also the main control parameter in varying $Re$. To distinguish $Re$ and $Fr$ effects would require experiments with a different (identical) viscosity and/or changes in pipe diameter. Here the main focus has been on changing $\beta$.

For $\chi > \chi_c$ the flows have a significant inertial component, driven by buoyancy, as well as a contribution from the mean flow. For parameters below the curve (4) we have classified these flows as non-diffusive and inertial, but in reality a wide range of behaviours is observed within our data. First, we have seen that for $Fr < 2$ in this range many of the flows have strong backflows and are dominated by buoyancy. We have termed these exchange flow dominated. As $Re \cos \beta/\hat{Fr}$ is decreased these flows persist down to $Re \cos \beta/\hat{Fr} \approx 50$, which is outside the range of our data here but covered by the data in Ref. 4. For $Re \cos \beta/\hat{Fr} \lesssim 50$ (and $\chi > \chi_c$) the flows remain exchange dominated but are now viscous exchange flows; see Refs. 3 and 4.

Within the exchange dominated range at lower $Re \cos \beta/\hat{Fr}$ the fluid streams remain fairly distinct and structured, say roughly for $50 \lesssim Re \cos \beta/\hat{Fr} \lesssim 270$. Interfacial instabilities occur, but mixing between the streams is relatively minor. As the flow rate is increased, for $Fr > 2$, buoyancy effects reduce and the countercurrent flow is progressively eliminated as we pass into the regime of instantaneous displacements. These flows laminarise as we cross $\chi = \chi_c$, leading to the counterintuitive observation that increases in flow rate can stabilize the flow; see Ref. 2. The novel result of this study is in determining that stabilisation of exchange dominated inertial flows by the mean flow extends up to $Re \cos \beta/\hat{Fr} \approx 270$. For $Re \cos \beta/\hat{Fr} \gtrsim 270$ exchange dominated flows persist. Increasing $Re \cos \beta/\hat{Fr}$ further destabilises these flows. We progressively observe more intermittency and instability, leading to a greater degree of transverse mixing. Eventually we cross (4) into the regime of fully diffusive flows. Equally we traverse (4) on increasing $Fr$, again becoming progressively unstable (i.e., in contrast to the stabilisation by the mean flow observed for $Re \cos \beta/\hat{Fr} \approx 270$).

For engineering design purposes it is of value to have an approximation to $\hat{V}_f$ governing the flows in the triangular region of Fig. 13, that we have classified as non-diffusive inertial flows. In the range of low $Re \cos \beta/\hat{Fr}$ our study coincides with that of Taghavi et al.,¹ who studied near-horizontal pipes and fitted the expression

$$\hat{V}_f/\hat{V}_t = 0.7 + 0.595Fr + 0.362Fr^2,$$

(5)

to the available data in these regimes; see Ref. 4. The rationale for the form of expression is that $\hat{V}_t$ is the appropriate velocity scale for inertial/bouyancy-driven flows. The first coefficient coincides with that from extensive exchange flow studies; see Ref. 8, and the next coefficients are logical for an expansion in terms of small $\hat{V}_0/\hat{V}_t (= Fr)$. We compared our experimental front velocity values scaled by inertial velocity ($\hat{V}_f/\hat{V}_t$), against the formula (5). Although reasonable, a discernible trend was the deviation from (5) at larger values of $Re \cos \beta/\hat{Fr}$, as well as for higher $Fr$. The deviation from (5) at higher $Fr$ is inevitable since we are moving away from exchange flow dominated regimes. Equally in our near-horizontal flow experiments we were restricted to $Re \cos \beta/\hat{Fr} < 120$. The leading coefficient 0.7 in (5) comes from exchange flow studies conducted over the approximate range $50 < Re \cos \beta/\hat{Fr} < 200$; see Ref. 8. We hypothesize that the relationship (5) should have a significant dependency on $Re \cos \beta/\hat{Fr}$, and have developed the following curve fit:

$$\hat{V}_f/\hat{V}_t = Fr - 0.002337(\frac{Re \cos \beta}{Fr} + 50)Fr - 500)(1 - 0.98Fr + 1.03Fr^2),$$

(6)

as an improved model. The form of the model is chosen to ensure that $\hat{V}_f/\hat{V}_t = Fr$ on (4), i.e., $\hat{V}_f = \hat{V}_0$. Figure 14(b) plots experimental $\hat{V}_f/\hat{V}_t$ values against $\hat{V}_f/\hat{V}_t$ predicted from (6), showing close agreement uniformly with $Re \cos \beta/\hat{Fr}$.

Turning now to the fully diffusive flows, these occur primarily in the region of the ($Fr$, $Re \cos \beta/\hat{Fr}$)-plane lying above (4). We first note that there are a good number of data points in...
Fig. 13 that are fully diffusive but not instantaneous displacements. These are found above (4) for $Fr < 2$ and $Re \cos \beta/Fr$ extending up to around 800. These are an extension of the exchange dominated flows into the fully diffusive regime. This differs significantly from pure exchange flow studies, driven only by buoyancy. For example, in the data of Debacq et al. we can see the onset of fully diffusive flows at $Re \cos \beta/Fr \gtrsim 1000$, for exchange flows in vertical tubes. However, the difference is not surprising as classification of such flows depends partly on the experimental timescale. Our classification scheme here simply means that there is displaced fluid present above the gate valve at the end of our experiment. Given that transverse mixing is very effective for these flows it is unlikely that displaced fluid would persist here over very long timescales, eventually being mixed and washed away.

We have seen earlier that our fully diffusive data (at $V_f = 0$) match well with that from Seon et al. data, leading to the expression (3) for the diffusivity $DM_{\text{Exchange}}$. In developing an approximation to the bulk diffusivity $DM$ we have seen in Fig. 11(b) that $DM \sim 0V_0$ for larger $Fr$. As we expect that the buoyant component is largely predicted by $DM_{\text{Exchange}}$, we examined the difference $DM - DM_{\text{Exchange}}$, against $0V_0$. It was seen that the data for each pipe inclination $\beta$ followed an individual curve (results are not shown here). Variation with $\beta$ should correspond to a buoyancy effect, rather than simply a mean flow effect. To incorporate this, we have looked for a relationship of form,

$$DM = DM_{\text{Exchange}} + 0V_0(c_0 + c_1(\beta)/\sqrt{Fr}).$$

At high $Re$ we expect $DM \sim 0V_0$ from the Taylor dispersion analysis, where $Re$ dependency through the friction factor is weak. We have therefore used our data for high Reynolds number experiments $Re > 2100$, to fit $c_0 = 0.6618$, and then have used the average data values over each inclination, to fit $c_1(\beta) = 0.9054 - 1.838 \tan \beta$.

The effectiveness of the expression in (7) in approximating $DM$ is illustrated in Fig. 15(a). Although reasonable as a prediction, there is some spread in the data. We believe this variability is at least partly due to variability in the measurement procedure, leading to fitting of $DM$. In each experiment we wait for long times to fit $DM$. However, as these are (mostly) instantaneous displacements the time of sampling of each experiment is related to the transit time along the tube (an advective timescale), i.e., if you wait too long the data has left the pipe. If the flow development is diffusively controlled, ideally one should sample at times that are equally significant diffusively.

Finally, we consider the appropriate front velocity for the diffusive regime. We have used $(\hat{c} - \hat{c}_0)/\sqrt{t}$ as a similarity scaling for the concentration profiles in fitting $DM$. This implicitly assumes a speed $\hat{V}_0$ for the advective transport. We can also, however, proceed in the reverse
direction for any given \( \hat{D}_M \), to find the front velocity. Our procedure for finding the front velocity relies on tracking the concentration profiles at later times in our experiment. However, for fully diffusive flows part of the evolution of the concentration is driven by axial diffusion. For given \( \hat{D}_M \) we therefore use the assumed erfc form of solution to compute the speed of the front at \( C = 0.1 \), purely due to diffusion as the mean front is about to exit the pipe, say \( \hat{V}_{f,D} \). We subtract this effect from the measured \( \hat{V}_f \) of fully diffusive experiments to recover the advective component of \( \hat{V}_f \). This is plotted against \( \hat{V}_0 \) in Fig. 15(b). For \( \hat{V}_{f,D} \) we have used both \( \hat{D}_M \) and an estimate of the effects of the initial transients in each experiment, also expressed as a diffusivity, \( \hat{D}_{M,i} \). We can see that the advective component of \( \hat{V}_f \) is very close to the mean velocity.

V. CONCLUSION

Displacement flow of two miscible iso-viscous Newtonian fluids in an inclined pipe has been investigated experimentally in the case where the displacing fluid is denser than the displaced fluid (i.e., density unstable). Our experiments have covered a broad range of the governing dimensionless parameter space \((\beta, Re, Fr)\), not covered before in any experimental study.

We have classified flows as fully diffusive, instantaneous, inertial and viscous, providing a qualitative description of each and delineating where each flow can be found in the dimensionless planes of \( Fr \) and \( Re \cos \beta/Fr \). It was also found that due to strong mixing at inclinations close to vertical, instantaneous displacements can still exist at values of \( \chi \) higher than the predictions of the lubrication model \((\chi_c = 116.32)\). For \( Re \cos \beta/Fr \) our results form a natural extension of those of Ref. 4, but the range of phenomena observed is significantly different here, as the pipe inclination is changed.

For each type of flow we have produced a closure approximation to the front velocity \( \hat{V}_f \) and to the bulk axial diffusivity \( \hat{D}_M \), that can be used for engineering calculations. When the flow is viscous the lubrication model\(^1\) can be an effective tool to predict \( \hat{V}_f \). In this case no bulk diffusion coefficient can be defined for the flow. On the other hand, when the flow is fully diffusive, the front velocity is very close to the mean imposed velocity \( \hat{V}_f \approx \hat{V}_0 \). It was also found that among the diffusive flows \( \hat{D}_M \) exceeds the Taylor dispersion coefficient, \( \hat{D}_T \), by up to an order of magnitude, over the full range of experiments.

Another important contribution of this work is in broadening our knowledge of the stabilizing/de-stabilizing effect of the mean flow on the well-studied exchange flows.\(^7\)–\(^\text{11}\) The effect of the mean flow is investigated over all inclination angles. Interestingly we have found that the imposed velocity can have quite different effects (stabilizing and/or de-stabilizing) on the flow, all controlled by the parameter \( Re \cos \beta/Fr \). In particular we found that the stabilizing effect of the mean flow found in Ref. 2 is valid up to \( Re \cos \beta/Fr \approx 270 \). Above this limit the imposed flow was found to progressively destabilize the flow up to \( Re \cos \beta/Fr \approx 500 \). Above this limit the imposed flow has a neutral effect on the flow since the mixing is already very high for exchange-dominated regime and the nature of the system is diffusive.

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