Incomplete fluid–fluid displacement of yield stress fluids in near-horizontal pipes: Experiments and theory

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ABSTRACT

We present results of a primarily experimental study of buoyant miscible displacement flows of a yield stress fluid by a higher density Newtonian fluid along a long pipe, inclined at angles close to horizontal. We focus on the industrially interesting case where the yield stress is significantly larger than a typical viscous stress in the displacing fluid, but where buoyancy forces may be significant. We identify two distinct flow regimes: a central-type displacement regime and a slump-type regime for higher density ratios. In the central-type displacement flows, we find non-uniform static residual layers all around the pipe wall with long-wave variation along the pipe. In the slump-type displacement we generally detect two propagating displacement fronts. A fast front propagates in a thin layer near the bottom of the pipe. A much slower second front follows, displacing a thicker layer of the pipe but sometimes stopping altogether when buoyancy effects are reduced by spreading of the front. In the thin lower layer the flow rate is focused which results in large effective Reynolds numbers, moving into transitional regimes. These flows are frequently unsteady and the displacing fluid can channel through the yield stress fluid in an erratic fashion. We show that the two regimes are delineated by the value of the Archimedes numbers (equivalently, the Reynolds number divided by the densimetric Froude number), a parameter which is independent of the imposed flow rate. We present the phenomenology of the two flow regimes. In simplified configurations, we compare computational and analytical predictions of the flow behaviour (e.g. static layer thickness, axial velocity) with our experimental observations.

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1. Introduction

There are many industrial processes in which it is necessary to remove a gelled material or soft-solid from a duct. Examples include bio-medical applications (mucus [27,34], biofilms [7,52]), cleaning of equipment and food processing [6,8], oil well cementing and waxy crude oil pipeline restarts. A wide range of material models are used to describe residual deposits in these situations. Some of these flows are turbulent, but equally often process limitations dictate that the flows be laminar. It is this case that we study here. Our industrial motivation comes from the oil industry, and we consider that the fluid to be removed is either a drilling ingredient of industrial design rules for oilfield cementing [10,29,40], and latterly also simulation based design models [3,41]. Further features of oilfield cementing are discussed in [36], but here our geometry is simpler.

In waxy crude oil pipeline restarts (see [5,12,42,49]) a large pressure is applied at one end of the pipe, to break the gel of the waxy oil. The waxy state has formed due to a drop in temperature below the wax appearance temperature, often related to stopping the pipeline for maintenance or other issues. Temperature is not particularly

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important in the restart process itself [51]. It is common to displace the in situ oil with an oil of different physical properties (often this is the same oil at higher temperature and thus with a Newtonian viscosity). In the displacement it is possible for static residual layers to form on the walls of the pipeline; see [17,50].

The phenomenon of a static wall layer in a plane channel was first studied by Allouche et al. [2] who studied symmetric displacement flows of two visco-plastic fluids. As is intuitive, a necessary condition for the existence of a static wall layer is that the yield stress of the displaced fluid exceeds that of the displacing fluid (if there is one). In [2] a constant displacement flow rate was imposed. A number of two-dimensional (2D) simulations were performed with no density difference, and the static layer thickness was measured from the results. Additionally, expressions were derived for the maximal static layer thickness (for vertical plane channels in the presence of a density difference), and for a prediction of the actual layer thickness (no density difference). Frigaard et al. [20] extended this approach, showing that in a steady displacement flow with a uniform static wall layer the thickness of the layer and the shape of the interface are non-unique for the steady displacement problem and consequently must result from transient aspects of the flow. The concept of maximal static wall layers was further explored in [18]. More recently in [53], an extensive computational study of static layer thickness in iso-density fluid displacements (Newtonian fluid displacing Bingham fluid) was performed, including the effect of flow rate oscillations. This has shed further light on the effects of the main three dimensionless parameters (Reynolds number, Bingham number and viscosity ratio), in the absence of density differences.

In contrast to the amount of computational work, there are relatively few experimental studies of displacement of yield stress fluids by other fluids. Gabard [21], and Gabard and Hulin [22] investigated iso-density miscible displacements in which a more viscous fluid is displaced by a Newtonian fluid. In their experimental investigation the geometry used was a vertical tube. They observed the effect of rheology of the displaced fluid and the flow velocity on the transient residual film thickness during the displacement process. They showed that in displacements of shear-thinning fluids the residual wall layer thickness decreases compared to Newtonian fluid displacements. The shear thinning fluid displacements were characterised by slowly evolving interfacial instabilities of (inverse) bamboo type, which further reduced the initially symmetric residual wall layer. For yield stress fluids static residual wall layers were observed of uniform thickness. Axisymmetric computations were also carried out in [21] with results which were qualitatively similar to the experimental results.

Other experimental studies involving two fluid flows of yield stress fluids in the pipe geometry include [11,32] who have studied the exchange flow problem (i.e. buoyancy driven flow in a closed ended pipe). The focus of these studies is stopping the motion using the yield stress of one of the fluids. Huen et al. [28] and Hormozi et al. [26] have studied core–annular flows, using a yield stress fluid for the outer lubricating layer and a range of different Newtonian and non-Newtonian fluids for the core. The start-up phase of these experiments is displacement-like, although the final steady state is a multi-layer flow.

In the Hele-Shaw geometry, Lindner et al. [30,31] studied the Saffman–Taylor (viscous fingering) instability while displacing yield stress fluids. They observed a yield stress dominated regime at low velocity and a viscous dominated regime when the velocity was higher. The former regime shows branched patterns because in simple words each finger does not really feel the presence of walls or other fingers due to the fluid’s yield stress. In the viscous dominated regime, yield stress does not play an important role. Other investigations of viscous fingering (with stability analyses) include [9] and the earlier Darcy-flow analogues of [37–39].

Finally, a number of authors have considered the displacement of yield stress fluids by a gas. De Souza Mendes et al. [13] investigated the displacement of viscoplastic flows in capillary tubes experimentally through gas injection. They showed that below a certain critical flow rate, the visco-plastic liquid is completely displaced by the displacing fluid. However above this critical flow rate small lumps of unyielded liquid will remain on the walls. For increased values of imposed flow rate a smooth liquid layer of uniform thickness forms. They reported that the thickness of this layer increases with the dimensionless flow rate. There have also been extensive computational studies of these flows; see [14,15,43,48]. Finally, there is a limited amount of analytical work concerning bubble propagation/displacement in Hele-Shaw geometries; see [1].

The aim of our study is to deepen our understanding of yield stress fluid displacements in pipes in a regime that has not been previously studied. Namely we study displacement of fluids with large yield stress and where buoyancy is significant. Near horizontal pipelines and wells lead to situations in which buoyancy forces promote slumping and asymmetry for Newtonian displacement flows. When a yield stress fluid is involved the displaced fluid rheology may counter the tendency to stratify, but these flows are poorly understood. The situations of primary industrial interest are those in which the yield stress is very large, so that static residual layers may persist.

The outline of our paper is as follows. In Section 2 we outline the scope of our study and the experimental methods used. Results are presented in Section 3. We first describe the main finding of the paper, namely the observation of two principal types of flow delineated by the ratio of Reynolds number to densimetric Froude number (equivalent to the square root of the Archimedes number). We then describe in more detail the features of the central (Section 3.2) and slump (Section 3.4) type displacements. The paper ends with a brief summary.

2. Scope of the study and methodology

As explained in the introduction, the aim of our study is to better understand displacement flows of visco-plastic fluids in near-horizontal pipes. The choice of a near-horizontal pipe follows from our previous work on Newtonian–Newtonian fluid displacements, where this range of pipe inclinations is found to exhibit interesting transitions between inertia-buoyancy and viscous-buoyancy dominated regimes; see [44]. In moving from an iso-viscous buoyant Newtonian–Newtonian fluid displacement to a displacement flow of a typical shear-thinning visco-plastic fluid by a Newtonian fluid, we have at least three more dimensionless parameters, e.g. for a Herschel–Bulkley model. Although we consider pipe inclinations $\beta \approx 90^\circ$, a comprehensive experimental study of flow variations with the remaining six dimensionless parameters is infeasible. Therefore, we focus on displacing visco-plastic fluids with large yield stress, with the motivation that these are the flows that are most problematic from an industrial perspective. However, the idea of large yield stress needs qualifying.

Suppose we displace a yield stress fluid (with yield stress $\tau_Y$) with a Newtonian fluid of viscosity $\mu$, by imposing a flow rate $Q = \pi V_0 D^2/4$, through a long pipe of diameter $D$, i.e. $V_0$ is the mean velocity. Inevitably the fluid will finger through some part of the pipe cross-section, potentially leaving behind residual layers as the displacement front propagates. Apart from close to the tip of the finger we might suppose that the Newtonian flow in the bulk of the finger becomes near-parallel and generates viscous stresses of order $\tau = \mu V_0 / D$. If we wish to study flows in which it is possible for the visco-plastic fluid to be left behind as the displacement
front propagates along the pipe, it is clear that a first requirement is:

$$\mu \dot{V}_0/D = \dot{\tau}_y \ll \tau_y.$$  \hfill (1)

We may reformulate this as

$$B_N = \frac{\tau_y D}{\mu \dot{V}_0} \gg 1$$  \hfill (2)

Here $B_N$ is a form of Bingham number, but with the viscous stress scale coming from the Newtonian fluid, as would be appropriate in this type of flow.

Secondly, since the flow close to the displacement front will be three-dimensional we must consider inertial stresses as well as viscous. The inertial stress scale is $\tau_\ell = \rho \dot{V}_0^2$. If we were to consider flows for which $\tau_\ell \sim \tau_y$, meaning $Re \sim O(1)$, then (2) would imply that $\dot{\tau}_y \ll \tau_y$. It is then unlikely that we would see much variation in our results as $\dot{V}_0$ is varied. Consequently, we have targeted our study at the range $Re > 1$, where inertial effects are dominant close to the displacement front. Further, we wanted to see how changes in flow rate might affect the type of displacement observed and so have selected flow parameters such that:

$$\rho \dot{V}_0^2 \gg \tau_y.$$  \hfill (3)

Note that if $\dot{\tau}_y > \tau_y$, then we enter a regime in which inertial stresses alone might be sufficient to yield and fully displace the viscoplastic fluid; hence the inequality above, which is equivalent to $Re/B_N \ll 1$.

A third consideration for our study is that we wish to observe buoyancy effects. The appropriate scale for buoyant stresses transverse to the pipe is $\tau_{\beta} = \Delta \rho g D \sin \beta$, where $\Delta \rho$ is the absolute density difference between fluids (axial buoyancy stresses are much smaller than this since $\beta = 90^\circ$). If we hope to observe significant effects of buoyancy on the type of displacement flow, we would expect that the buoyancy stress contributes to yielding at the front.

Thus, we have selected fluid parameters such that:

$$\Delta \rho g D \sin \beta \gg \tau_y.$$  \hfill (4)

If instead $\dot{\tau}_b \ll \tau_y$ it is likely that there would be no effect of buoyancy. The three conditions (2)–(4) frame the parameter space of our experiments.

2.1. Experimental description

Our experimental study was performed in a 4 m long, 19.05 mm diameter, transparent pipe with a gate valve located 80 cm from one end; see Fig. 1. The pipe was mounted on a frame which could be tilted to any angle. Initially, the lower part of the pipe is filled with water in a mixing tank, while the mixing blade was rotating. The displacing fluid was fed by gravity from a large elevated tank. The flow rate was controlled by a valve and measured by both a rotameter and a magnetic flowmeter, located downstream of the pipe. At the start of the experiment the gate valve is opened. Images of the displacing fluid are recorded using two cameras, and subsequently analyzed to characterize different aspects of the flow. Velocity is also measured through the central plane of the pipe at a position downstream of the gate valve, using an Ultrasonic Doppler Velocimeter (UDV). These methods are described below in Section 2.1.2.

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1 For interpretation of colour in Figs. 2–4 and 10–19, the reader is referred to the web version of this article.
and the power law index ($n$) were below 7% and 12% respectively.

Table 1

<table>
<thead>
<tr>
<th>Carboxol solution</th>
<th>Carboxol % (wt/wt)</th>
<th>NaOH % (wt/wt)</th>
<th>$\tau_y$ (Pa)</th>
<th>$n$</th>
<th>$\kappa$ (Pa s$^n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.1125</td>
<td>0.0322</td>
<td>1.17</td>
<td>0.60</td>
<td>0.49</td>
</tr>
<tr>
<td>B</td>
<td>0.12</td>
<td>0.0343</td>
<td>3.05</td>
<td>0.60</td>
<td>8.24</td>
</tr>
<tr>
<td>C</td>
<td>0.15</td>
<td>0.0429</td>
<td>6.51</td>
<td>0.39</td>
<td>4.63</td>
</tr>
<tr>
<td>D</td>
<td>0.14</td>
<td>0.040</td>
<td>3.12</td>
<td>0.26</td>
<td>20.44</td>
</tr>
</tbody>
</table>

cone tip. The cone and plate were roughened with a thin layer of sand paper (400 grit roughness), to avoid slip. Identical loading procedures were followed in all tests. Temperature was controlled to match that in the experimental room. Supplementary experiments were carried out with Carboxol solution D, nearly 18 months after the original experiments with A–C. Rheological measurements were carried out using the same procedure but now with a parallel plate geometry and a different rheometer (HR Nano from Malvern). A rheological model that fits well the shear behaviour of Carboxol behavior is the Herschel–Bulkley model:

$$\tau = \tau_y + \kappa \gamma^n. \quad (7)$$

This includes the simpler Bingham, power law and Newtonian models and is defined by three parameters: a fluid consistency index $\kappa$, a yield stress $\tau_y$, and a power law index $n$. From our rheometer data, we determined the yield stress value through the shear stress value at the global maximum of the viscosity. Afterwards, we subtracted the yield stress value from the remaining shear stress data and then we found the best fit to a power law curve. The error in determining the yield stress value of the Carboxol solution in this way is estimated to be in the range 5–27%. The errors in the consistency ($\kappa$) and the power law index (n) were below 7% and 12% respectively.

In fact the Carboxol rheology plays little apparent role in our experiments, as we target the range $B_0 \gg 1$ where much of the displaced fluid will be unyielded, i.e. it is important to have a large yield stress, but the rheology after yielding is probably irrelevant for our particular experiments. Determined values of the rheological constants for each of the four Carboxol solutions that we have used are shown in Table 1. An example flowcurve from the rheometer compared with the fitted Herschel–Bulkley model data is shown in Fig. 2.

2.1.2. Image processing and local velocity measurement

In order to help the visualization of the two fluids, the pipe was illuminated from behind by a light box containing fluorescent light tubes filtered through a diffusive paper to give a homogeneous light. Light absorption calibration was carried out for both cameras. During the experiment (after opening the gate valve), images were obtained at regular time intervals, which enabled us to create spatiotemporal diagrams of the averaged concentration profiles along the length of the pipe. The fronts were marked on these diagrams by a sharp boundary between the different relative concentrations of the fluids. The front velocities were obtained from the slope of this boundary.

We also measured the velocity profile at 80 cm below the gate valve, using an Ultrasonic Doppler Velocimeter DOP2000 (model 2125, Signal Processing SA) with 8 MHz, 5 mm (TR0805LS) transducers (with a duration of 0.5 μ s). This velocimetry technique suits our experimental needs well since it does not require transparent fluids and is completely non-intrusive. The measuring volume has a cylindrical shape and its the axial resolution in our fluids is around 0.375 mm and the lateral resolution is equal to the transducer diameter (5 mm) slightly varying with depth. The slightly diverging ultrasonic beam enters the fluids by passing through a 3.175 mm-thick plexiglass pipe wall. This technique is based on the pulse-echo technique and allows measurement of the flow velocity projection on the ultrasound beam, in real time [25]. For the tracer, we used polyamid seeding particles with a mean particle diameter of 50 μ m, with volumetric concentration of 0.2 g l$^{-1}$ in both fluids. Following [4], there is a trade off between a good signal to noise ratio and small ultrasonic signal reflections, achieved by mounting the probe at an angle in the range 68–72° relative to the axis of the pipe.

3. Results

Two qualitatively distinct flows were observed in our experiments. In some flows the displacing fluid propagated approximately centrally along pipe, leaving behind residual layers on all walls. We call this a central type displacement and describe its characteristics below in Section 3.2. In other displacements the heavier fluid appeared to slump to the lower part of the pipe and propagate along the lower wall. As far as could be observed, the interface was approximately horizontal as measured in a transverse plane and the flow stratified progressively in the length-wise direction. We call this a slump type displacement and describe its characteristics below in Section 3.4.

3.1. The transition between central and slump displacements

It was not surprising that the slump displacements appeared to occur for larger density differences. However, we sought a more quantitative description for their occurrence. All our experiments were purposefully designed to satisfy (2)–(4), meaning a large yield stress. This suggested that the yield stress itself would not play a significant role in determining flow type. Equally, since all flows had significant residual layers it appeared that the Carboxol must be yielded only close to the front and far ahead of the displacement front in thin wall layers (i.e. Poiseuille flow). Thus, it also seemed unlikely that the sheared rheology of the Carboxol would be particularly relevant to our experiments.

With the above considerations, we were led to consider only those dimensionless parameters relevant to the Newtonian displacing fluid: the Atwood number, $A_t$, the Reynolds number $Re$ and the densimetric Froude number $Fr$. In buoyant displacement flows, $A_t$ independently influences only the inertial terms in the momentum balance. For density differences of less than 10% this effect can be largely ignored between the fluids, and our $A_t$ falls in this range. Neglecting $A_t$, buoyancy still has a significant influence through the densimetric Froude number $Fr$. On analyzing our data we discovered that the transition between regimes was governed by the ratio $Re/Fr$ and was largely independent of all other dimensionless groups we considered. A selection of plots,
showing this dependency on $\text{Re}/\text{Fr}$, and independency with respect to other parameters, is shown in Fig. 3.

The parameter $\text{Re}/\text{Fr}$ is interesting in that it is independent of the mean velocity $V_0$. For small $At$, $\text{Re}/\text{Fr}$ is equivalent to the square root of the Archimedes number, $Ar$:

$$
Ar = \left[ \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} g D^3 \right]^{1/2} \cdot \frac{\text{Re}}{\text{Fr}} = \frac{\rho_1 (\rho_1 - \rho_2) g D^3}{(\rho_1 + \rho_2)^{1/2} \mu_1} \cdot \left[ \frac{1}{2} \frac{Ar}{2} [1 + O(At)] \right]^{1/2}
$$

(8)

One could also include a $\sin \beta$ term with the gravitational constant above, but for simplicity this is neglected. The Archimedes number occurs commonly in flows where both forced and natural convective forces are involved: large $Ar$ indicates dominance of the buoyancy forces, as indicated here by the stratified slumping.

A dependency on $\text{Re}/\text{Fr}$ was also evident in our studies of Newtonian fluid displacements; see [44]. In those experiments, different flow regimes are delineated in the plane of $Fr$ versus $\text{Re} \cos \beta / Fr$. Here we have not explored variation with $\beta$, using only two pipe inclinations. However, it is worth pointing out that the effects are anyway markedly different to the Newtonian displacements. Firstly, there is little apparent effect of the velocity (captured here in $Fr$). Secondly, in the Newtonian fluid studies, stratified viscous regimes were associated with smaller values of $\text{Re}/Fr$, whereas here the reverse is observed: the central regime is found for smaller $\text{Re}/Fr$.

3.2. Central-type displacements

Examples of central-type displacements are shown in Figs. 4 and 6, at different imposed flow rates, inclinations and for different Carbopol solutions. Figs. 4a and 6a show sequences of images as the displacing fluid advances steadily through the Carbopol. The front shape is skewed towards the top of the pipe, which suggests inertial dominance at the tip/front. Purely viscous effects would lead to slumping. The bottom image on each figure shows the scale for the images, which can be interpreted as a mean concentration at each position. After the displacement front has passed we see darker regions at the top and bottom of the pipe, but also at mid-height the images indicate that there is residual displaced fluid. Darkness at the top and bottom is simply because we are viewing from the side of the pipe, essentially looking through the residual layer rather than perpendicular to the layer. This pattern is consistent with the presence of a residual wall layer all around the pipe and the images suggest that the layer is not uniform.

Fig. 4b shows the spatiotemporal plot corresponding to the displacement of Fig. 4a. The boundary separating dark and light regions indicates the position of the propagating front. It is relatively easy to calculate the front velocity from such images. Behind the

![Fig. 3. Classification of our experiments: - slump type displacement; - central type displacement.](image-url)
front, we observe vertical streaks in the spatiotemporal plot, indicating that the residual layers are static. Computation of the mean concentration at different positions in the region behind the displacement front suggests that 30–40% of the Carbopol is not displaced. A similar range is found for the displacement of Fig. 6a.

Concentration-based estimates of the residual layer thickness can be compared with estimates of the mean layer thickness, made from the front velocity measurements. For example, in Fig. 6a the imposed flow velocity is \( V_0 = 44 \text{ mm s}^{-1} \) whereas the measured front velocity is \( V_f = 64 \text{ mm s}^{-1} \). Assuming the residual layers are static, this suggests a mean layer thickness

\[
\hat{h} = \frac{D}{2} \left[ 1 - \sqrt{\frac{V_0}{V_f}} \right] \approx 1.63 \text{ mm},
\]

with \( \approx 31\% \) of the Carbopol remaining. In general we have found that such estimates are self-consistent, but as we can see in both Figs. 4a and 6a there is variation in layer thickness along the pipe.

To quantify this variation in layer thickness we have analysed data from the rectangular time-space region indicated by the broken line in Fig. 4b. First of all we have averaged with respect to the axial distance \( \hat{x} \) and with respect to time, to give a mean concentration \( \overline{c}(y) \) at each depth in the pipe; see Fig. 5a. Looking in the mid range of depths, we can see that \( \overline{c}(y) \) is skewed about the centre-line, meaning that the centrally propagating fronts are not axisymmetric (presumably a buoyancy effect). Close to the top and bottom of the pipe the data in Fig. 5a is hard to interpret quantitatively, since curvature effects also come into play.

In Fig. 5b we look at the axial variation of the concentration. Here we average with respect to depth \( y \) and also with respect to time. This plot and the various insets show that the main larger amplitude fluctuations are occurring on the order of 50–100 mm. The small high frequency oscillations correspond approximately to the pixel scale, 1 pixel \( \approx 1 \text{ mm} \), and are likely to be image-related. Thus, the variations evident in figures such as Fig. 4 are long-wavelength.

Fig. 6a presents a second example of a central displacement, and has been discussed above. A representative example of the velocity profiles obtained from the UDV measurement is shown in Fig. 6b, for the same experiment as Fig. 6a. The UDV probe is fixed at 80 cm below the gate valve angled at 68° to the surface of the pipe. The velocity readings are taken through the pipe centreline in a vertical section. The vertical axis shows depth measured from the top of the pipe. The velocity contours are averaged time-wise over 25 velocity profiles (3 s). We can see that the main flow

---

Fig. 4. Central displacement for \( \beta = 83^\circ \). At \( t = 3 \times 10^{-1} \), \( \bar{V}_0 = 32 \text{ mm s}^{-1} \) with Carbopol solution A: (a) images of the displacement at \( t = 4, 6, \ldots, 20, 22 \text{ s} \) after opening the gate valve; (b) spatiotemporal image of the same displacement. The plot shows a 833 mm long section of the pipe a few centimeters below the gate valve. The rectangular region marked by the broken line is explained in the text and is used in the following figures.

Fig. 5. Variation of (a) \( \overline{c}(y) \) and (b) \( \overline{c}(x) \) in the rectangular region (illustrated by a broken line) in Fig. 4b. The insets in (b) show that the interfacial modes of the mean static layer apparently have long wavelength variations, of the order of 50–100 mm.
is more or less in the center of the pipe. Also looking carefully we can see very thin static residual layers close to the walls. Although the resolution of the readings falls off close to the walls, we shall see later that identical readings are obtained for slump-like displacements where the upper layer is very thick. Thus, we consider these layers to be static.

The dark broken curves superimposed on the UDV contours indicate where a perfectly axisymmetric concentric flow would have its static layer, (based on the same mean concentration as that measured). The numbers within the velocity field indicate the speed (mm/s) that this same axisymmetric concentric flow would measure). The numbers within the velocity field indicate where a perfectly axisymmetric concentric flow would have its static layer, (based on the same mean concentration as that measured). The numbers within the velocity field indicate the speed (mm/s) that this same axisymmetric concentric flow would have.

Although we have not observed much sensitivity of the central-slung transition to changes in \( V_o \), there are other qualitative changes in the flow as \( V_o \) is increased. An example sequence of central displacements is shown in Fig. 7, for increasing \( V_o \). The flow becomes progressively unsteady during this sequence. The residual layer thickness generally decreases.

### 3.2.1. Maximal static residual layers

Some insight into the presence of residual static layers is gleaned from a simple one-dimensional model. Suppose that the flow is steady, laminar, axisymmetric and that the outer layer of fluid is static. If the interface between the Newtonian inner fluid and outer static layer is at radius \( r_i \), then it is straightforward to show that the magnitude of the shear stress at the outer wall of the pipe is given by:

\[
\tau_w = \frac{8\mu \dot{V}_0}{D} \frac{1}{\lambda_i} \frac{1}{4} (1 - \lambda_i)(\dot{\rho}_1 - \dot{\rho}_2) \bar{D} \hat{g} \cos \beta.
\]

where \( \lambda_i = 2r_i/D < 1 \). Provided that \( \tau_w \leq \tau_Y \) then the fluid layer may remain static.\(^2\) We readily see that (9) can be interpreted in alternate way. For a given yield stress \( \tau_Y \) an outer layer may remain static only for \( \lambda_i \geq \lambda_{i,\text{min}} \), for which:

\[
\tau_Y = \frac{8\mu \dot{V}_0}{D} \frac{1}{\lambda_{i,\text{min}}} \frac{1}{4} (1 - \lambda_{i,\text{min}})(\dot{\rho}_1 - \dot{\rho}_2) \bar{D} \hat{g} \cos \beta.
\]

We see that \( \lambda_{i,\text{min}} \) is given by:

\[
\lambda_{i,\text{min}} = 2 \frac{\phi_b}{\phi_b} \left(1 + \frac{\phi_b}{4}\right) \left[1 - \sqrt{1 - \frac{8\phi_b}{B_N(1 + \frac{\phi_b}{4})}}\right],
\]

where the two dimensionless groups are:

\[
\phi_b = \frac{(\dot{\rho}_1 - \dot{\rho}_2) \bar{D} \hat{g} \cos \beta}{\tau_Y \dot{V}_0}, \quad B_N = \frac{\tau_Y \bar{D}}{\mu \dot{V}_0}.
\]

The first of these is the ratio of axial buoyant stress to the yield stress of the fluid. Since we have designed our experiments so that (4) is satisfied, we typically have \( \phi_b \sim \cot \beta \ll 1 \). The second parameter \( B_N \) appears in (2), which states \( B_N \gg 1 \) for our experimental design.

Fig. 8 shows contours of \( 1 - \lambda_{i,\text{min}} \) in the \( \phi_b-B_N \) plane. In this figure the shaded area marks the limit where no static wall layers are possible. We can interpret \( 1 - \lambda_{i,\text{min}} \) as a dimensionless maximal

\(^2\) Strictly speaking, some additional assumptions are needed to ensure that the second term on the right-hand side above is not too large, or we may have a buoyancy driven flow backwards against the mean flow. These assumptions are anyway met by our experimental conditions.
static layer thickness (scaled with the pipe radius). For our range of experiments ($\phi_B \ll 1$ and $B_N \gg 1$), we have the approximation:

$$\lambda_{\text{min}} \approx 2 B_N^2 (1 - \frac{\phi_B}{4} - \frac{24 \phi_B}{B_N} + \cdots)$$  \hspace{1cm} (12)

Thus, the maximal layers in our experimental range are always predicted to be close to 1, i.e. a very thin radial channel down the centre of the pipe. This is very far from what we observe: the actual layer thicknesses (residual volume fractions) are far less than the maximal possible. This is not unexpected and has been observed in other geometries, e.g. [2,21,53]. The cause of the over-prediction of the residual layer thicknesses (residual volume fractions) are far less than the predicted to be close to 1, i.e. a very thin radial channel down the centre of the pipe. In any case these maximal stresses are minimal here. The jump in shear stress gradient is barely perceptible across the interface. In the eccentric case we have slightly larger stresses at the wall, but in any case these maximal stresses are far below the yield stress (expressed dimensionlessly by $B_{Ny}$ and $B_{Nw}$).

### 3.3. Axial flow computations

To gain insight into the observed phenomena, and in particular to give evidence at the computational level that residual layers are indeed static, we compute the flow in a simplified configuration, assuming the flow to be steady and uniaxial along the pipe. The cross-section of the pipe is assumed to be divided into two domains: $\Omega_1$ (for the displacing fluid) and $\Omega_2$ (for the displaced fluid). We scale the axial velocities with the mean imposed velocity $V_0$, lengths with $D$ and adopt a stress-scale $\mu V_0/D$, based on the viscous shear stress in the Newtonian fluid. The scaled velocity $w(z,y)$ satisfies the following problem:

$$-f = \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial y^2}, \quad (z,y) \in \Omega_1,$$

$$b - f = \frac{\partial}{\partial z} \tau_{xz} + \frac{\partial}{\partial y} \tau_{xy}, \quad (z,y) \in \Omega_2,$$

$$\left(\tau_{xz}, \tau_{xy}\right) = \left[k\left|\nabla w\right|^{\alpha-1} + \frac{B_N}{\nabla w}\right] \nabla w, \quad \Leftrightarrow \left|\left(\tau_{xz}, \tau_{xy}\right)\right| > B_N,$$

$$\left(z,y\right) \in \Omega_2,$$

$$\left|\nabla w\right| = 0, \quad \Leftrightarrow \left|\left(\tau_{xz}, \tau_{xy}\right)\right| \leq B_N, \quad \left(z,y\right) \in \Omega_2,$$

where $k = k V_0^{1-\alpha}/(\mu D^{\alpha-1})$ is a dimensionless consistency, $f$ is the dimensionless modified pressure gradient and $b > 0$:

$$b = \phi_B B_0 = \frac{\left(p_1 - p_2\right) D^2 \cos \beta}{\mu V_0} = \frac{2\pi \mu D^2 \cos \beta}{\nu V_0},$$

which is a buoyancy parameter. Here $\nu$ is defined using the mean density $\bar{\rho} = (\rho_1 + \rho_2)/2$ of the two fluids and the viscosity $\mu$ of the Newtonian fluid. At the pipe wall, $w(z,y) = 0$. Both the shear stress and velocity are continuous at the interface. Finally, since the flow rate is fixed the following flow rate constraint is satisfied:

$$\int_{\Omega_1} w(z,y) dy = \frac{\pi}{4}$$  \hspace{1cm} (17)

This constraint is used to find $f$ iteratively.

Problems of this nature are considered theoretically in [19] and computationally in [35]. The computations shown below and later in the paper have been computed using a finite element method, as described in detail in [35]. The numerical code solves the flow problem for a general uniaxial flows of two Herschel–Bulkley fluids (although in our case one fluid is always Newtonian). The algorithm used is the augmented Lagrangian algorithm (ALG2) of Glowinski and co-workers (see [16,23,24]), with slight modifications.

We present two example computations, for the same experimental parameters as in Fig. 4. Using the mean concentration from the experiment we have defined the area fraction of the Newtonian displacing fluid and then computed the velocity and stress field, assuming that the interface is circular. In Fig. 9a and b, the interface is concentric and the velocity solution can be validated against the analytical solution (easily obtained). In Fig. 9c and d we have eccentered the interface towards the top of the pipe. In this figure, the eccentricity $e$ is defined as the distance between the centre of the circular Newtonian fluid region and the centre of the circular pipe, divided by the difference in radii of these two circles.

The advantage of using the augmented Lagrangian method for this type of problem, with a yield stress fluid, is that below the yield stress the strain rate is exactly zero. We see that the residual layers in Fig. 4a and c are unyielded and have zero velocity. The shear stresses increase radially outwards from the centre of the Newtonian fluid domain, even when eccentric, and the maximal shear stresses are found at the wall. The effects of (axial) buoyancy are minimal here. The jump in shear stress gradient is barely perceptible across the interface. In the eccentric case we have slightly larger stresses at the wall, but in any case these maximal stresses are far below the yield stress (expressed dimensionlessly by $B_{Ny}$ and $B_{Nw}$). When the yield stress is so much larger than the wall shear stresses generated, the axial flow of the inner fluid 1 is completely decoupled from that of fluid 2, i.e. fluid 2 is simply a solid.

### 3.4. Slump-type displacements

At the beginning of Section 3 we presented the main finding of our study, namely that central and slump displacement regimes are sharply delineated by the value of $Re/Fr$. The interesting feature of this parameter is that it is independent of the imposed flow rate (i.e. $V_0$). Although this is the case, we do in fact see significant qualitative changes in the flow as $V_0$ is increased. We describe these changes below, via examination of the results of two experimental sequences of increasing $V_0$.

#### 3.4.1. Sequence 1: $\beta = 85^\circ$, $At = 10^{-2}$, Carbopol C

In essentially all of our slump-type displacements the displacing fluid was observed to propagate along the bottom of the pipe in a fairly thin but fast moving layer. A significantly slower second front followed, typically with a much thicker layer of displacing fluid and an orientation that was (at least initially) approximately perpendicular to the pipe axis. Fig. 10 shows the velocity of the two fronts for different imposed $V_0$ within this sequence. In this sequence, neither front stops during the time of the experiment.

At lower flow rates the second displacement front moves very slowly. Fig. 11a illustrates such a displacement. The initially perpendicular second front becomes progressively sloped as the displacement continues, but still moves very slowly. Presumably
inertial forces are relatively weak here and the buoyancy gradients are weakened as the front elongates, i.e. there is some form of slow relaxation.

We see that the interface between fluids appears to slowly move upwards, i.e. the residual layer is thinned over time. In Fig. 11b we show the velocity contours from the UDV system for the same experiment, superimposing also the interface height (estimated from the measured mean concentration). We observe that the thick layer towards the top of the pipe is static. This is not surprising: the flow and interface are pseudo parallel and since \( \frac{B_0}{C_{176}} \) we expect the viscous stresses (transmitted across the interface) to be insufficient to yield the upper layer. Curious however is that the interface still moves and the upper layer thins. We can only speculate that this might be due to erosion from the interface over time. We can see that the initial velocity in the lower layer is quite high and only relaxes later as the residual layer thins. The effective Reynolds number in the lower layer is of order 10^3 so that we might expect some unsteadiness in the flow, as is suggested by the UDV. Unsteady interfacial stress fluctuations could weaken the gelled upper layer allowing mixing and dilution of the Carbopol solution.

Two further examples are shown at increased \( \hat{V}_0 \) in Fig. 12. We show again snapshots of the displacement and also the UDV contours. The most notable difference is that here the second interface moves steadily along at a significant speed. Although there is evidence of some residual fluid near the top of the pipe, behind the second front, the displacement is quite effective. In the early part of the experiment, when only the fast moving lower front has passed the UDV position, we observe that the upper residual layer is apparently moving in a plug like fashion with decay in velocity close to the upper wall. Since all the flow does not pass
in the lower layer with the first front, it is necessary that the upper layer moves, in order to conserve mass. As the second front passes the upper residual layer is thinned considerably, but now becomes static. The entire imposed flow rate is now passing below the interface. At the larger flow rate the final residual layer is thinned.

Fig. 11. Displacement of Carbopol C for $\beta = 85^\circ$, $At = 10^{-2}$ at $V_0 = 26$ mm s$^{-1}$: (a) a sequence of snapshots showing a 990 (mm) long section of the pipe a few centimeters below the gate valve at $t = 30, 60, \ldots, 570$ s600 after opening the gate valve; (b) contours of velocity from the UDV at 80 cm below the gate valve: readings taken through the pipe centreline in a vertical section. The vertical axis shows depth measured from the top of the pipe. Velocity data is averaged time-wise over 25 velocity profiles (3 s), but no spatial averaging/filtering is applied. The broken line illustrates the depth of the interface, as inferred from the normalized concentration across the pipe at the UDV position.

Fig. 12. Displacement of Carbopol solution C for $\beta = 85^\circ$, $At = 10^{-2}$: (a and b) show data for $V_0 = 42$ mm s$^{-1}$; (c and d) show data for $V_0 = 57$ mm s$^{-1}$. (a) Snapshots of the displacement at $t = 1, 3, \ldots, 31$ 33 s in a 990 mm long section of the pipe a few centimeters below the gate valve. (b) UDV contours and superimposed interface position estimated from the mean concentration. (c) Snapshots of the displacement at $t = 1, 2, \ldots, 16$ 17 s in a 990 mm long section of the pipe a few centimeters below the gate valve. (d) UDV contours and superimposed interface position estimated from the mean concentration.
3.4.2. Sequence 2: $\beta = 85^\circ$, $At = 1.6 \times 10^{-2}$, Carbopol B

The second sequence we examine is for Carbopol B, and with a slightly larger density difference. The main interesting feature of this sequence, by comparison to the previous one is that although there are two fronts, the second front stops at long times. It appears that there is some form of relaxation of the stresses close to the second front, as the displacement progresses. Possibly the buoyant stresses diminish as the interface slope changes. A second possibility is that the initial front channels through making a sufficiently wide channel to accommodate the entire flow rate without yielding the upper layer. This then would have the effect of reducing the inertial stresses close to the second front. We have seen in the central displacements that at large $B_0$, the maximal static layer thickness is very large. Below we shall perform a similar calculation for slump-like configurations.

An example of a slump-like displacement where the second front stops is shown below in Fig. 13. The deceleration of the second front is evident in the images of Fig. 13a. For this displacement the second front does not reach the UDV position before stopping. In Fig. 13b the velocity contours therefore show the mobile lower layer only, which is increasing gradually in thickness over this time frame. We observe as the layer expands the maximum velocity decreases slightly. Spatiotemporal diagrams shown in Fig. 13c and d indicate that the second front, although static, has displaced all the Carbopol. We can see that further down the pipe there are light spots in the spatiotemporal diagram. These indicate unsteadiness in the flow, which we discuss later. However, we also observe that the concentration lines in spatiotemporal diagram becomes progressively vertical as the displacement progresses, suggesting that the residual layer is static.

Qualitatively similar displacements are found at other flow rates ($V_0$) in this sequence. As the velocity is increased we observe that the initial front speed also increases and that the depth of static layer (say $d_{\text{static}}$, measured from the top of the pipe), also decreases. These effects are shown in Fig. 14. We also find that the distance that the second front travels before stopping increases with $V_0$. These effects are perhaps intuitive in that larger displacement velocities generally give rise to larger stresses.

In Fig. 15 we show the axial flow solutions for the final static layer depths of two of the experiments in the sequence of Fig. 14. We have assumed a perfectly horizontal interface of the same height as that in the experiments and computed the velocity and stress profiles. We observe firstly that in both cases the computed solutions are indeed static. However, compared to our earlier computations with the circular interface, the stresses induced in the static layer appear significantly larger. Part of this is an effect of restricted flow area, but part is due to the interface shape. Note in particular the high stresses in the upper layer close to the interface at each side wall. This suggests that, for equal flow areas, the stratified configuration will yield before the central configuration.

Fig. 13. An example of slump-like displacement for which the second front stops; ($V_0 = 30\text{mm.s}^{-1}$, $\beta = 85^\circ$, $At = 1.6 \times 10^{-2}$, Carbopol solution B). (a) Images at $t = 2.6, 34, 62, 66$ s after opening the gate valve for a 990 (mm) long section of the pipe a few centimeters below the gate valve. (b) Velocity contours from the UDV measurement, at 80 (cm) below the gate valve. The normalized concentration across the pipe is interpreted as an interface height (shown by the broken line). (c and d) Spatiotemporal diagrams, both close to the gate valve and further downstream.
We should note that in the case that the upper fluid is static, the stress field computed is simply an admissible candidate stress field.

By iterating the 2D computational solution we may compute the maximal depth of the static layer $d_{\text{max}}$ (measured down from the top of the pipe, scaled with the pipe diameter). We have computed this for three values of $\phi_B$ that span the range of our experiments; see Fig. 16a. To interpret this figure, at fixed layer depth $d$ the layer is static for any $B_\eta$ above the curve, or alternatively for any fixed $B_\eta$ the layer is static for any depth $d \leq d_{\text{max}}$. We observe a slight increase in $d_{\text{max}}$ with $\phi_B$. Note that $\phi_B$ represents a buoyancy force upwards along the pipe, opposing the mean flow and...
thus reduces the shear stress in the upper layer, increasing the layer thickness. However, for larger values of $\phi_B$ this buoyancy force would eventually yield the upper layer, moving it backwards against the mean flow. This also happens in the concentric interface case considered earlier. In Fig. 16b we have numerically computed the flow rate through the displacing fluid layer (expressed as

\[
\frac{B_N}{B} = 0.01, \quad \frac{B_N}{B} = 0.1, \quad \frac{B_N}{B} = 0.5
\]

and $C_0$, all computed at fixed buoyancy parameter, $B = B_{0\phi_B} = 250$.

Fig. 16. (a) Maximal static layer depth $d_{max}$ measured from the top of the pipe, computed iteratively from the 2D computational solution: $\phi_B = 0.01 - \square; \phi_B = 0.1 - \bigcirc; \phi_B = 0.5 - \triangle$. (b) Fraction of total flow rate flowing in the lower layer as a function of layer depth $d$ and $B_N$, all computed at fixed buoyancy parameter, $B = B_{0\phi_B} = 250$.

Fig. 17. Unsteady slump-like displacement for $\beta = 85^\circ$, $At = 10^{-2}$, $V_0 = 36$ mm s$^{-1}$ with Carbopol solution C. (a) From top to bottom we show images for $t = 1, 2, 3, \ldots, 11, 12, 13, 14, 15$ s after opening the gate valve. The figure shows a 990 mm long section of the pipe a few centimeters below the gate valve. The last image at the bottom is the colourbar of the concentration values. (b) Spatio-temporal diagram for the same experiment. (c) Velocity contours as measured by the UDV, situated at 80 cm below the gate valve.
a fraction of the total flow rate), for large $B_N$ and for a range of layer depths $d$. For these computations we have fixed the buoyancy parameter, $b = B_N/B = 250$ (roughly an upper bound for our experimental range). We can see that even when the displaced fluid layer does yield, the fraction of flow rate in the upper layer is relatively small (0–25%).

### 3.4.3. Unsteady displacements

Within the slump-like displacements there were a number of experiments that gave rise to distinctly unsteady flows. Typically this unsteady behaviour was found in the layer close to the base of the pipe, where the first front is displacing. This type of flow is characterised in less extreme cases by the occurrence of irregularly shaped regions of displaced Carbopol above the narrow lower layer. We have already observed this type of effect (e.g. see Fig. 12a and c (quite faint), and Fig. 13a).

In more extreme cases the unsteady flow can leave the lower part of the pipe and channel through in a disorderly fashion. An example of this is shown in Fig. 17a, with spatiotemporal plot in Fig. 17b and UDV data in Fig. 17c. In this flow the front channels through the Carbopol initially higher up in the pipe, before settling down again to the lower part of the pipe. We can observe from the spatiotemporal plot that after the front has passed by any fixed position in the pipe the spatiotemporal pattern becomes largely stationary in time, indicating that the residual layers and shape of yielded regions are static even though quite irregular. The UDV signal indicates temporal fluctuations in the velocity field.

What is interesting about this type of flow is that we have observed some of the most unsteady flows at intermediate flow rates. We are uncertain of the causes of this type of flow, but make the following comments. Firstly, the apparent bias towards the lower part of the channel suggests that buoyancy is perhaps important in these flows, exerting a stabilizing influence on the orientation of the channel. Secondly, we suspect that the yield stress fluid is largely passive in determining the direction of the propagating front. We are in the regime $B_N \gg 1$ for which it is possible for a Newtonian fluid to channel through a pipe in any variety of shaped channels, leaving the outer fluid static. To illustrate this we have taken the experimental values from Fig. 17 and computed from the mean concentration a representative area fraction corresponding to the channeling Newtonian fluid. In Fig. 18 we show the computed 2D axial velocity for a selection of different channel cross-section shapes. Although the stress field is different in these different cases, in no case are we close to yielding the outer fluid.

Thirdly, although our experiments are conducted at significant $Re$, these $Re$ values are below transition for a single fluid pipe flow. However, this led us to speculate that the effective Reynolds number could be much larger within the narrow channel formed at the base of the pipe as the first front propagates. To explore this we have calculated a relative hydraulic Reynolds number $Re_h$ for three sets of experiments in slump type displacement regime at $\beta = 85^\circ$.

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**Fig. 18.** Velocity profiles, $w(z,y)$, obtained through 2D computation with parameters of the experiment shown in Fig. 17, $(B_N = 3405.4, \phi_a = 0.04906)$: (a) an eccentric circular interface with $e = 0.48$; (b) a centred square interface; (c) a plane interface. In all plots the area fractions are equal, corresponding approximately to a channel area fraction based on the mean concentration in Fig. 17. Broken white lines indicate the interface.
Our experimental apparatus before any instability is observed. If at the bottom of the pipe, it may well be that this front exits rapidly and stably. Although there is an initial very fast precursor face, the area and perimeter of the fast moving channel is estimated must be acknowledged. Using the fast front \( \frac{V_f}{V_b} \), which is typically very thin. Therefore, we compute \( \frac{h_1}{h_0} \) though a control volume method, using \( \frac{h_1}{h_0} \), \( \frac{V_f}{V_b} \), and \( V_0 \). Although the values of \( \frac{h_1}{h_0} \), \( \frac{V_f}{V_b} \) and \( V_0 \) can be obtained with acceptable accuracy, estimation of \( \frac{V_f}{V_b} \) from the spatiotemporal contains is less accurate, so that significant errors in our estimate of \( h_1 \) must be acknowledged. Using \( h_1 \) and assuming a horizontal interface, the area and perimeter of the fast moving channel is estimated and \( R_{b0} \) is computed. The results are shown in Fig. 19. Fig. 19 indicates that \( R_{b0} > 1000 \) is quite common in the propagating channel, in each of these three experimental sequences. As is evident from the spatiotemporal plots that we have seen earlier, we have significant long-wave axial variation in the geometry of the displacing fluid channel. At these high \( R_{b0} \) and with a varying geometry we speculate that the flow is quite possibly turbulent or transitional. Potentially this is the cause of the erratic direction taken by the advancing front. We have seen earlier that the displacing fluid seems to intermittently rupture the Carbopol, opening up into a larger irregular channel. This larger channel will evidently slow the flow, perhaps re-laminarising locally. Although speculative, we feel this is a possible stabilising influence, countering the transitional/turbulent flow regions in the narrower channels, and that overall this is a plausible propagation mechanism for unsteady displacements.

On the other hand we can note some anomalies. Considering for example the sequence at \( \frac{h_1}{h_0} = 1 \times 10^{-2} \) and Carbopol solution C, shown in Fig. 19, we can see that \( R_{b0} \) increases with \( R_{re} \). On the other hand, the lower flow rates (hence \( R_{re} \)) appear more unstable than the higher flow rates. Compare Figs. 12 and 17. At the higher flow rates the secondary fronts in Fig. 12 appear to propagate fairly rapidly and stably. Although there is an initial very fast precursor front at the bottom of the pipe, it may well be that this front exits our experimental apparatus before any instability is observed.

4. Summary

We have explored displacement flows of a yield stress fluid by a Newtonian fluid, with a laminar imposed flow along a long pipe in-clined close to horizontal. The study has been focused at the regime where the yield stress is far larger than the characteristic viscous stress and, although laminar, the Reynolds numbers are significant. We have observed two distinct flow regimes: a central-type regime where the displacing fluid propagates in a finger along the centre of the pipe, and a slumping configuration, where the displacing fluid moves along the bottom of the pipe. The transition between these two characteristic flow types appears to occur at a critical ratio of the Reynolds number to the densimetric Froude number, and has been found largely independent of other dimensionless groups.

In both regimes, due to the large yield stress, we find residual layers of displaced fluid present at long times. Both our UDV measurements and computations from a simplified axial flow model, suggest that these residual layers are fully static. In each case we see slow axial variation in layer thickness along the pipe axis and it appears that the residual layers are significantly thinner than the computed maximal static layers.

At larger displacement flow rates we have generally seen a decrease in the residual layer thickness. For the central displacements the flows became progressively unstable as the flow rate increases. For the slump like displacements we have observed a number of different evolutions. Firstly, there are typically two fronts: a rapidly propagating lower front along the bottom of the pipe, followed by a slower front that displaces a larger fraction of the Carbopol. The second front may in some cases stop completely. The first front has been observed to destabilise and propagate erratically along the pipe.

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