Dynamics of the removal of viscoplastic fluids from inclined pipes

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\textbf{A B S T R A C T}

The dynamics of the removal of a viscoplastic fluid by a Newtonian fluid is investigated experimentally and theoretically in an inclined pipe based on our previous studies on near-horizontal and highly inclined configurations. The fluids are miscible. The displacing Newtonian fluid is heavier than the displaced viscoplastic one i.e. the configuration is density-unstable. In our earlier work it was found that two major flow regimes, namely center-type and slump-type, might occur depending on the density difference. These flows are explored in great details through measurements of the displacement speeds and hydraulic Reynolds numbers. The residual viscoplastic layer unevenness is characterized revealing that the flows are in the range of large roughness regimes. Through an integrated experimental-theoretical approach, estimates of the interfacial and wall shear stresses are given which is of great importance in designing the displacement and cleaning processes involving fluids with yield stress. Accompanied by Ultrasonic Doppler Velocimetry (UDV) data, the dynamics of the removal of the viscoplastic fluid from a pipe is elucidated suggesting three distinct phases in the displacement process namely a plug flow, inertial multi-dimensional flow at the displacing front and steady multi-layer developed flow. Finally, the viscoplastic displacement flow results are compared against the predictions of the closure model, previously proved successful for Newtonian and shear-thinning fluids displacement in pipe. It is found that in the case of viscoplastic fluids the closure model always over predicts the displacing front velocity due to the inertial stresses present at the front not captured in the model.

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1. Introduction

There exist many industrial processes in which it is necessary to remove a gelled material from a duct. Some examples include oil & gas well cementing [1], waxy crude oil pipeline restarts [2], Enhanced Oil Recovery (EOR) [3], Gas Assisted Injection Molding (GAIM) [4], biomedical applications (mucus [5], biofilms [6]), cleaning of equipment and stubborn soil [7], food processing [8] and personal care [9]. A wide range of models are developed to describe residual deposits in these situations. Some of these flows are turbulent [10], but equally often process limitations dictate that the flows be laminar. We study this laminar-imposed-flow case in the current manuscript.

In the context of cleaning-in-place (CIP) processes, numerous researchers have attempted to characterize the cleaning time as a function of the flow parameters. Fryer et al. [8] found that in CIP processes, increased flow rates induced greater surface shear on the deposit shortening the cleaning time. On the other hand the cost of pumping the cleaning fluid may become excessive at higher flow rates. Depending on the flow conditions, there might exist a threshold below which the mechanical effect of flow is negligible. In some specific cases no significant change in cleaning rate was reported when moving from laminar to turbulent flow. Using temperature, conductivity and turbidity probes, Cole et al. [9] conducted experiments to study the cleaning of toothpaste in a pipe. They found that the removal occurs mainly in three steps: In the first stage named as core removal, a core of the viscoplastic fluid is displaced from the pipe. Most of the remaining layer of the material on the wall is then slowly sheared away by fluid action in the second stage known as thin film removal. Finally in the third stage called patch removal, patches of the deposit left on the surface are gradually removed. While the core removal stage develops over the same time scale as that of the advective flow, thin film removal can take a very long time (majority of the cleaning time). The overall cleaning time was also found to be influenced by temperature and velocity of the cleaning fluid. A recent study by Palabiyik et al. [10] showed that the amount of product recovered in core removal is not a function of flow conditions. However, they interestingly
found that the conditions during the core removal can significantly affect the two latter thin films and patch removal stages and thus the overall cleaning time. The overall cleaning time could be reduced by at least 25% upon selecting the best removal conditions at different stages.

There are a number of studies in the literature on the displacement of a viscoplastic fluid by an immiscible fluid (mostly gas). Although the current manuscript focuses on a miscible displacement, many of the underlying mechanism can be similar to an immiscible case. One of the early studies in this field is that of Polinski et al. [11] which experimentally studied isothermal displacement of a viscoplastic liquid by gas in a horizontal tube with applications to GAIM process. They found that the thickness of the viscoplastic coating at high gas penetration rates approaches 0.35 of the tube radius, the asymptotic limit which was previously observed for Newtonian liquids [12]. However, at low gas penetration rates, the viscoplastic coating can be much thinner than its Newtonian counterpart. Shear-thinning and viscoelastic effects were discussed in [13,14] afterwards. Dimakopoulos and Tsamopoulos [15] later numerically studied the high-capillary number, inertialess displacement of a viscoplastic material by air in straight and suddenly constricted tubes motivated by GAIM and EOR processes. In the case of straight tubes, they reported similar film thickness values to those found in [11]. Counter-intuitively, local small decrease of the film thickness with increasing Bingham number was also found. When there is flow constriction unyielded regions appeared near the recirculation corner, but not around the entrance of the narrower tube. More complex geometries than constricted flows were considered later in [16].

de Souza Mendes et al. [17], experimentally found that the viscoplasticity in capillary tubes alters the flow kinematics which in turn changes the amount of mass left attached at the tube wall dramatically compared to the Newtonian case [12]. It was also observed that there is a critical dimensionless flow rate below which the displacement can be perfect i.e. no observable liquid left attached to the wall. Similar results were observed numerically by de Sousa et al. [18]. They depicted that the fraction of the mass of shear-thinning and viscoelastic materials deposited on the tube wall decreases and the shape of the interface becomes flatter compared to the Newtonian case. Concerning flow patterns, in the shear-thinning and viscoplastic cases de Sousa et al. [18] also found an interesting type of intermediate flow regime which is not present in the transition from bypass to fully recirculating flows of the Newtonian case [12]. Fields of yielded and unyielded zones for the viscoplastic case were shown later in more details in [19]. In most of the immiscible flow studies, the displacement of a viscoplastic fluid by a gas has been considered. Recently, Freitas et al. [20] studied the immiscible displacement of a viscoplastic liquid by a viscoplastic liquid in a plane channel using finite element method. The fluids were modeled as Bingham fluids with a regularized viscosity function proposed by Papanastasiou [21]. Increasing the yield number of the displacing fluid or the displaced one induced a thinner film of mass attached to the wall. For the cases analyzed, it was shown that changing the yield number of the displaced fluid has more impact on the flow patterns than changing the yield number of the displacing fluid. The transition flow configurations of the viscoplastic–viscoplastic displacement were also found to be very different from those of the Newtonian–Newtonian displacement problem.

There are also a number of studies in the literature focusing on the displacement of a viscoplastic fluid by a miscible liquid which is of more relevance to the current paper. Due to the industrial applications we are interested in high Péclet number regimes where the degree of molecular diffusive transport compared to advective transport is very small. Gabard and Hulin [22] studied the miscible displacement of non-Newtonian fluids in a vertical tube experimentally. For shear-thinning fluids (water–Xanthan solutions), the residual film thickness was found to be 0.28–0.30 of the radius. For viscoplastic fluids (water–Carbopol solutions) this value decreased down to 0.24–0.25 of the radius. However, instabilities at the interface between the two fluids developed downstream, leading to a reduction of the final thickness of the film at longer times which was larger for lower viscosity ratios and higher velocities.

Motivated by the fluid–fluid displacement in a narrow eccentric annulus in primary cementing process, Allouche et al. [23] modeled the displacement of two viscoplastic fluids in a plane channel predicting the static wall layers thickness for various range of parameters. Through a lubrication approximation they calculated maximum possible static wall layer thickness. Upon numerically solving the lubrication model developed, it was shown that the interface asymptotically approached the maximum static layer thickness predicted analytically. However, results from fully two-dimensional displacement computations indicated that the displacement front propagates at a steady speed along the channel, leaving behind a static layer significantly thinner than the predicted maximal static layer thickness. This is related to the existence of the 2D flow and inertial stresses at the displacement front. Similar to an immiscible displacement [15], Allouche et al. [23] counter-intuitively found that the computed static layer thickness decreased with an increase in the dimensionless yield stress of the displaced fluid. Recently Swain et al. [24] studied the buoyant displacement flow of a viscoplastic material by a Newtonian fluid using lattice Boltzmann simulations. For the ranges studied, it was revealed that increasing the Bingham number and flow index increases the size of the unyielded region of the fluid in the downstream section of the channel and increases the thickness of the residual layer of the residual fluid left. It was shown that the presence of the unyielded material in the residual film led to the suppression of the interfacial instability at higher Bingham numbers, which in turn, reduced the speed of finger propagation. Preliminary studies of the stability of two-layer flow of a viscoplastic fluid and a laminar and turbulent Newtonian fluid are given in [25] and [26] respectively.

Biel et al. [27] studied the effects of pulsating turbulent flows on wall shear stress components in a straight pipe using a series of microelectrodes as a non-intrusive electrochemical method. Their analysis revealed that pulsating flows induce an increase of local velocity gradient at the pipe wall through periodical renewal of the boundary layers and redistribution of eddies size near the wall. They later studied the effect of this pulsating flow on CIP of the adhered bacterial spores, in addition to the effect of the mean velocity of the flow [28]. A high level of the cleaning rate was observed despite the reduction of the magnitude of the mean velocity. Through 2D computations, Wielage-Burchard and Frigaard [29] later studied the effect of inertia and flow pulsation on static wall layer thickness. Thinner layers were found at higher Reynolds numbers due to the increased energy production locally around the finger. Both small and large amplitudes were considered to study the effect of the flow rate pulsation. The conditions on the pulsation frequency and amplitude for mobilizing the static layers were presented for different range of flow and rheological parameters.

The displacement of a yield-stress (viscoplastic) fluid along a pipe or duct can be influenced by a large number of parameters, making a comprehensive study impractical. Our focus in this paper is on situations in which the yield stress of the displaced fluid is high and the displacement is thus difficult. Briefly, this means that the yield stress should be the dominant stress in the flows considered. We study flows where the typical viscous stresses generated by the imposed flow are far less than the yield stress. Both inertial and buoyant stresses can be comparable to the yield stress locally, e.g. close to the displacement front. We therefore study the effects
of pipe inclination on the displacement of a yield-stress fluid (here Carbopol) by a Newtonian fluid of higher density. This study is a sequel to that of [30,31], which covered various inclination angles from near-horizontal to fully vertical. The main observation made in these studies was that displacements fell into one of two main categories, referred to as center-type displacements and slump-type displacements. Briefly, in center-type displacements the displacing fluid advances through the center of the pipe, leaving an approximately uniform layer of Carbopol on the wall. The slump-type displacement involved the displacing fluid slumping underneath the lighter Carbopol. A more detailed description of these type displacements is given in Section 3.6 and 3.7 respectively. Estimates of the inertial stresses at the displacing front are characterized in Sections 3.6 and 3.7, respectively. Estimates of the inertial stresses at the displacing front velocities are given in Section 3.2. The hydromagnetic and hydraulic and Reynolds number based on the displacing fluid properties and Fr is the densimetric Froude number (see Section 2.2 for definitions).

The parameter $Re/Fe = \hat{\rho}_h/(\hat{\rho}_h - \hat{\rho}_l)D^{1/2}/(\hat{\rho}_h + \hat{\rho}_l)^{1/2}\hat{\mu}$, naturally represents a buoyancy-driven transition. Here $\hat{\mu}$ is the viscosity of the Newtonian fluid, $\hat{\rho}_0$ is the mean velocity, $\hat{D}$ is the diameter of the pipe and $\hat{\rho}_h, \hat{\rho}_l$ are the densities of the displacing and displaced fluids respectively. The destabilizing buoyant stresses $\{\hat{\rho}_h - \hat{\rho}_l\}\hat{D}$ may be balanced locally by any combination of yield, viscous or inertial stresses. In balancing with inertia, an appropriate velocity scale for buoyancy driven motion is $\hat{V}_B = [\hat{\alpha}_B \hat{D}]^{1/2}$, where $\hat{\alpha}_B = \hat{\rho}_h/\hat{\rho}_l)./(\hat{\rho}_h + \hat{\rho}_l)$ is the dimensionless Atwood number. We see that for small $\hat{\alpha}_B$, $Re/Fe \approx (\hat{\rho}_h - \hat{\rho}_l)^{1/2}/(2\hat{\mu}\hat{V}_B/\hat{D})$ In [30,31], it was observed that above some threshold in $Re/Fe \approx 800$ the displaced fluid yielded in a way that promoted stratification (i.e. slumping). This had the effect of reducing the local buoyant stresses.

The novelty of our study includes the following. (i) Quantitatively characterizing the displacing front velocity, $\hat{V}_f$, which is useful as an indicator of the displacement efficiency over full range of our experiments. In a long duct, with imposed mean flow velocity, $\hat{V}_0$, the displacement efficiency is proportional to $\hat{V}_0/\hat{V}_f$; see [32]. (ii) Characterizing the displacing fluid layer flow through measurements of its hydraulic diameter and Reynolds number. (iii) Quantifying the effective relative roughness in the residual viscoplastic layers. The surface roughness plays a very important role in flow dynamics characterization and displacement/cleaning processes design. Palabiyik et al. [10] postulated that changes in the wall layer induced during core removal of the toothpaste in pipe by water could significantly affect the overall cleaning time (a very wavy residual layer led to rapid removal in subsequent thin film and patch removals). Despite its importance, there unfortunately does not exist any study investigating and quantifying the magnitude of the surface unevenness. (iv) Through a simple and effective control volume analysis and based on the experimental measurements of the displacing layer area and front velocity, estimates of the interfacial and wall shear stresses in buoyant displacement flow of a viscoplastic fluid are given which are extremely important in designing the displacement and cleaning processes. (v) Responsible mechanisms for gel break-up are discussed in details backed up by Ultrasonic Doppler Velocimetry (UDV) results. (vi) The validity and effectiveness of the analytical closure model in [33], previously found successful in capturing the displacement flow behavior in pipe for Newtonian and shear-thinning fluids, is finally tested against experimental results for viscoplastic fluid displacements.

An outline of our paper is as follows. In Section 2 we review the experimental methodology and define the parameter space. The main results follow in Section 3. We first review general characteristics of center and slump-type flows in Section 3.1. Measurements of the displacing front velocities are given in Section 3.2. The hydraulic diameter $D_h$ and Reynolds number $Re_h$ of the displacing fluid layer are presented in Section 3.3 followed by the characterization of the relative roughness values $\hat{\varepsilon}/D_h$ within the viscoplastic residual layer given in Section 3.4. Regime classifications in $(Re_h - \hat{\varepsilon}/D_h)$-plane is then specified in Section 3.5. Through a control volume analysis, the interfacial and wall shear stresses for average and constricted flows are characterized in Sections 3.6 and 3.7, respectively. Estimates of the inertial stresses at the displacing front are presented in Section 3.8. Finally the results are compared against the predictions of the pipe stratified two-layer flow closure model of [33] in Section 3.9. The paper concludes with a brief summary in Section 4.

2. Displacement in a pipe

2.1. Experimental description

Fig. 1 shows a schematic of the experimental set-up; see [30,31] for details. In summary, the displacement flow experiments were performed in a 4 m long, 19 mm diameter, transparent pipe with a
gate valve located 80 cm from one end. The pipe was mounted on a frame which could be tilted to an angle, \( \beta \), measured from the vertical. Initially, the lower part of the pipe was filled with a less dense fluid (fluid 2) colored with a small amount of ink. The upper part of the pipe, above the gate valve, was filled with the denser fluid (fluid 1) colored with an equal amount of ink. The upper and lower parts of the pipe were separated by a thin horizontal line. Initially, the upper part of the pipe was filled with a less dense fluid (fluid 2) colored with a small amount of ink. The upper part of the pipe, above the gate valve, was filled with the denser fluid (fluid 1) colored with an equal amount of ink. The upper and lower parts of the pipe were separated by a thin horizontal line.

2.1. Parameter ranges studied

Table 2 Parameter ranges for the experiments reported in this paper.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ) (deg)</td>
<td>0–85</td>
</tr>
<tr>
<td>( V_0 ) (m/s)</td>
<td>0–120</td>
</tr>
<tr>
<td>( t_v ) (Pa)</td>
<td>0–26</td>
</tr>
<tr>
<td>( At )</td>
<td>0.0035–0.016</td>
</tr>
<tr>
<td>( B_N )</td>
<td>550–72000</td>
</tr>
<tr>
<td>( Re )</td>
<td>0–2300</td>
</tr>
<tr>
<td>( Fr )</td>
<td>0.1–6</td>
</tr>
<tr>
<td>( Re/\Pi )</td>
<td>260–1080</td>
</tr>
</tbody>
</table>

2.2. Parameter ranges studied

The parameter ranges covered by our experiments are given in Table 2; as in [30,31]. Note that 8 dimensionless groups describe the parameter space for a Newtonian Herschel–Bulkley fluid displacement, so that restriction of the parameter space is needed to focus on specific phenomena of interest. Our intention is to investigate flows for which some fraction of the yield-stress fluid could be left behind in the pipe, i.e. difficult displacements. Constraints imposed are as follows.

- We assume the viscous stresses \( \tau_v \) are small relative to the yield stress \( \tau_Y \). i.e. \( \mu \dot{V}_0 / D = \tau_v < \tau_Y \). or alternately:

\[ B_N > 1, B_N = \frac{\tau_v D}{\mu \dot{V}_0} \]

Here \( B_N \) is a Bingham number, but with the viscous stress scale coming from the Newtonian fluid.

- We allow the inertial stresses \( \tau_i \) to be comparable to the yield stress, i.e.

\[ \rho \dot{V}_0^2 = \tau_i < \tau_Y \]

where \( \rho = (\rho_H + \rho_L) / 2 \). Generally, we consider displacement inertial flows: \( Re = \rho \dot{V}_0 D / \mu > 1 \), but (2) also restricts \( \tau_i \) if \( \tau_i > \tau_Y \) then inertial stresses alone might be sufficient to yield and fully displace the yield-stress fluid. The inequality in (2) is equivalent to \( Re/B_N \approx 1 \).

- We consider displacements flows in which buoyancy stresses \( \tau_b \) may be significant in affecting yielding:

\[ \Delta \rho \bar{g} \bar{D} = \tau_b \sim \tau_Y \]

where \( \Delta \rho \) is the difference in density between the fluids.

The 3 conditions (1)–(3) define the parameter space of our experiments. Apart from the dimensionless groups introduced above, the density difference is characterized by the dimensionless Atwood number. The densimetric Froude number is defined as \( \Pi = \dot{V}_0 / (At \bar{g} \bar{D})^{1/2} \). The quantity \( \Pi^2 \) is the ratio of inertial and buoyancy forces.

3. Results

We first briefly review the general features of center and slump-type flows. Then we focus on a more detailed characterization of the dynamics of each of these flows in a series of sections.

3.1. General flow patterns observed

Fig. 2 shows typical experimental results for a center and slump-type displacement flow. Fig. 2(a) shows time sequences of a typical center-type displacement flow experiment. The displacing front travels approximately centrally through the pipe, leaving a (darker) residual layer of Carbopol all around the pipe. These residual layers are non-uniform with irregular, slow spatial variations in thickness. Fig. 2(b) shows snapshots of a slump-type displacement with all the governing parameters the same as in Fig. 2(a) except the density difference (or Atwood number). The parameter that delineated the qualitatively different types of displacement flows observed in [30,31] is the ratio \( Re/\Pi \), which is independent of flow rate. Within the experimental setup, varying \( Re/\Pi \) is accomplished by density variation and the center and slump-type displacements correspond to \( At = 0.0035 \) and \( At = 0.016 \), respectively (here and throughout the rest of the paper). It can be seen that slump-type flows are associated with an initial displacing front moving under the viscoplastic fluid at relatively high speed. In all slump-type displacements, whether for near-horizontal [30] or more inclined situations [31], we observed a breakage of the viscoplastic layer due to this fast-moving front. For nearly horizontal displacements the breakage is weaker than in the more inclined cases, and it happens at later times and further downstream since the streamwise component of buoyancy is smaller. Alba et al. [31] also described a range of more exotic sub-categories under the category of slump-type flows, that were not observed in the nearly horizontal experiments (e.g. corkscrew, blockage-type, etc.).

Table 1 Rheological properties of different Carbopol solutions used as displaced fluid in our experiments. Note that the displacing fluid is always a Newtonian salt-water solution with \( \mu = 0.001 \) Pa s.

<table>
<thead>
<tr>
<th>Carboxol solution</th>
<th>( \tau_Y ) (Pa)</th>
<th>( n )</th>
<th>( \kappa ) (Pa s^n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>11.74</td>
<td>0.58</td>
<td>2.63</td>
</tr>
<tr>
<td>B</td>
<td>19.65</td>
<td>0.56</td>
<td>4.27</td>
</tr>
<tr>
<td>C</td>
<td>6.92</td>
<td>0.71</td>
<td>1.56</td>
</tr>
<tr>
<td>D</td>
<td>22.58</td>
<td>0.53</td>
<td>6.32</td>
</tr>
<tr>
<td>E</td>
<td>14.89</td>
<td>0.67</td>
<td>2.66</td>
</tr>
<tr>
<td>F</td>
<td>4.26</td>
<td>0.60</td>
<td>1.32</td>
</tr>
<tr>
<td>G</td>
<td>15.96</td>
<td>0.53</td>
<td>3.94</td>
</tr>
<tr>
<td>H</td>
<td>3.82</td>
<td>0.61</td>
<td>1.23</td>
</tr>
<tr>
<td>I</td>
<td>3.05</td>
<td>0.60</td>
<td>8.24</td>
</tr>
</tbody>
</table>

Table 2 Parameter ranges for the experiments reported in this paper.
3.2. Displacing front velocity measurements

Time sequences of the depth-averaged concentration profiles can be used as a tool to measure the velocity of the displacing front. The image-processing procedure used to calculate the front velocity is similar to the iso-viscous Newtonian flows in [34,35]. Fig. 3(a) and (b) shows the dependence of $\hat{V}_f$ on $V_0$ and inclination angle, $\beta$, for (a) $At = 0.0035$ (center-type flows) and (b) $At = 0.016$ (slump-type flows). The solid lines show $\hat{V}_f = \hat{V}_0$. The dash-dotted lines show $\hat{V}_f = 1.62\hat{V}_0$ in (a) and $\hat{V}_f = 1.92\hat{V}_0$ in (b) obtained from curve-fitting to the data.
might approach efficient mixing at smaller $Re$ than the reported studies with density-stable configuration [9,10]. Note that due to our range of parameters studied, the effect of flow destabilization at higher $V_0$ resulting in $\hat{V}_f \approx V_0$ cannot be seen clearly from Fig. 3. Further experiments are required to fully prove this hypothesis. If this turns out to be true, it shows how important buoyancy can be in processes involved with cleaning of viscoplastic materials. In other words, (for downwards flows) by densifying the displacing fluid, one can displace the gel layer much more efficiently at a much lower pumping cost.

### 3.3. Displacing fluid layer area, hydraulic diameter and Reynolds number

Although a detailed regime classification has been presented in our previous studies [30,31], the dynamics of the displacement flow in each regime have not been analyzed, as we now do via the displacing fluid layer. In order to assess whether the flow within the Newtonian layer is laminar, transitional or turbulent we would need to know the hydraulic Reynolds number $Re_h$, defined as $Re_h = \hat{\rho} \hat{V}_f \hat{D}_h / \mu$, within this layer. For calculating $Re_h$ we need to know the speed of the displacing layer $\hat{V}_f$ (which we presented in Fig. 3) and the hydraulic diameter of the layer. In order to estimate the hydraulic diameter, we need to quantify the concentration of the fluids as a function of time and location. A convenient way of analyzing the concentration field and front velocity is by depth-averaging the concentration values at different streamwise locations. Fig. 4(a) and (b) shows the evolution of the depth-averaged concentration profiles with time along the streamwise direction, for the same snapshots as shown in Fig. 2(a) and (b), respectively. It can be seen that in the case of center-type flows, the depth-averaged value of the concentration field increases noticeably once the displacing front passes (Fig. 4(a)). On the other hand, for slump-type flows, there is only slight increase observed in the depth-averaged concentration after passing the displacing front (Fig. 4(b)).

The depth-averaged value of concentration profile $C$ can be now used to find a representative dimensionless area fraction, $\alpha$, through which the displacing layer flows. We have calculated $\alpha$ as the displacing fluid largest passage area fraction: $\bar{C} + \sigma (C)$, where $\bar{C}$ is the averaged concentration along the depth and length of the pipe at a time the flow is well developed, and $\sigma$ is the corresponding standard deviation. Note that in the case of slump flows we have only focused on the displacing front finger to assess the flow dynamics, and not the entire flow which might include ripped, corkscrew, ... patterns [31].

We find that $\alpha$ is larger for center-type flows than for slump-type flows, with neither flow changing significantly with inclination angle, $\beta$. The average values of $\alpha$ for center and slump-type flows are 0.55 and 0.17, respectively. For center-type flows the average value of 0.55 corresponds to a residual film layer with $\approx 0.26$ radius of the pipe which is in the range of values reported in [22] for upwards vertical displacement flows of viscoplastic fluids by Newtonian fluids. There is no discernible variation of $\alpha$ with $Re$ for slump type displacements and only a very slight increase of $\alpha$ with $Re$ in center type displacements. This corresponds to a decrease in the residual film thickness with inertia, which is in line with the findings of the numerical study [29].

Having measured the displacing fluid layer area $\alpha$ and its speed $\hat{V}_f$, we can now assess in what proportion the mean flow speed $\hat{V}_0$ is divided between the displacing and displaced layers. Assuming that the total flow goes through the displacing layer (i.e. a completely static displaced layer) the continuity equation requires $\hat{V}_f = \hat{V}_0 / \alpha$. Therefore, the deviation of $\hat{V}_0 / \alpha$ from the measured $\hat{V}_f$ is a measure of how fast the displaced layer is moving throughout the experiment. Fig. 5(a) and (b) compares $\hat{V}_0 / \alpha$ against the front velocity $\hat{V}_f$, at different inclination angles, $\beta$, for center and slump-type flows respectively. The solid lines represent $\hat{V}_0 / \alpha = \hat{V}_f$. Naturally $\hat{V}_0 / \alpha$ increases monotonically with $\hat{V}_f$. Close agreement found between $\hat{V}_0 / \alpha$ and $\hat{V}_f$ in Fig. 5(a) suggests that most of the flow goes through the Newtonian layer and any residual viscoplastic layer remains static throughout the duration of the experiment. Fig. 5(b) in turn shows that there is a significant difference between $\hat{V}_0 / \alpha$ and $\hat{V}_f$ in the case of slump-type flows. Thus, in slump type flows the displaced fluid layer is also generally moving. This corresponds to the observation in [30] of a slower moving (trailing) displacement front.

The center-type displacement shown in our case is very similar to the core removal mechanism shown in [9] for cleaning of toothpaste by water in a pipe. In [9] the authors found that once the core removal phase is over, the viscoplastic layer is gradually removed through thin film removal which can take a long time. A predictive formula for dimensionless cleaning time $t_c = \hat{V}_0 \hat{t}_c / D$ as a function of the Reynolds number has been given in [9] as

$$t_c = \frac{9 \times 10^7}{Re^{0.78}}. \quad (4)$$

Applying (4) to our experiments gives dimensionless cleaning times $t_c \approx 4.1 \times 10^2$ for $Re \approx 1000$. As our flow loop is $\approx 200$ diameters, these timescales would correspond to the circulation of $\approx 10^3$ pipe volumes in our experiment. Hence, it is not surprising that we find the static layers stationary in our experiments. It should
be noted that (4) was derived for density stable configurations in a horizontal pipe at $Re > 8000$. Thus, its strict applicability to our experiments is unclear.

Upon measuring the displacing layer area $\alpha$, the hydraulic diameter can be conventionally defined as $D_h = \frac{4A}{P}$ where $A$ and $P$ are cross-sectional area and wetted perimeter respectively. It is not difficult to show that the hydraulic diameter for center-type flows can be obtained as $D_h = \sqrt{\alpha D}$ and for slump-type flows as $D_h = \frac{\pi \alpha D}{(c + s)}$, where $c$ and $s$ are the dimensionless cord and arc lengths of the circular segment respectively; see later illustration in Fig. 12(b). Note that both the cord and arc lengths have been used in calculating the wetted perimeter in the case of slump-type flows since there can exist a non-zero shear rate/stress over these lengths. Fig. 6(a) and (b) shows the dependency of $Re_h$, defined as $Re_h = \frac{\rho \dot{V}_f D_h}{\mu}$, on $Re$, at different inclination angles, $\beta$, for center and slump-type flows respectively. First note that similar to Fig. 3, there is no apparent dependency of the results on $\beta$ in either center or slump cases. Although there is a monotonic increase in $Re_h$ with $Re$ in the case of center-type flows, this dependency seems to be non-monotonic for slump-type flows (initial increase and then decrease in $Re_h$ with $Re$). Also interestingly we observe that the $Re_h$ values in center-type flows are overall larger than slump-type ones, which is counter-intuitive given the fact that the speed of the fast displacing front in the latter is larger than that of the former.

Note that a much larger range of parameters is considered here, than for the near-horizontal measurements in [30]. Also compared to [30] we have used an area-based measurement technique for $\alpha$ and $D_h$, rather than estimating the interface height.

### 3.4. Effective roughness in viscoplastic residual layers

In the previous section, measurements of the hydraulic Reynolds number $Re_h$ within the Newtonian displacing layer were presented. As observed in Fig. 6, most of $Re_h$ values fall below a nominal transition Reynolds number $Re \approx 2100$. However, we cannot yet conclude that flow within the displacing layer is laminar because of the non-uniformities observed in the viscoplastic residual layer (e.g. see Fig. 4). In order to fully understand the dynamics of the flow within the Newtonian layer and ultimately understand the mechanism of the removal of the viscoplastic layer, we need to quantify the roughness formed at the interface between the layers. Fig. 7 shows a schematic diagram of the rough interface formed after the displacing fluid front has passed. There are a number of methods proposed in the literature regarding the characterization of the surface roughness, most of which yielding to largely
similar answers; see [36] for a review. Adopting a fairly similar approach in [36], the hydraulic diameter diameter $D_h$, the constricted flow diameter $D_{cf}$, and the roughness $\bar{C}$ can be defined as shown in Fig. 7. Here $\bar{D}_h$ and $\bar{D}_{cf}$ are simply calculated based on areas $\bar{C} + \sigma (C)$ and $\bar{C} - \sigma (C)$ respectively, where $\bar{C}$ is the averaged concentration along the pipe and $\sigma$ is the corresponding standard deviation. The roughness is then expressed as $\bar{\epsilon} = (\bar{D}_h - \bar{D}_{cf})/2$.

Roughness of the residual viscoplastic layer plays a very important role in the dynamics of the displacing layer flows within. Palabiyik et al. [10] hypothesized that changes in the wall layer induced during core removal of the toothpaste in pipe by water can significantly affect the overall cleaning time. In their case a very large relative roughness values observed in our study. They found that in the transitional regime the friction factor is approximated by:

$$f = \frac{\bar{\epsilon}}{D_h} + 64/R_h$$

where $R_h$ is the corresponding standard deviation of the roughness $\bar{\epsilon}$.

The relative roughness values measured in our viscoplastic displacement flow experiments fall in the range $0.03 \leq \bar{\epsilon}/D_h \leq 0.25$, which is mostly outside the range of a standard Moody chart for pipe flows. In other words, we can consider the flow of the displacing fluid within the (largely static) rough residual layer as a flow in a pipe with large roughness [37]. The problem of flow in pipes/channels with large relative roughness has recently received a lot of attention particularly in the context of micro-fluidics flows [36–40] due to the comparable size of the surface roughness to the length scales of interest. The comprehensive recent study of Huang et al. [37] experimentally measures the friction factor in pipes with large roughness for $0.03 < \bar{\epsilon}/D_h < 0.3$ which covers the range of the roughness values observed in our study. They found that in the laminar regime the friction factor is approximated by:

$$f = \left(10210 \left(\frac{\bar{\epsilon}}{D_h}\right)^2 - 529.66 \frac{\bar{\epsilon}}{D_h} + 64\right) / R_h$$

(5)

It can be seen that for large roughness, even in the laminar regime, the friction factor $f$ is a function of $\bar{\epsilon}/D_h$ (contrasting with the assumed independence of the laminar friction factor $f = 64/R_h$ for $\bar{\epsilon}/D_h \leq 0.03$). Huang et al. [37] also showed that the threshold Reynolds number from laminar to transitional flows can be expressed as

$$Re_L = \frac{6340.3 \bar{\epsilon}}{D_h} + 2304.4,$$

(6)

i.e. significantly reduced by roughness. The threshold Reynolds number from transitional to fully rough turbulent flows is approximated in [37] as

$$Re_T = \frac{3542 \log \left(\frac{\bar{\epsilon}}{D_h}\right) - 3739.6}$$

(7)

Using (5) in the laminar regime and expressions from [37] for the fully rough regime (where Re-independence is exhibited), we have reconstructed the friction factor map in Fig. 9(a) for values of $\bar{\epsilon}/D_h$ covering the range of our experiments. A linear interpolation has been used for the friction factor in the transitional regimes. The friction factor curve for a smooth pipe $\bar{\epsilon}/D_h = 0$ is also plotted for comparison using the formula given in [41]. We can see that in the presence of the large relative roughness values, the friction factor can be significantly larger than that of a smooth pipe.

3.5. Classification of the flow within displacing fluid layer

We are now able to make a regime classification. Fig. 9(b) shows the dynamics of the displacing fluid layer flows classified in terms of laminar, transitional and fully rough turbulent in the plane of $R_h$ and $\bar{\epsilon}/D_h$. The solid line indicates the laminar-transitional threshold (eq:ReL) and the dashed line the transitional-turbulent threshold (eq:ReT), both from [37]. Both center (solid data) and slump-type (hollow data) experiments are included. It can be seen that most data points fall into the laminar regime and a few data points (mostly belonging to center-type flows) in the transitional region. The figure suggests that the displacing fluid layer mostly flows in the laminar regime which might be counter-intuitive given the irregular shape of the interface formed by the viscoplastic layer, see Figs. 4 and 10 for instance. Recall that in the case of slump-type flows once the displacing fast front forms and develops over time it can cause ripping and breakage in the viscoplastic layer [31]. Here we have only attempted to classify the dynamics of the flow within the finger region. The flow in the vicinity of the rapped sections and/or at the front will be more complex. Although the picture given in Fig. 9(b) largely suggests that the displacing fluid layer flows in laminar regime, an important question remains unanswered: how reliable is the regime classification picture given i.e. can it be validated upon other methods? In order to assess this, we have examined our Ultrasonic Doppler Velocimetry (UDV) data to see if they also confirm the general trends shown in Fig. 9(b). A DOP2000 velocimeter (model 2125) from Signal Processing SA
Fig. 9. (a) Friction factor, \( f \), plotted versus the hydraulic Reynolds number, \( Re_h \), for various relative roughness values \( \hat{\epsilon}/\hat{D}_h \) based on Huang et al.'s study [37]. Friction factor for a smooth pipe case with \( \hat{\epsilon}/\hat{D}_h = 0 \) is also plotted for comparison using the formula given in [41]. b) Dynamics of the displacing fluid layer in the viscoplastic experiments classified in terms of laminar, transitional and fully rough in the plane of \( Re_h \) and \( \hat{\epsilon}/\hat{D}_h \). The solid line indicates \( Re_L = -6340.3\hat{\epsilon}/\hat{D}_h + 2304.4 \) and the dashed line \( Re_T = -3542\ln\hat{\epsilon}/\hat{D}_h - 3739.6 \) as the critical threshold Reynolds numbers [37]. Different markers represent \( \beta = 0^\circ \) (□), 15° (△), 30° (♦), 45° (▲), 60° (▼), 75° (◁) and 85° (○) a n d8 5° (◦). Solid and hollow symbols refer to center-type (At = 0.0035) and slump-type flows (At = 0.016) respectively. The UDV profiles for the experiments marked by black squares and red square are given in Fig. 10 and 11(a), respectively.

Fig. 10. Ultrasonic Doppler Velocimetry (UDV) profiles obtained for (a) \( \beta = 83^\circ \), At = 0.0035, \( \hat{V}_0 = 71 \text{ mm/s} \) and Carbopol solution H at \( \hat{t} = 24 \text{ s} \) (inset shows a snapshot of the experiment over a length of 1383 mm located 1696 mm below the gate valve at \( \hat{t} = 23.5 \text{ s} \)). (b) \( \beta = 60^\circ \), At = 0.0035, \( \hat{V}_0 = 63.92 \text{ mm/s} \) and Carbopol solution E at \( \hat{t} = 30 \text{ s} \) (inset shows a snapshot of the experiment over a length of 1341 mm below the gate valve at \( \hat{t} = 31.5 \text{ s} \)). (c) \( \beta = 85^\circ \), At = 0.016, \( \hat{V}_0 = 30.03 \text{ mm/s} \) and Carbopol solution I at \( \hat{t} = 99 \text{ s} \) (inset shows a snapshot of the experiment over a length of 1096 mm located 1000 mm below the gate valve at \( \hat{t} = 17.5 \text{ s} \)). The dashed lines show the approximate position of the interface in each experiment. The dotted lines correspond to the laminar Poiseuille profiles fitted to the displacing fluid layer velocities. The UDV probe is placed at \( \hat{x} = 1560 \text{ mm} \) in (a), (b), (d) and \( \hat{x} = 300 \text{ mm} \) in (c).

was used with 8 MHz, 5 mm (TR0805LS) transducers (with a duration of 0.5 \( \mu \text{ s} \)). The UDV profiles for the experiments marked in Fig. 9(b) by black squares and red square are given in Figs. 10 and 11(a) respectively.

Fig. 10 (a),(b) and (c),(d) shows the streamwise velocity profile for center and slump-type flows respectively, predicted to be laminar in Fig. 9(b). The snapshots in the figures show the corresponding flow pattern in each case. Due to the nature of the UDV measurements there is always some refraction errors present close to the lower wall of the pipe (note the non-zero velocity values in Fig. 10(b)–(d)). The downward skewness in the velocity profiles observed in our previous studies [30,31] has been corrected...
in this paper. The dashed lines show the approximate position of the interface between the displacing and displaced fluids in each experiment. Through the area in which the displacing layer flows, a parabolic laminar Poiseuille velocity profile has been fitted which has equal integral over the layer depth $y$ to the UDV velocity profiles (see dotted lines). This is meant only as a guide and it is understood that in the slug flow the vertical distribution of velocity will not be precisely parabolic. In all the figures it can be seen that the agreement between the fitted and UDV profiles is reasonable, suggesting that the flow within the displacing layer is indeed laminar as earlier predicted in Fig. 9(b). Fig. 10(a) shows a center-type flow case where there is almost a symmetric static layer surrounding the Newtonian displacing fluid layer, whereas in Fig. 10(b) the static layer has formed mostly towards the upper wall of the pipe. In fact, due to the density-unstable configuration in our studies, it has been found that the thickness of the static layer close to the lower pipe walls is slightly less than the upper walls. Also note that for slug-type flow cases shown in Fig. 10(c), the viscoplastic layer is completely static, whereas in Fig. 10(d) it is slowly flowing.

To further confirm that the displacing fluid velocity profiles shown in Fig. 10 are indeed laminar, we examine transitional or turbulent/mixed profiles. Fig. 11 shows 3 examples of such flows. Fig. 11(a) corresponds to the data point which was predicted to be in the transitional regime in Fig. 9(b). Note the unsteadiness evident in the velocity profile of the displacing layer. The viscoplastic layer in this case is also flowing (not static) similar to Fig. 10(d). Two examples of turbulent velocity profiles are shown in Fig. 11(b) and (c) for a Newtonian-Newtonian [35] and Newtonian-viscoplastic (current study) displacement flow, respectively. The insets clearly show efficient mixing across the pipe in both cases. As is evident from the figures, the velocity profile flattens towards the center in these cases [both in Newtonian and viscoplastic cases] which is typical of turbulent flows. A parabolic profile is also included but has apparently been not fitted successfully to the UDV results in these cases. We conclude that most of the flows within the displacing fluid layer flow in the laminar regime. Finally note that in general, presenting the UDV profiles for transitional and turbulent flows are challenging due to the sensitivity of the UDV measurements to flow oscillations. If the degree of the unsteadiness in the flow is very high, the flow might mix and become completely turbulent. In this case defining a value of the surface roughness becomes impossible and the data points cannot be shown comparatively on plots such as Fig. 9(b); see also [31] for more details on turbulent/mixed flows.

Note that the results presented in this section are valid for a developed section of the flow which is away from the initial transitions and/or displacing front. The dynamics of the flow at the displacing front may be completely different; see also Section 3.8. The results presented in this section are of value in paving the way for further linear/non-linear stability analysis of the residual layers in multi-layered center and slump-type flow configurations as future work in this field (see e.g. [25] and [26]).

3.6. Control volume analysis

As far as designing cleaning or displacement processes is concerned, determining the shear stresses at the interface between the fluids and at the duct wall is absolutely critical. Cole et al. [9] found that the cleaning time correlates well with the inverse of the wall shear stress. In this section through a simple control volume (CV) analysis, we aim to estimate these stresses. Fig. 12(a) and (b) shows the schematic of the displacing fluid area (white region) and the displaced fluid area (black region) chosen as CV’s of interest in finding the interfacial and wall shear stresses for center and slump-type flows; see Sections 3.6.1 and 3.6.2, respectively.

3.6.1. Center-type flows

Given the small density differences in our study, we can assume a Boussinesq approximation is valid implying $\rho L \approx \rho$. The momentum balance in the displacing heavy Newtonian layer reads

$$\left(\bar{\rho}_i - \bar{\rho}_o\right)\hat{\alpha} + \bar{\rho}_i\hat{\alpha} \hat{L} \cos \beta = 2\pi \bar{\tau}_i \hat{L} \hat{t}_i,$$

where

$$\hat{t}_i = \frac{\int \bar{\rho}_i \hat{V}_i^2}{8}.$$  \hspace{1cm} (9)

is the shear stress at the interface between Newtonian and viscoplastic fluids and $\hat{t}_i = \sqrt{\hat{\alpha}/\pi}$. On scaling with the viscous stress $\hat{\tau}_v = \mu \hat{V}_o / \hat{D}$. equation (9) becomes: $\tau_i = f \bar{\rho} L \hat{V}_i^2 / 8$.

Now let us look at the momentum balance in the surrounding viscoplastic layer

$$\left(\hat{\rho}_i - \hat{\rho}_o\right) \left(\frac{\pi \hat{D}^2}{4} - \hat{\alpha}\right) + \hat{\rho}_i \left(\frac{\pi \hat{D}^2}{4} - \hat{\alpha}\right) \hat{L} \cos \beta$$

$$= \pi \hat{D} \hat{t}_w - 2\pi \hat{t}_i \hat{L} \hat{t}_i.$$  \hspace{1cm} (10)

The pressure difference in (10) can now be substituted by the value calculated from (8) in the Newtonian layer. After simplification one arrives at the following dimensionless equation for the wall shear stress

$$\tau_w = \hat{t}_i \sqrt{\hat{V}_i} - \frac{X}{4} \left(1 - \frac{1}{\hat{V}_i}\right).$$  \hspace{1cm} (11)
where \( \chi = 2Re \cos \beta / Fr^2 \). Note that \( V_f \) can also be written as \( V_f = 1/\alpha \) based on the continuity equation and the fact that the viscoplastic layer is completely static and all the imposed flow passes through the Newtonian layer. Given the range of the relative roughness values in our experiments we have used the friction factor formula given in [37], for calculating \( \tau_i \) and eventually \( \tau_w \).

3.6.2. Slump-type flows

Following the same CV approach as above in Section 3.6.1, let us now find the interfacial stress \( \tau_i \) and wall stress \( \tau_w \) for slump-type flows. The momentum balance within the Newtonian layer is:

\[
\left( \hat{p}_m - \hat{p}_{out} \right) \hat{\alpha} + \hat{p}_g \hat{\alpha} \hat{L} \cos \beta = \hat{r}_i \left( \hat{\xi} + \hat{s} \right) \hat{L},
\]

(12)

where \( \hat{\alpha} \) is the dimensional displacing fluid area, \( \hat{\xi} \) and \( \hat{s} \) are the cord and arc lengths of the circular segment respectively (see Fig. 12(b)), which may all be defined as functions of the segment angle \( \theta \):

\[
\hat{\xi} = \theta \hat{D}/2,
\]

\[
\hat{s} = \sin(\hat{\theta}/\hat{D}),
\]

\[
\hat{\alpha} = \hat{D}^2(\theta - \sin \theta)/8.
\]

Note that the interfacial stress expression in the case of slump-type flow remains the same as center-type flow (9). The momentum balance in the viscoplastic layer is:

\[
\left( \hat{p}_m - \hat{p}_{out} \right) \left( \frac{\pi \hat{D}^2}{4} - \hat{\alpha} \right) + \hat{p}_g \left( \frac{\pi \hat{D}^2}{4} - \hat{\alpha} \right) \hat{L} \cos \beta = \hat{r}_w \left( \hat{D} - \hat{s} \right) \hat{L} - \hat{\tau}_i \hat{L} \hat{L}.
\]

(13)

The pressure difference in (14) can now be substituted by the value calculated from (12) in the Newtonian layer. After simplification one arrives at the following dimensionless equation for the wall shear stress

\[
\tau_w = \frac{\sin(s(\alpha)) + (1 - \alpha)s(\alpha)}{(\pi - s(\alpha))\alpha} \tau_i - \frac{\pi \chi}{4} \frac{1 - \alpha}{\pi - s(\alpha)},
\]

(15)

where \( \alpha \) and \( s \) are dimensionless versions of the geometric parameters defined in (13).

The procedure adopted for calculating \( \tau_w \) and \( \tau_i \) is such that knowing \( \alpha \) from our experimental measurements, one can compute \( \theta \) numerically and then find the corresponding \( s \) value. The hydraulic Reynolds number and friction factor can then be computed based on the geometric parameters found, which finally give the interfacial and wall shear stresses. Also note that the \( V_f \) values used for calculating \( \tau_i \) are those measured experimentally and not from the continuity equation based on \( \alpha \). In other words we have not imposed any assumption on the viscoplastic layer being static in the case of slump-type flows.

Fig. 12 shows the calculated interfacial \( \tau_i \) and wall \( \tau_w \) shear stresses for our experiments in order to see how they compare against the dimensionless yield stress of the displaced fluid \( B_N \). The results will help us better understand how far these stresses are from yielding the viscoplastic fluid in the pipe. Fig. 13(a) and (b) shows the calculated \( \tau_i \) values while (c) and (d) represents \( \tau_w \) for center and slump-type flows, respectively. The interfacial stress \( \tau_i \) can only be positive (Fig. 13(a) and (b)) due to the induced shear by the Newtonian layer. However, the wall shear stress can be either positive or negative (Fig. 13(c) and (d)) due to the presence of the buoyancy (11) and (15). If \( \tau_w \) values are close to the \( B_N \), we can conclude that the viscoplastic fluid at the wall is about to be yielded towards downstream direction due to the viscous stress of the Newtonian layer and if it is close to \( -B_N \) then it is at the threshold to be yielded towards upstream direction due to the buoyancy. Fig. 13(c) and (d) interestingly shows that the wall shear stress for near-horizontal angles \( \beta = 75^\circ, 83^\circ, 85^\circ \) do not become negative due to the weaker component of streamwise buoyancy force for this range of angles.

From Fig. 13 it can also be observed that the magnitude of \( \tau_w \) is larger than \( \tau_i \) and these stresses are greater for slump flows than center flows. The figure suggests that the interfacial and wall shear stresses can be anywhere from 1–4 order of magnitudes less than the dimensionless yield stress of the viscoplastic fluid. It is only in the case of slump flows that the wall stress approximately approaches the yield stress. Whilst this confirms the picture we have, of the residual layers being fully static in the center type displacements, this calls into question how the slump-type residual layers move even though slowly? This is unclear, within the extent of our experimental measurements. One possibility is that the roughness constricts the flow (see below in Section 3.7). A second possibility is that not only shear stresses are present in the residual layer, but also extensional stresses. Certainly, ahead of the displacing front the entire displaced fluid should move as a Poiseuille flow, implying streamwise velocity gradients (and consequent extensional stresses) are present at least at the displacement front. Thirdly, residual layer roughness may itself induce other stresses at the interface, which persist within the layer and contribute to yielding. Note that here we are solely discussing the yielding mechanisms at the interface between the fluids and the pipe wall. Details of the discussion on the yielding at the displacing front are given below in Section 3.8.

3.6.3. Note on the maximal static layer thickness calculation

A final quick note in this section is that instead of solving for the wall shear stress \( \tau_w \) from (11) and (15) in center and slump-type flows, one may set \( \tau_w = B_N \) in these equations and find the maximal static layer thickness; see also [23]. Accordingly we will...
3.7. Flow constriction and obstruction

The results presented on the regime classification of the displacing fluid layer in Section 3.5 and on the interfacial and wall shear stresses in Section 3.6 were based on the assumption of uniform steady flow along a pipe with diameter $\hat{D}_h$. However, in the case of large relative roughnesses, the irregular surface elements reduce the actual diameter of the flow locally, an effect known as flow constriction [37,38]. The effective constricted flow diameter can be obtained as (see also Fig. 7)

$$\hat{D}_{cf} = (\hat{D}_h - 2\hat{\epsilon})$$

(18)

From the continuity equation it is not difficult to show the relation between the constricted and average flow velocity and Reynolds number as:

$$V_{cf} = \frac{1}{(1 - 2\hat{\epsilon}/\hat{D}_h)^2} V_f$$

$$Re_{cf} = \frac{1}{(1 - 2\hat{\epsilon}/\hat{D}_h)^2} Re_h$$

(19)

At this stage we can assess how the regime classification picture given in Fig. 9(b) for displacing fluid layer changes if constricted flow Reynolds number $Re_{cf}$ is to be used instead of $Re_h$. Using (19), it can be found that $Re_{cf} \in [200–3500]$ (compare with $Re_h \in [175–2700]$). Given this range of $Re_{cf}$ values, the data points in Fig. 9(b) will slightly be shifted up on the y-axis. Due to this, a few more data points will fall into the category of the transitional and turbulent flows. Note that the results are not presented here for brevity. Also note that the analysis carried out here is in local sense meaning that the flow might only locally reveal the dynamics shown. The global behavior of the flow still remains the same as that given in Fig. 9(b).

The constricted flow friction factor, $f_{cf}$, can now be calculated using the modified Reynolds number in the narrower sections $Re_{cf}$. Consequently, the interfacial stress $\tau_i = fReV_f^2/8$ can be re-written...
This effect is known as stress due to area constriction, surface irregularity might also dis-}

\[ \tau = \frac{f_c V_c^2}{8}, \]  

which can be used for both center and slump-type flows. Similarly, the wall shear stress for center flows becomes

\[ \tau_w = \tau_s \sqrt{\frac{g}{2}} \left( \frac{1}{4} - \frac{1}{V_c} \right), \]

and for slump flows

\[ \tau_w = \frac{c_i \alpha_i - \alpha_i s_i}{\alpha_i (\pi - s_i)} \times \tau_s \left( \frac{1 - \alpha_i}{4} - \frac{\pi - s_i}{\pi} \right). \]

where \( \alpha_i, s_i \) and \( c_i \) are the constricted flow geometric parameters based on \( D_c f \).

The constricted flow interfacial \( \tau_{i,c} \) and wall \( \tau_{w,c} \) shear stresses can now be calculated using (20) and (21) and (22). We have realized that compared to Fig. 13 the magnitude of the interfacial and wall shear stresses are larger in the case of the constricted flows than the average flow (results not presented for conciseness). Although these stresses become larger, in the case of center-type flows they are still smaller than the dimensionless yield stress of the displaced layer \( \hat{\tau}_Y \). Therefore, there should be a stress of comparable size to the yield stress of the viscoplastic fluid. The key question here is what is the responsible mechanism for the yielding and breakage of the viscoplastic fluid layer. In order to answer this question, the yield stress of the viscoplastic fluid, \( \hat{\tau}_Y \), for different experiments were compared against the inertial stress of the Newtonian layer at the front, \( \hat{\rho} \hat{V}_f^2 / 2 \).

We showed in Section 3.5 that the displacing fluid in the developed section of the flow away from the front is mostly in laminar regime. We also found that the inertial stresses at the displacing front can cause yielding. A more general picture of the displacement flows can now be deduced. Fig. 14(a) shows schematically that the displacing fluid flows mostly in the laminar regime past residual wall layers that remain static over long times. The flow at the front of the displacement is inertial and most likely 3D. Furthermore, downstream of the flow we recover the Poiseuille flow of a viscoplastic fluid with the plug region formed in the central region of the pipe. The patterns discussed for center-type flows are similar for slump-type flows shown schematically in Fig. 14(b), except in contrast to center flows, the viscoplastic layer in the displaced section in slump flows does sometimes move slowly which can enhance the displacement quality and cleaning efficiency. It is not very clear what mechanisms govern the displacing front and what the flow pattern is exactly in this region. We can however argue that the flow is inertial and most likely 3D. There might also be some local tip splitting instability similar to [15]. This region of the flow needs to be studied in more detail later through high resolution imaging and perhaps the use of 2D and 3D UDV and Particle Image Velocimetry (PIV).

Fig. 15(a)–(c) and (d)–(f) shows two sequences of UDV results for center and slump-type flows, confirming the schematic patterns depicted in Fig. 14. The figures are recorded at three stages: (a),(b) show the laminar profile within the displacing layer. Note that for center-type flow (a) the viscoplastic layer is static whereas for slump-type flow (d) it is slowly moving; (b) and (e) depict the velocity profile when the displacing front reaches the UDV probe location. For center-type flow (b) the profile within the Newtonian layer is flattened towards the center which is reminiscent of a turbulent flow profile. The unsteadiness is also clear for slump-type flow (e). Interestingly, the inertial flow in the displacing layer has affected the velocity profile within the viscoplastic layer. In center-type flow (b) the viscoplastic fluid layer is not static. Also for slump-type flow (e) there is instability evident in the viscoplastic fluid layer velocity profile; (c) and (f) show the plug type velocity profile downstream of the flow which is fully occupied by the viscoplastic fluid. Note that the flow patterns observed in Fig. 15 are qualitatively in line with those reported previously in the
literature for viscoplastic–viscoplastic displacement (Fig. 10 in [23]) and gas–viscoplastic displacement (Fig. 10 in [11]) flows.

3.9. Predictions from the closure model of [33]

In [33] through a lubrication theory, the authors developed a semi-analytical closure model for 3D pipe flows of both Newtonian and non-Newtonian (Herschel-Bulkley) fluids with a stratified interface. Although the model has been compared favorably with experimental results for iso-viscous Newtonian fluid displacements [42], and with viscosity ratio and shear-thinning fluid displacement flows [43], its predictions have not been evaluated for viscoplastic displacement flows. Note that the model can only be compared against slump-type flows due to the stratified flow assumption made. The readers are referred to [33] for the details of the pipe flow closure model (see [32,44] for similar analytical models on viscoplastic displacement flows in 2D channel flow geometry).

Fig. 16(a) shows the prediction of the displacing front velocity \( \hat{V}_{f,\text{closure}} \) from the closure model [33] against the experimentally measured ones \( \hat{V}_{f,\text{exp}} \). Note that only slump-type flows are included for comparison since a center-type flow configuration is irrelevant in this context. The solid line represents \( \hat{V}_{f,\text{closure}} = \hat{V}_{f,\text{exp}} \). It can be observed that the closure model values are much higher than the experimental values suggesting that the closure model overpredicts the front velocity. Note that in the case of the Newtonian (iso-viscous and/or differing-viscosity) and shear-thinning displacement flows a much closer agreement in the front velocity values was found with the predictions of the closure model [33]. Note that due to the mass conservation, the over prediction in the front velocity by the closure model is equivalent to under prediction of the displacing fluid area, \( \alpha \), i.e., the smaller the displacing fluid area is, the faster it has to move. A possible explanation for the discrepancies with the closure model can be that the inertial multi-dimensional effects at the displacing front cannot be captured in the lubrication-based closure model i.e. the flow is dealt with as viscous throughout the domain which is not realistic in the case of viscoplastic displacements. Note that for displacement of fluids with no yield stress there is also an inertial tip region [45] but perhaps the effect in this region is much smaller compared to the viscoplastic case.

Apart from the displacement efficiency (proportional to the inverse of the front velocity), another very important criterion in design of the displacement flows is estimating whether displacement occurs instantaneously or not; see [46]. In an instantaneous displacement, the buoyancy force is not strong enough to cause (back) flow of the displaced fluid in the upstream direction. In the context of primary cementing of oil & gas wells avoiding the back flow of the drilling mud during pumping cement slurry is of great importance. We have experimentally demonstrated that if the buoyancy parameter \( \chi = 2Re \cos \beta / Fr^2 > \chi_c \) a backflow can occur causing a non-instantaneous flow, for both Newtonian [35] and non-Newtonian [31] fluids. Fig. 16(b) shows the prediction of the critical buoyancy parameter from the closure model \( \chi_c, \text{closure} \) against \( \chi \) for slump-type experiments. It can be seen that the closure model predicts all the flows to be in instantaneous regime since \( \chi_c > \chi \). Experimentally we had found that most of the flows are indeed in instantaneous regime [31] as predicted by the closure model. However, there were also a few experiments in which non-instantaneous flows were observed. These experiments are marked by black squares in Fig. 16(b). Also note that the required \( \chi_c \) increases monotonically (almost linearly) with \( \chi \). Although variations with inclination angle \( \beta \) appear to be significant, there is no easily definable trend, which is similar to our earlier findings.
The displacement of a viscoplastic fluid by a miscible Newtonian fluid in an inclined pipe has been investigated experimentally in the case where the displacing fluid is denser than the displaced fluid (i.e. the density-unstable configuration). Our experiments have covered a broad range of the governing dimensionless parameter space ($\beta$, $Re$, $Fr$, $Bi$). We previously found two major regimes namely center and slump type flows [30,31]. The transition between the two flow regimes occurred in the range $600 < Re/Fr < 800$.

The velocity of the displacing front has been measured and characterized over our range of parameters, which is useful as an indicator of the displacement efficiency [32]. It is found that the front velocities $V_f$ are mainly a function of the mean flow speed $V_0$ and not strongly dependent on the inclination angle $\beta$ and the yield stress of the displaced fluid $\tau_Y$. Also in the case of center-type flows, $V_f$ is smaller than that of slump-type flows. Predictive formulas based on curve fitting of the data are given as $V_f = 1.62V_0$ and $V_f = 1.92V_0$ for center and slump-type flows respectively, but the data shows some deviations from these.

The residual layers in the case of center-type flows are found to remain static over long times. In the case of slump-type flows the residual layer can move with a speed much slower than that of the displacing front finger. The dynamics of the displacing fluid (i.e. the density-unstable configuration). Our experiments have covered a broad range of the governing dimensionless parameter space ($\beta$, $Re$, $Fr$, $Bi$). We previously found two major regimes namely center and slump type flows [30,31]. The transition between the two flow regimes occurred in the range $600 < Re/Fr < 800$.

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Fig. 16. (a) Comparison of the displacing front velocity obtained from the closure model [33] against experimentally measured $V_f$ for slump-type flows showing overprediction. (b) Critical buoyancy parameter predicted by closure model $\chi_c$, closure versus $\chi$ of slump-type experiments. The solid line indicates $\chi_c = \chi$. The data points marked by the squares were experimentally found to be in non-instantaneous displacement regime [31].

4. Summary

The displacement of a viscoplastic fluid by a miscible Newtonian fluid in an inclined pipe has been investigated experimentally in the case where the displacing fluid is denser than the displaced fluid (i.e. the density-unstable configuration). Our experiments have covered a broad range of the governing dimensionless parameter space ($\beta$, $Re$, $Fr$, $Bi$). We previously found two major regimes namely center and slump type flows [30,31]. The transition between the two flow regimes occurred in the range $600 < Re/Fr < 800$.

The velocity of the displacing front has been measured and characterized over our range of parameters, which is useful as an indicator of the displacement efficiency [32]. It is found that the front velocities $V_f$ are mainly a function of the mean flow speed $V_0$ and not strongly dependent on the inclination angle $\beta$ and the yield stress of the displaced fluid $\tau_Y$. Also in the case of center-type flows, $V_f$ is smaller than that of slump-type flows. Predictive formulas based on curve fitting of the data are given as $V_f = 1.62V_0$ and $V_f = 1.92V_0$ for center and slump-type flows respectively, but the data shows some deviations from these.

The residual layers in the case of center-type flows are found to remain static over long times. In the case of slump-type flows the residual layer can move with a speed much slower than that of the displacing front finger. The dynamics of the displacing fluid layer and its impact on the removal of the viscoplastic fluid is investigated through calculation of the hydraulic Reynolds number $Re_b$ and residual layer surface roughness. Large relative roughness values were found especially in the case of slump-type flows. The flows within the displacing layer are mostly in laminar regime. A few cases are also identified in transitional regimes. Knowing the right flow dynamics within the displacing layer enables us to find the correct friction factor and shear stresses in the domain. Through a control volume based theoretical analysis integrated with experimental findings, estimates of the interfacial and wall shear stresses are given disclosing that they can be $1–4$ orders of magnitude lower than the dimensionless yield stress of the viscoplastic fluid, $B_N$. On using a constricted flow model, it was observed that some slump flows were able to yield the displaced fluid layers at the wall, consistent with the observed slow motion of this layer. Comparable sized inertial stresses to $B_N$ can exist at the very displacing front area which are responsible for gel breakup. Ultrasonic Doppler Velocimetry (UDV) data confirm the effects observed.

Finally, the results presented are compared against the predictions of the closure model in pipe [33], which was previously shown to successfully predict the displacement flow behavior for Newtonian and shear-thinning fluids. It is found that in the case of viscoplastic fluids, the closure model always over predicts the displacing front velocity values due to the fact that the inertial stresses present at the front are not captured in the model. However, the closure model predicts the flows to be instantaneous regime i.e. no back flow which is overall in line with our experimental observations.

Future work on density-unstable displacement flows of viscoplastic fluids will focus on carefully examining the transition between center-type and slump-type flows in the range $600 < Re/Fr < 800$ to understand the key underlying mechanisms and physics behind the changeover. The flow at the very displacing front can also be visualized more accurately through 2D, 3D UDV and/or PIV measurements. The center-type displacement flow experiments with laminar imposed flow can be run over long times to see if similar thin film and patch removal regimes to [9,10] will be found after the core removal phase is over. The displacement and cleaning process can also be investigated for turbulent imposed flows with $Re \gg 2100$ in the presence of buoyancy to see how the current findings and trends will be affected by inertia.

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