

A Model of Statutory Policy Making under Uncertainty

In the previous chapter, I presented an intuitive and example-based framework for understanding how uncertainty and vote buying shape statutory policy making. This theme is developed further in this chapter with the introduction of a relatively simple mathematical model. Its main goal is to shed light on how party loyalty, conflicting influences, and vote buying affect chief executives' legislative performance.

There is obviously no such thing as the right degree of abstraction for all analytic purposes. In the mathematical representation of the lawmaking process that I present in this chapter, I ignore many details and make a series of simplifications. The model is thus tractable, yet rich enough to generate interesting testable propositions.¹

Throughout the chapter, I attempt to keep the exposition of the main ideas as simple as possible, without losing mathematical precision and generality. In some passages, I rely on figures and numerical examples to convey intuition about the results. Technical details are left to the footnotes, and formal proofs can be found in Appendix A. For expositional purposes, I adopt the following convention: Legislators are identified as female and chief executives as male.

¹ The importance of generalization and abstraction in thought and science is cleverly illustrated in *Funes the Memorious*, a short story by the Argentine writer Jorge Luis Borges: "Without effort [Funes] had learned English, French, Portuguese, Latin. I suspect, nevertheless, that he was not very capable of thought. To think is to forget a difference, to generalize, to abstract. In the overly replete world of Funes there was nothing but details, almost contiguous details." (Borges 1962: 115). In a similar vein, Paul Krugman (1994) argues that the utility of modeling stems from useful simplification.

3.1 LEGISLATIVE STAGE

I assume the legislative process starts when the chief executive chooses between two alternatives. He can either send a proposal x^* to the legislature or keep the status quo policy x^{sq} , both elements of the set of real numbers, \mathbf{R} . If the chief executive decides to do the former, then the legislature chooses between adopting the bill and making x^* the new policy, or killing it and keeping the status quo policy x^{sq} .

I analyze first the second stage of the game, namely legislative behavior once the chief executive has sent a proposal to the legislature. In the next section, I turn to the proposal stage.

Environment and Players. Two kinds of players exist: the chief executive and the legislators. The legislature is composed of an odd number of legislators, $i = 1, 2, \dots, n$ ($n \geq 3$). Each legislator casts a vote v_i for or against the proposal. Thus, the action space for every legislator is the set $V = \{\text{yes}, \text{no}\}$. Let v be the vector of cast votes $[v_1, v_2, \dots, v_n] \in V^n$. This voting profile determines the legislature’s decision through a decision rule $r(v)$. I assume that this decision rule is simple majority.

I make the assumption that if the bill is adopted, the chief executive derives utility $u_E(x^*) > u_E(x^{sq})$; however, if the bill is rejected, he pays a political cost $c > 0$, and gets $u_E(x^{sq}) - c$. Therefore, the chief executive strictly prefers the new policy x^* to the status quo.

If the proposal does not command a majority, the chief executive may be able and willing to bribe some legislators to affect the outcome. Let $\tau_i(v) \geq 0$ denote the bribe offer by the chief executive to legislator i under the realized voting profile v . The chief executive’s payoff can be then written as:

$$u_E(x, c) + [\Pi(v) - \sum_{i=1}^n \tau_i(v)] \tag{3.1}$$

where $\Pi(v)$ is his fixed budget to buy off individual legislators.

Let $\theta_E = u_E(x^*) - u_E(x^{sq})$ denote the value of policy change for the chief executive. It follows that if $\theta_E > \sum_{i=1}^n \tau_i(v) - c$, then the chief executive would rather pass the new policy and make bribes than tolerate the status quo policy, as well as pay a cost c and keep his vote-buying budget.²

² But, if the total cost of securing these votes exceeds the value of policy change, the chief executive may be better off by conceding defeat.

For a legislator i , let $\theta_i = u_i(x^*) - u_i(x^{sq})$ denote the value of policy change. I assume that legislators belong to legislative parties, and that, absent any further pressures, they would follow the party line when deciding how to vote. I do not take a position regarding the particular sources of legislators' partisan alignment. Whether legislators vote with their parties because of ideological affinity (Krehbiel 1993) or for other reasons, such as protecting the party's brand name or to enjoy privileged access to legislative posts (Cox and McCubbins 1993, 2005) is not germane to the argument. Regardless of their motivation, I assume that legislators' ideal policy, \hat{x}_i , corresponds to their party's preferred alternative:

$$\hat{x}_i = \delta_i x^* + (1 - \delta_i) x^{sq} \quad (3.2)$$

where the parameter δ indicates legislator i 's party, and takes the value of $\delta = 1$ if legislator i belongs to a government party, and $\delta = 0$ otherwise. Therefore, θ_i is identical for all legislators who belong to the same party.

As I argued in Chapter 2, legislators vote in the legislature, but they secure support, campaign resources, and electoral rewards outside the legislative arena. Therefore, I assume that legislators act as agents of particular constituencies (their principals), and that these principals may not only induce preferences in the legislators but also constrain their mode of behavior. For simplicity, I make the assumption that each legislator has a single principal $j \in J$; each principal j is characterized by the intensity of his/her preference of x^* over x^{sq} :

$$\omega_j = u_j(x^*) - u_j(x^{sq}) \quad (3.3)$$

I further assume that both principal and legislator have the same information throughout the relationship. That is, a legislator and her principal share common information as to all relevant characteristics of the chief executive's proposal, and legislator i 's behavior is verifiable, so every principal can check if she has voted in accordance to its views. Since the legislator's behavior and the final result of the relationship are observable, the principal can introduce these variables explicitly into the terms of the contract. I assume that the payoff that legislator i receives from her principal j is contingent on how she casts a vote. These payoffs may take either negative or positive values (i.e., legislators can either receive a punishment or a reward), and they materialize in different forms. One can think of them as the reaction of the principal in the next election, campaign contributions, media exposure, and so on.

In terms of her utility from choosing x , I assume that it is additively separable between: (i) the utility the legislator derives from the policy that

is collectively chosen by the legislature, denoted by x ; (ii) the bribes (τ_i) offered to her by the chief executive; and (iii) the utility she derives from her principal’s reaction to how she votes, denoted by s_i .

Therefore, the payoff of legislator i can be written as:

$$u_i(x, \tau_i, s_i|v_i) = u_i(x) + \tau_i + u_i(s_i|v_i) \tag{3.4}$$

where v_i stands for legislator i ’s vote; and s_i , the principal’s punishment or reward is given by:

$$s_i = \begin{cases} |\omega_j| & \text{if } \omega_j > 0 \text{ and } v_i = \text{yes or } \omega_j < 0 \text{ and } v_i = \text{no} \\ -|\omega_j| & \text{if } \omega_j < 0 \text{ and } v_i = \text{yes or } \omega_j > 0 \text{ and } v_i = \text{no} \end{cases} \tag{3.5}$$

For example, the utility of legislator i when the collective outcome would be x^* regardless of how she votes, and she casts a vote in favor of x^* is represented by $u_i(x^*, \tau_i, s_i|\text{yes})$. The utility of legislator i when the collective outcome would be x^{sq} regardless of how she votes, and she votes for x^* , is $u_i(x^{sq}, \tau_i, s_i|\text{yes})$.³

One final assumption is that within each party, legislators are only distinguishable by the intensity of their principals’ preferences. Let m denote the size of the government’s legislative contingent. Legislators can then be ordered separately according to the intensity of their principals’ preference of x^* over x^{sq} . Government legislators can be ordered in the following form: $\omega_1^{\delta=1} \leq \omega_2^{\delta=1} \leq \dots \leq \omega_m^{\delta=1}$. Opposition legislators can be ordered as $\omega_1^{\delta=0} \leq \omega_2^{\delta=0} \leq \dots \leq \omega_{n-m}^{\delta=0}$. Note that for each legislator i , ω_j defines her ω_i , so herein I use the latter notation.

Sequence. The sequence of play is as follows.

1. *Bill Introduction.* The chief executive sends a bill to the legislature without knowing the ideal policy of legislators’ principals but has some prior on $\omega_i^{\delta=1} \sim N(\mu_1, \sigma_1^2)$ and $\omega_i^{\delta=0} \sim N(\mu_0, \sigma_0^2)$.
2. *Vote Buying.* Once the bill is sent to the legislature, lawmakers receive a mandate from their principals that defines ω_i for each one of them. At this point, everyone knows everything (i.e., the ω_i s become common knowledge). Once this information is revealed, a voting profile v is realized. The chief executive can now “count noses” and decide whether to offer each legislator a schedule $\tau_i(v)$ of payments for voting for x^* .

³ The analysis in Rasmusen and Ramseyer (1994) characterizes legislative voting behavior in a similar way.

3. *Legislative Voting.* Each legislator simultaneously casts a vote $v_i \in \{yes, no\}$. The chief executive observes a voting profile v and delivers payments according to $\tau_i(v)$. The collective outcome x is determined by majority rule.

Strategies. I assume that once the chief executive sends a bill x^* to the legislature, he cannot change its content.⁴ Therefore, his strategy at this stage consists of an n -dimensional bribe vector \mathbf{p} with components $\tau_i \in \wp \equiv \{V : \{yes, no\}^n \rightarrow 0 \cup \mathbf{R}_+\}$ mapping a voting profile v into payments for voting for x^* . Legislator i 's strategy $v_i : x \times \wp^n \rightarrow \{yes, no\}$ maps the chief executive proposal x^* and the payment schedules into a vote.

Given the sequentiality of the decisions by the chief executive and by the legislature, I use *subgame perfection* as my solution concept. A subgame perfect equilibrium requires each player's strategy to be optimal, given the other players' strategies, not only at the start of the game but after every possible sequence of events (Osborne 2004). First, I analyze equilibrium outcomes where the chief executive does not offer any bribes; then, I examine vote-buying situations.

3.1.1 Equilibrium Outcomes without Bribes

As I discussed in Chapter 2, I rule out the possibility that legislators can make binding agreements to cooperate with one another (i.e., correlate strategies) when casting a vote. Instead, I consider that in any given vote, each legislator chooses her own optimal action while holding the choices of all other legislators fixed.

I also assume that legislators do not use mixed strategies. I believe that this is an appropriate assumption for any voting body that allows legislators to change their votes after seeing all other votes, and does not allow the presiding officer to end the voting period when there are still legislators who want to record or change their vote (i.e., to invoke a "quick gavel"). Voting models that allow mixed strategies are consistent with the quick-gavel norm. Most legislatures around the world, however, specify a minimum amount of time for voting, and legislators can continue to vote even after the official time has expired (Groseclose and Milyo 2009). Hence, in this model, votes are the legislators' pure strategies, which I

⁴ As I discussed in Chapter 2, if the chief executive can change the content of the legislation, then he would be able to tailor it to accommodate the policy preferences of a majority of legislators and avoid being defeated.

take to be history-independent (i.e., voting strategies only depend on the existing proposal).

Recall that legislators have preferences over which alternative wins. Therefore, legislator i 's vote will only have an effect on the utility she derives from the collective outcome if she is decisive. Otherwise, legislator i will only be concerned with her principal's reaction to how she votes. Taking this into account, legislators can be arranged according to how much their principals like the proposal relative to their own taste.

For each proposal x^* , six legislative *factions* or voting blocks can be defined. The composition for each is a function of: (i) whether a legislator shares the same party as the chief executive; (ii) how much a legislator's principal values policy change; and (iii) the possible collective outcomes whenever legislator i 's vote is decisive.

Formally, each legislator can be characterized as being of the type $t_i : \{\omega_i, \delta_i\}$. Let n_t be the measure of a faction composed by type- t legislators, where n_t is positive and integer-valued, so $\sum_{t=1}^6 n_t = n$. If C is any voting coalition, let $N_t(C)$ be the set of type- t legislators in C . Let $n_t(C)$ be the total number of type- t legislators in C , and let $n(C) = \sum_{t=1}^6 n_t(C)$ be the total number of legislators in C . I assume that all legislators in the same faction act the same. Specifically, unless they receive any bribes from the chief executive, all legislators who belong to the same faction have the same dominant voting strategy.⁵

Consider now the situation that legislator i faces when the chief executive proposes $\tau_i(v) = 0$ for all legislators. In this case, her utility depends on the policy that is collectively chosen and the individual payoff that she would get from her principal, which is contingent on how she votes. Her vote, as discussed in Chapter 2, can have two effects. It can be instrumental in determining the collective outcome, or it can be a "voice" that reflects her principal's preferences without altering the outcome. To determine whether the former is more important to legislator i than the latter, we need to consider the interdependence of legislators' decisions. Table 3.1 summarizes legislators' voting strategies. The first column identifies each faction. The fourth column shows legislator i 's utility from casting her vote in a particular way. Finally the last two columns list the equilibrium strategies of legislator i when her individual vote does not change the collective outcome, and when her vote is decisive.

⁵ For a somewhat similar approach, see Rasmusen and Ramseyer (1994) and Snyder, Ting, and Ansolabehere (2005).

Table 3.1. *Voting strategies when no bribes are offered*

Faction	Party	Ideal policy of principal	Utility from casting a particular vote	Strategy if she is not decisive	Strategy if she is decisive
$N_1(C)$	Government	$\omega_i > 0$	$u_i(x, s_i yes) > u_i(x, s_i no)$	Vote Yes	Vote Yes
$N_2(C)$	Government	$\omega_i < 0$	$u_i(x^*, s_i yes) > u_i(x^{sq}, s_i no)$	Vote No	Vote Yes
$N_3(C)$	Government	$\omega_i < 0$	$u_i(x, s_i yes) < u_i(x, s_i no)$	Vote No	Vote No
$N_4(C)$	Opposition	$\omega_i < 0$	$u_i(x, s_i yes) < u_i(x, s_i no)$	Vote No	Vote No
$N_5(C)$	Opposition	$\omega_i > 0$	$u_i(x^*, s_i yes) < u_i(x^{sq}, s_i no)$	Vote Yes	Vote No
$N_6(C)$	Opposition	$\omega_i > 0$	$u_i(x, s_i yes) > u_i(x, s_i no)$	Vote Yes	Vote Yes

Notes: This table shows how the interdependence of legislators' decisions affects their individual voting strategies when no bribes are offered. Given proposal x^* , six factions can be defined. The composition of each faction depends on: (i) a legislator's party; (ii) the ideal policy of her principal $j \in J$, denoted by $\omega_j \equiv u_j(x^*) - u_j(x^{sq})$; and (iii) the collective outcome if her vote is decisive. Without bribes, a legislator's utility from choosing x is additively separable between the utility she derives from: (i) the policy that is chosen by the legislature (x); and (ii) her principal's reaction to how she votes (s_j). For example, her utility when the outcome is x^* and she casts a vote in favor of x^* (i.e., she votes yes) is represented by $u_i(x^*, s_i|yes)$. Legislators in factions $N_1(C)$ and $N_6(C)$ will always vote in favor of x^* . Conversely, legislators in factions $N_3(C)$ and $N_4(C)$ will never vote in favor of x^* . In the case of legislators in factions $N_2(C)$ and $N_5(C)$, their vote will depend on whether they are decisive. The former constitute the set of *potentially decisive* government legislators, whereas the latter are *potentially decisive* opposition legislators.

Legislators in faction $N_1(C)$ are government legislators whose principals like the proposal. To gain some intuition, consider Republican president George W. Bush's free-trade legislation discussed in Chapter 1. Most Republicans supported the president's measure, whereas most Democrats opposed it. Still, some Democrats from export-dependent districts were in favor of the bill, whereas Republicans from protectionist districts railed against it. In this particular example, free-trade Republicans from export-dependent districts should thus be considered members of this faction.

Faction $N_6(C)$ is composed of the opposition legislators whose principals like the government proposal so much that they are more than compensated for the disutility they incur by voting in favor of x^* . One can think of these legislators as the Democrats from export-dependent districts in the free-trade legislation example.

Faction $N_3(C)$ is composed of government legislators whose principals strongly oppose the proposal. In keeping with the same example, one can think of these legislators as Republicans from protectionist districts. Republican representative Chris Smith of New Jersey's 4th congressional district is a case in point. He voted against Bush's proposal, as well as other free-trade agreements such as NAFTA, CAFTA, and PNTR for China and Vietnam.

Legislators in faction $N_4(C)$ are opposition legislators whose principals also oppose the proposal. In the case of Bush's free-trade legislation, most Democrats opposed the bill arguing that it would bleed American jobs. These legislators should thus be considered members of this faction.

Faction $N_2(C)$ is composed of legislators who favor x^* but whose principals mildly oppose the proposal. The strategy of these legislators depends on whether their vote changes the collective outcome. If their vote is not decisive, they will always vote in favor of x^{sq} . This will make them better off because they will be able to reap the benefits from the policy that is collectively chosen and at the same time act on behalf of their principals. Robin Hayes of textile-rich North Carolina, who voted against Bush's free-trade legislation after its victory was assured, embodies the legislator who belongs to this faction.

Finally, the $N_5(C)$ faction comprises those legislators who are opposed to x^* but have principals who mildly support the measure. This support, however, is not sufficient to compensate for the disutility these legislators may incur by voting in favor of x^* , should such vote change the collective outcome. If their vote is not decisive, they will vote in favor of x^* , otherwise they will vote for x^{sq} . Turning back to the example, one can think

of these legislators as the twenty five Democrats who supported Bush’s free-trade proposal.

In the absence of bribes, given a proposal x^* , the outcome depends entirely on the distribution of types in the legislature. Legislators in factions $N_1(C)$ and $N_6(C)$ will always vote in favor of x^* . Conversely, legislators in factions $N_3(C)$ and $N_4(C)$ will never vote in favor of x^* . In the case of legislators in factions $N_2(C)$ and $N_5(C)$, their vote will depend on whether they are decisive. The former constitute the set of *potentially decisive* government legislators, whereas the latter are *potentially decisive* opposition legislators.

Obviously, a proposal x^* will be adopted without any bribes if a majority of *unconditional supporters* in the legislature exists; namely, if:

$$n_1(C) + n_6(C) \geq \frac{n + 1}{2} \tag{3.6}$$

In this case, legislators in factions $N_1(C)$ and $N_6(C)$ will vote in favor of x^* and the remaining legislators would cast their votes according to their principals’ preferences. Legislators in factions $N_2(C)$, $N_3(C)$, and $N_4(C)$ will vote against x^* . Legislators in $N_5(C)$ will cast a vote in support of the bill.

Conversely, a proposal x^* will be not adopted without any bribes if there is a majority of *unconditional opponents* in the legislature; namely, if:

$$n_3(C) + n_4(C) \geq \frac{n + 1}{2} \tag{3.7}$$

In this case, legislators in factions $N_3(C)$ and $N_4(C)$ will vote against x^* and the remaining legislators will cast their votes according to their principals’ preferences. Legislators in factions $N_1(C)$, $N_5(C)$, and $N_6(C)$ will vote for x^* . Legislators in $N_2(C)$ will cast a vote against the bill.

Suppose now that neither unconditional supporters nor unconditional opponents constitute a majority. Let k be the number of additional votes needed to pass x^* , once the votes from the unconditional supporters are accounted for. Formally, let:

$$k = \frac{n + 1}{2} - n_1(C) - n_6(C) \tag{3.8}$$

The outcome depends on how the potentially decisive legislators cast their votes. This reasoning leads to the following proposition.

Proposition 1 *A pure-strategy equilibrium outcome, where the chief executive offers no bribes and x^* wins, exists. In every equilibrium, legislator i will never vote against her principal unless her vote is decisive.*

Proof. All proofs are in Appendix A.

An important feature of these equilibrium outcomes is that each legislator's vote will affect the collective decision and hence her own payoff. This is the case because legislators have preferences over which policy alternative wins. A government legislator, however, would only take the "bitter pill" and vote contrary to her principal's wishes if her vote is the decisive one.⁶

Another characteristic of these equilibria is their symmetry. This is another significant feature because it implies that strategic behavior on the part of individual legislators may lead to Pareto-inefficient outcomes. Specifically, whenever the set of potentially decisive government legislators or the set of potentially decisive opposition legislators is empty, the existence of "surplus" votes generates a kind of multilateral prisoners' dilemma. By playing dominant strategies, all potentially decisive legislators end up voting in favor of a policy outcome that is undesirable for them. In consequence, when more than one additional vote is needed to change the outcome, the chief executive may not be able to pass the new policy x^* without offering bribes.⁷

When there are both government and opposition potentially decisive legislators, two situations arise. Either the legislative stage of the game has no pure strategy equilibrium outcomes, or in equilibrium every legislator votes with her principal. As such, the size and partisan make-up

⁶ As Denzau et al. (1985) note, when legislators are monitored and evaluated by constituents who care both about legislative outcomes and about legislative behavior, "... result-oriented strategic calculation and sophisticated behavior in the legislative arena may require actions that run contrary to the nominal preferences of important constituents ..." (Denzau et al. 1985: 1118). The authors examine these situations and find that when voting behavior is consistent with a pure strategy Nash equilibrium, legislators should only act sophisticatedly in the knife-edge case of a tie. Similarly, Groseclose and Milyo (2009) demonstrate that legislators with preferences over policy and the positions that they take (i.e., the way in which they vote when they are not pivotal), will rarely engage in sophisticated voting.

⁷ When the chief executive does not offer any bribes, there are several pure strategy equilibrium outcomes where this is not the case. However, in all of these situations, the "strongest" voting profile is the one where all potentially decisive legislators vote as if they were not casting the decisive vote, as it involves (weakly) dominant strategies. See Dal Bo (2007) for a similar treatment.

of unbribed winning coalitions can be characterized in the following way:

Comment 1 *In the absence of bribes, minimum-winning voting coalitions where the chief executive wins for sure only occur if:*

1. $n_1(\mathbf{C}) + n_6(\mathbf{C}) = \frac{n+1}{2}$, and $n_5(\mathbf{C}) = 0$;
2. $n_1(\mathbf{C}) + n_6(\mathbf{C}) < \frac{n+1}{2}$, $k > 1$, $n_2(\mathbf{C}) > 0$, and $n_5(\mathbf{C}) = k + 1$;
3. $n_1(\mathbf{C}) + n_6(\mathbf{C}) < \frac{n+1}{2}$, $k = 1$, $n_2(\mathbf{C}) > 0$, and $n_5(\mathbf{C}) = 0$.

If unconditional supporters constitute a majority of the legislature, or if unconditional supporters plus one potentially decisive opposition legislator constitute a majority (given $k = 1$), then x^* wins for sure. In both cases, however, all members of faction $N_5(\mathbf{C})$ will vote for x^* . And as these legislators join the winning coalition, its size becomes larger than minimum.

We know that an equilibrium outcome where all legislators vote with their principals always exist in pure strategies when $n_5(\mathbf{C}) > k$. In this case, a minimum-winning coalition will only exist if $n_5(\mathbf{C}) = k + 1$. Without bribes, a cross-partisan minimum-winning coalition will be an equilibrium outcome only if unconditional supporters are exactly a majority of the legislature and there are no potentially decisive opposition legislators, or if: (1) neither unconditional supporters nor unconditional opponents constitute a majority; (2) there are some potentially decisive opposition legislators; (3) more than one additional vote is needed to change the outcome; and (4) the set of potentially decisive opposition legislators is equal to the number of additional votes needed to pass x^* plus one.

An all-government – or *nonflooded*, in Groseclose and Snyder (1996) terms – minimum-winning coalition will only happen if: (1) neither unconditional supporters nor unconditional opponents constitute a majority; (2) there are some potentially decisive government legislators; (3) exactly one additional vote is needed to change the outcome; and (4) there are no potentially decisive nor unconditional supporters in the opposition. The prediction that nonflooded minimum winning coalitions would only rarely occur (i.e., under a very restrictive set of circumstances) is consistent with the literature (Groseclose and Snyder 1996; King and Zeckhauser 1999) and evidence from roll-call votes in the U.S. Congress.

3.1.2 Equilibrium Outcomes with Bribes

I now turn my attention to the analysis of the chief executive's strategy. The chief executive would only offer bribes if he can achieve a better outcome than being defeated at a sufficiently low cost. In principle, every chief executive would like to make an offer such that each legislator would actually receive the reward if and only if her vote happens to be decisive. As Dal Bo (2007) shows, by promising to reward at least one more voter than she needs to win, all legislators become nondecisive, and therefore the chief executive would not need to make any payments at all. Nonetheless, when legislators are cross-pressured, chief executives cannot use these so-called *pivotal offers* to manipulate the collective decision at virtually no cost (Rasmusen and Ramseyer 1994; Dekel et al. 2005; Dal Bo 2007).

It should now be clear why this is the case. A legislator who votes in favor of a policy that damages her principal would suffer a loss equal to her principal's punishment, no matter what the other legislators do. In other words, the agency relationship with her principal shields every legislator from the externalities caused by other legislators' votes. The chief executive will therefore be unable to accomplish much by offering pivotal contracts.

To reiterate, unless a legislator is actually decisive, she will never cast a vote that contradicts her principal's preferences. By the same logic, whenever she is not decisive, the policy component of her utility is irrelevant to her voting decision. As a result, a chief executive could exploit this aspect of legislative behavior and buy legislators' votes at a cost of $\tau_i(v) = 2s_i$ each. This "price" reflects the fact that in order to cast a vote against her principal, a legislator would have to be compensated for contradicting her (which entails forgoing her reward and suffering her punishment).

If this stage of the legislative game has been reached, it means that every legislator's ω_i has become common knowledge. In consequence, the chief executive can use this information to compute the optimal n-dimensional bribe vector using the following algorithm. Denote by \mathbf{B} the set of bribed legislators. Let $\mathbf{N}_t(\mathbf{B})$ be the set of type-t legislators in \mathbf{B} . Let $n_t(\mathbf{B})$ be the total number of type-t legislators in \mathbf{B} , and let $n(\mathbf{B}) = \sum_{t=2}^5 n_t(\mathbf{B})$ be the total number of legislators in \mathbf{B} . The cost of a winning coalition increases in $n(\mathbf{B})$, thus, the optimal bribing strategy consists in picking the "cheapest" legislators such that $n(\mathbf{B}) \leq \frac{n+1}{2} - n_1(\mathbf{C}) - n_6(\mathbf{C})$.

The following proposition identifies this strategy, which I call the Least Expensive Bribed Majority strategy (LEBM) for the chief executive:

Proposition 2 *The LEBM strategy, whereby the chief executive buys k or fewer votes, guarantees a victory to x^* .*

This proposition presents one of the main findings in this book. A chief executive may be able to buy a minimum winning coalition, or even fewer votes than those needed to pass x^* (once the votes from the unconditional supporters are accounted for) and get his initiative approved. Proposition 2 also leads to the following observation about the size of “bribed” majorities:

Comment 2 *In the presence of vote buying, winning coalitions are either strictly minimal or include $(\frac{n+3}{2})$ legislators.*

This conclusion follows directly from the fact that legislators are cross-pressured. Given legislators’ policy preferences and responsiveness to their principals, these types of coalitions are the least expensive ones available. Whether the coalition is minimum-winning or includes one additional legislator depends on the distribution of types in the legislature. Occasionally, it will be cheaper for a chief executive to buy some votes and add enough legislators to the winning coalition so that no opposition legislator is actually decisive.

Example 1 *Let $n = 101$, the decision rule $r(v)$ be simple majority, $\omega_i^{\delta=1} \sim N(\mu_1, \sigma_1^2)$, and $\omega_i^{\delta=0} \sim N(\mu_0, \sigma_0^2)$.*

A possible distribution of the ω_i s in this legislature is represented in Figure 3.1. Suppose that the government and opposition legislative contingents are almost of equal size (i.e., the government has fifty one seats and the opposition has fifty). As shown in Figure 3.1, legislators can be ordered according to their ω_i s. All legislators in $N_1(C)$, $N_5(C)$, and $N_6(C)$ are located to the right of $\omega_i = 0$. All legislators in $N_2(C)$, $N_3(C)$, and $N_4(C)$ are located to the left of $\omega_i = 0$. With regard to legislators’ utilities, assume that for government legislators, $u(x^*) = 1$ and $u(x^{sq}) = -1$. Likewise, for opposition legislators, $u(x^*) = -1$ and $u(x^{sq}) = 1$. Hence, everything is measured in the metric of $u(x^*) - u(x^{sq})$.

Given these parameter values, a legislator in $N_2(C)$ could have a $\omega_i = -.8$, and if the outcome is already decided in favor of x^* , she will vote against x^* , receive no bribes, and still have a positive utility of 1.8. Suppose, however, that she would only vote in support of x^* if she receives a bribe. The chief executive would have to compensate her for casting a vote in favor of x^* and being punished by her principal. Therefore, the cost of buying this legislator’s vote becomes $2s_i = 1.6$. In the case of a

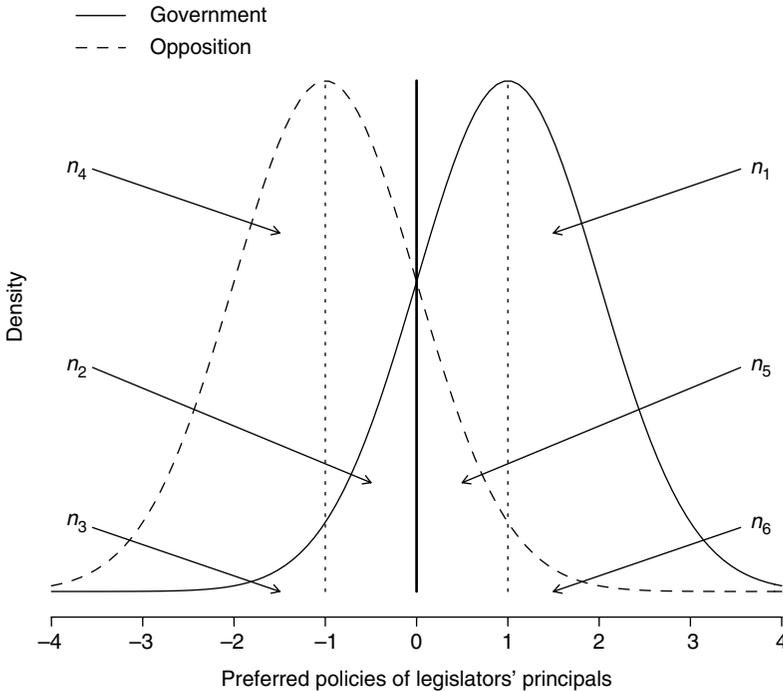


Figure 3.1. Preferred policies of legislators' principals – An example.

Notes: This figure illustrates how principals' policy preferences shape the size and composition of legislative factions. The example is based on a 101-member legislature operating under majority rule, where the government has 51 seats and the opposition has the remaining 50. The ideal policies of legislators' principals (ω_i s) are given by $\omega_i^{\delta=1} \sim N(1, 1)$ for government legislators and by $\omega_i^{\delta=0} \sim N(-1, 1)$ for those in the opposition. Legislators are ordered according to their ω_i s. To the right of $\omega_i = 0$ are $n_1(C)$, $n_6(C)$, and $n_5(C)$ legislators. To the left of $\omega_i = 0$ are $n_2(C)$, $n_3(C)$, and $n_4(C)$ legislators. Regarding legislators' utilities, everything is measured in the metric of $u(x^*) - u(x^{sq})$, where $u(x^*) = 1$ and $u(x^{sq}) = -1$ for government legislators and $u(x^*) = -1$ and $u(x^{sq}) = 1$ for opposition legislators. In this example, a $\omega_i = -.8$ may correspond to a *potentially decisive* government legislator. Whenever her vote is not decisive and the new policy is supported by a legislative majority, this legislator would vote against the proposal, receive no bribes, and still have a positive utility of 1.8.

legislator in $N_5(C)$, she could have a $\omega_i = .5$. If the outcome is already decided, she will vote for x^* and get a utility of 1.5 if x^{sq} is the winning policy and -.5 otherwise. If her vote is needed to change the outcome, then she can sell it and her vote will be worth $2s_i = 1$.

Because voting coalitions in support of x^* can be conformed by buying different types of legislators, a chief executive will always buy the least expensive votes to achieve x^* ; but their individual cost is given by $2s_i$, and these “prices” depend on the realization of the ω_i s. Therefore, to complete the analysis, the total cost of a bribed winning coalition needs to be calculated.

Let the “approval” cost function be $Y: g(\tilde{\omega}) \rightarrow \mathbf{R}$ where $g(\tilde{\omega}) = g(\omega^{\delta=1}, \omega^{\delta=0})$ denotes the realization of $\omega_i^{\delta=1}$ and $\omega_i^{\delta=0}$. Then, the expected total cost of bribes can be expressed as:

$$\hat{Y} = \int \int g(\tilde{\omega}) f_{\omega^{\delta=1}} f_{\omega^{\delta=0}} d\omega^{\delta=1} d\omega^{\delta=0} \tag{3.9}$$

The optimal bribe vector \mathbf{p} depends on the total cost of a winning coalition and on $\Pi(\nu)$, the chief executive’s budget to buy those individual votes. If the total cost of bribes is larger than his budget ($Y > \Pi(\nu)$), then the status quo will prevail. Likewise, if the total cost of securing these votes exceeds the value of policy change plus the political costs ($Y > \theta_E + c$), then the chief executive would actually be better off by conceding defeat.

3.2 PROPOSAL STAGE

Turning to the proposal stage, the chief executive’s decision entails sending a proposal x^* to the legislature or keeping the status quo policy x^{sq} . Because each legislator’s reservation value depends on a proposal’s content, the chief executive would like to send a bill x^* that maximizes his policy objectives and minimize his payments (i.e., by compromising on policy). Nonetheless, as I argued in Chapter 2, because the chief executive sends his bills to the legislature without knowing the ideal policy of legislators’ principals, he cannot calculate *ex-ante* such optimal proposal.

Given some prior on $\omega_i^{\delta=1}$ and $\omega_i^{\delta=0}$, the chief executive adopts a *sending* strategy of the form: $\rho : [\underline{Y}, \bar{Y}] \rightarrow \{1, 0\}$, where $\rho(\hat{Y}) = 1(0)$ means that the chief executive sends (does not send) a bill to the legislature if the expected cost of securing majoritarian support for the bill is \hat{Y} . Therefore, the sending strategy ρ depends on the probability that the total cost of bribes would not exceed either the chief executive’s vote-buying budget or the sum of the value of policy change and the political costs of defeat to him.

These considerations lead to the following proposition:

Proposition 3 *The game has an equilibrium in pure strategies where the chief executive sends a bill to the legislature and is defeated.*

The chief executive would only send a bill to the legislature when he estimates that his vote-buying budget is large enough to secure majoritarian support. Depending on the particular realization of Y , the chief executive may be able, if needed, to gather sufficient support for such bill or not. Even with well-defined priors, the draws of Y may be quite different. If the chief executive sends a bill to the legislature, and the realization of Y is larger than $Y = \theta_E + c$, then he may have to concede the issue. In addition, even if it is worthwhile for the chief executive to buy some additional votes and make x^* the new policy, if $\Pi(v) < Y < \theta_E + c$, the chief executive is unable to secure the bill's passage.

Defeat, of course, is not the only possible equilibrium outcome. As the total cost of a bribed winning coalition decreases, the chief executive would be able to pay the necessary compensations to achieve his preferred policy. He also would not need to resort to his "pocketbook" in order to handle the effects of cross-voting if the total cost of bribes drops to zero. If that is the case, a bill would pass with majority support and no payments would be necessary.

3.3 EMPIRICAL IMPLICATIONS

As discussed in Chapter 2, the model stresses the role of uncertainty regarding legislative voting behavior as the key factor shaping the capacity of chief executives to successfully enact policy changes through government acts that carry the force of law by winning legislative majorities. The importance of this distinction can be seen by explicitly examining how agenda setting and the number of legislators who belong to the chief executive's party/coalition affect the passage of legislation in this model.

Numerical Examples. To get a sense of the model's predictions, it is helpful to first work through several numerical examples. Let the chief executive be the proposer of legislation (i.e., the agenda-setter), $n = 101$, and the decision rule $r(v)$ be simple majority. Let $\omega_i^{\delta=1} \sim N(\mu_1, \sigma_1^2)$, and $\omega_i^{\delta=0} \sim N(\mu_0, \sigma_0^2)$. Also $u(x^*) = 1$ and $u(x^{sq}) = -1$ for government legislators; and $u(x^*) = -1$ and $u(x^{sq}) = 1$ for opposition legislators. Let $c = .5$.

Example 2 Suppose the chief executive's party has 45 seats in the legislature. Let $\Pi(v) = 2.5$, and $\hat{Y} = 2.5$.

In this case, the chief executive would adopt a sending strategy $s(\hat{Y}) = 0$. Therefore, the final outcome is that no bill is sent to the legislature, the status quo policy x^{sq} remains unchanged, and the chief executive does not pay any political costs of defeat.

Example 3 Suppose the chief executive's party has 53 seats in the legislature. Let $\Pi(v) = 3$, $\hat{Y} = .5$, and $Y = 2.6$.

Suppose the chief executive adopts a sending strategy $s(\hat{Y}) = 1$. Once the bill is sent to the legislature, he has enough resources to buy additional votes. However, buying the additional votes to make x^* the new policy is not a dominant strategy for the chief executive. Therefore, the bill x^* is defeated, the status quo policy x^{sq} remains unchanged, and the chief executive pays the political costs of defeat.

Example 4 Suppose the chief executive's party has 55 seats in the legislature. Let $\Pi(v) = .5$, $\hat{Y} = 0$, and $Y = .45$.

Suppose the chief executive adopts a sending strategy $s(\hat{Y}) = 1$. In this case, despite the miscalculation, the chief executive can buy the additional votes to make x^* the new policy. As a result, the bill x^* becomes the new policy after some payments are made.

Example 5 Suppose the chief executive's party has 55 seats in the legislature. Let $\Pi(v) = .5$, $\hat{Y} = 0$, and $Y = 1$.

Suppose the chief executive adopts a sending strategy $s(\hat{Y}) = 1$. In this case, buying the additional votes to make x^* the new policy is worthwhile for him (e.g., $\theta_E + c > Y$). Yet, as the chief executive miscalculated the total cost of bribes, his vote-buying budget is not large enough to make such payments. Therefore, the bill x^* is defeated, the status quo policy x^{sq} remains unchanged, and the chief executive pays the political costs of defeat.

Simulation Results. I now examine the model's predictions using simulated data. As before, let the chief executive be the proposer of legislation (i.e., the agenda-setter), $n = 101$, and the decision rule $r(v)$ be simple majority. Legislators' ω_i s were randomly generated assuming that $\omega_i^{\delta=1} \sim N(1, 1)$, and $\omega_i^{\delta=0} \sim N(-1, 1)$.

Table 3.2 presents how majority sizes, composition, and the total cost of winning coalitions vary as a function of ω_i for these simulated values. I restrict attention to situations where the party of the chief executive controls between 45 and 61 seats in the 101-member legislature. The first six columns indicate the number of legislators in each faction. The following two columns indicate the average ω_i for that draw, for both the government and opposition legislators. The ninth column indicates the composition of the voting coalition in support of x^* . The following column indicates the size of the coalition. The eleventh column indicates how many legislators have to be bribed. The final column indicates the total cost. These estimates are also examined according to the total number of government and opposition seats. The first three panels correspond to situations where the party of the chief executive fails to have a majority of seats in the legislature. The remainder of the table presents situations where government legislators have a majority of seats.

Take, for example, the first entry in Table 3.2. Government legislators hold 45 seats in the legislature, $n_1(C) = 36$, $n_2(C) = 6$ and $n_3(C) = 1$. The opposition is composed of 56 legislators; 42 legislators in $N_4(C)$, 11 in $N_5(C)$ and 3 legislators in $N_6(C)$. The average ideal policy for government legislators' principals, $\bar{\omega}^{\delta=1}$ is 0.73, and the average ideal policy for opposition legislators' principals, $\bar{\omega}^{\delta=0}$ is -0.69. The least expensive coalition includes 52 legislators from factions $N_1(C)$, $N_2(C)$, $N_4(C)$, $N_5(C)$, $N_6(C)$. The logic goes as follows: the chief executive can buy just two legislators to make all legislators in $N_5(C)$ nondecisive. Given the reservation value of the two least costly legislators (in this case, one legislator in $N_2(C)$, and one legislator in $N_4(C)$), the total cost of putting together a voting coalition in support of x^* is 0.34.

Consider now what happens in the scenario represented in the fourth entry of Table 3.2. There are also forty five government legislators and fifty six opposition legislators. But in this case, the government has to bribe eight legislators (excluding one potentially decisive government legislator) to make all legislators in $N_5(C)$ nondecisive. In this case the total cost of a winning coalition in support of x^* (including three legislators in $N_2(C)$, and five in $N_4(C)$) is 3.78. Alternatively, it can buy all five legislators in $N_5(C)$ plus six more legislators and make a legislator in $N_2(C)$ decisive. In this case, the cost would be 5.5. To have a sense of what this means in substantive terms, the cost of a winning coalition that accounts for the value of policy change for the chief executive is $\theta_E = 2$. In this last case, the cost of the least expensive coalition is almost twice as large than the utility loss for the chief executive from keeping the status quo.

Table 3.2. *Simulated coalitions and total cost*

n_1	n_2	n_3	n_4	n_5	n_6	$\bar{\omega}^{\delta=1}$	$\bar{\omega}^{\delta=0}$	Types included	Size	Bribed Legs.	Total Cost ($\sum_{i=1}^n \tau_i(v)$)
36	6	3	42	11	3	0.73	-0.69	1,2,4,5,6	52	2	0.34
39	6	0	50	5	1	0.98	-1.06	1,2,4,5,6	52	7	1.97
37	7	1	50	6	0	1.13	-1.19	1,2,4,5	52	9	1.88
39	6	0	51	5	0	1.08	-1.09	1,2,4,5	52	8	3.78
34	11	0	46	8	2	1.11	-0.99	1,2,4,5,6	52	8	2.26
41	4	2	41	13	0	1.19	-1.07	1,5	54	0	0
34	12	1	42	11	1	0.72	-1.05	1,2,4,5,6	52	6	1.93
38	7	2	46	7	1	1.02	-1.12	1,2,4,5,6	52	6	2.25
38	8	1	43	9	2	1.06	-0.75	1,2,4,5,6	52	3	0.77
39	8	0	47	7	0	1.11	-1.04	1,2,4,5	52	6	1.11
43	4	2	42	8	2	1.07	-1.16	1,5,6	53	0	0
41	8	0	46	5	1	1.09	-1.11	1,2,4,5,6	52	5	0.59
39	10	0	45	7	0	0.94	-0.96	1,2,4,5	52	6	3.01
42	6	1	44	7	1	1.05	-1.17	1,2,4,5,6	52	2	0.28
40	7	2	46	5	1	0.91	-1.12	1,2,4,5,6	52	6	1.31
42	8	1	42	8	0	0.83	-0.99	1,2,5	52	2	0.55
42	9	0	40	9	1	0.82	-0.88	1,5,6	52	0	0
41	9	1	43	6	1	0.66	-1.02	1,2,4,5,6	52	4	0.73
43	8	0	45	3	2	0.88	-1.11	1,2,4,5,6	52	4	0.65
43	8	0	43	7	0	0.91	-0.98	1,2,4,5	52	2	0.16
40	12	1	42	5	1	0.97	-0.98	1,2,4,5,6	52	6	0.51
47	5	1	39	8	1	1.17	-0.94	1,5,6	56	0	0
42	10	1	41	5	2	0.96	-0.79	1,2,4,5,6	52	3	0.11
41	11	1	42	6	0	0.87	-0.97	1,2,4,5	52	5	1.62
45	7	1	37	10	1	0.92	-0.98	1,5,6	56	0	0
49	4	2	43	2	1	0.99	-0.98	1,5,6	52	0	0
45	8	2	39	5	2	0.83	-0.92	1,5,6	52	0	0
49	6	0	39	7	0	1.01	-1.01	1,5	56	0	0
47	6	2	39	5	2	0.87	-0.93	1,5,6	54	0	0
45	10	0	36	9	1	1.11	-0.74	1,5,6	55	0	0
53	4	0	39	4	1	1.07	-1.19	1,5,6	58	0	0
44	10	3	38	6	0	0.64	-0.95	1,2,4,5	52	2	0.45
51	5	1	34	10	0	1.02	-0.99	1,5	61	0	0
46	11	0	38	5	1	0.95	-1.35	1,5,6	52	0	0
46	10	1	36	7	1	0.93	-1.07	1,5,6	54	0	0
45	13	1	41	1	0	0.88	-1.42	1,2,5	51	5	1.55
52	6	1	33	9	0	1.02	-0.88	1,5	61	0	0
50	8	1	35	6	1	1.05	-1.12	1,5,6	57	0	0
47	10	2	37	5	0	0.89	-1.24	1,5	52	0	0
49	8	2	34	6	2	0.88	-0.93	1,5,6	57	0	0
51	7	3	29	8	3	0.89	-0.70	1,5	62	0	0
54	7	0	34	5	1	0.96	-1.05	1,5,6	60	0	0
50	9	2	36	4	0	1.06	-1.22	1,5	54	0	0
53	6	2	37	3	0	1.05	-1.11	1,5	56	0	0
48	13	0	32	6	2	1.13	-1.07	1,5,6	56	0	0

Notes: This table shows how majority sizes, composition, and the total cost of winning coalitions vary as a function of the ideal policies of legislators' principals. The outcomes are based on simulated data for a 101-member legislature operating under majority rule. Legislators' mandates from their principals (ω_i s) were randomly generated assuming that $\omega_i^{\delta=1} \sim N(1, 1)$ for government legislators and $\omega_i^{\delta=0} \sim N(-1, 1)$ for those in the opposition. The first six columns indicate the number of legislators in each faction. Columns 7 and 8 indicate the average ω_i for that draw, for the government and opposition legislators, respectively. Column 9 indicates the composition of the voting coalition in support of x^* . Column 10 indicates the size of the coalition. Column 11 indicates how many legislators have to be bribed. Column 12 indicates the total cost. The first three panels correspond to situations where the party of the chief executive fails to have a majority of seats in the legislature. The remainder of the table presents situations where government legislators have a majority of seats.

Moving down in Table 3.2, one can see more generally how the composition of the legislature and the distribution of the ω_i s affect the cost of a winning coalition. For instance, when the chief executive's party is in the minority (i.e., it has fewer than fifty one seats) the average cost of a winning coalition is 1.43, including a maximum of 3.78 and a minimum of 0 (i.e., no bribes are needed). In all these cases, the chief executive receives all the votes from the potentially decisive opposition legislators for free without giving out any bribes or buying legislators from other factions.

Notice what occurs when the chief executive's party controls a majority of seats in the legislature. Take the cases where it has fifty one or fifty three seats. The average coalition cost is 0.43, with a maximum of 1.62 and a minimum of 0. In some cases, despite having a majority of seats, the chief executive may need to buy some of his own legislators to win with certainty. In some other cases, bribes will not be necessary given a "surplus" of potentially decisive opposition legislators. The variance in $\omega^{\delta=1}$ and $\omega^{\delta=0}$, which reflects the principal's influence, plays an important role both in favor and against the government. Finally, when the party of the chief executive has an ample majority of seats, the effects of the principals' influence are mitigated. When it has more than 55 seats, voting coalitions in support of x^* are costless or very cheap. Still, in some cases where legislators' principals really dislike a bill, some payments will have to be made to get x^* approved. For example, in one of the cases, government legislators hold 59 seats in the legislature, but unconditional supporters constitute a minority: $n_1(C) = 45$, $n_2(C) = 13$, and $n_3(C) = 1$. In the opposition, there are 41 legislators in faction $N_4(C)$, 1 in $N_5(C)$, and no legislators in $N_6(C)$. In this case, the legislator in $N_5(C)$ is always decisive. The chief executive, can thus buy 6 legislators (4 in $N_2(C)$, and 2 in $N_4(C)$) at a cost of 1.82. Alternatively, it can bribe the potentially decisive opposition legislator and buy 4 more legislators (all in $N_2(C)$) at a cost of 1.55, and make a legislator in $N_2(C)$ decisive. The latter option is the least expensive one, but still considerably high when considering that the government controls an ample majority of seats.

Figure 3.2 presents a graphical representation of the distribution of Y , the costs of securing majoritarian support for a bill, based on the simulated values. As Figure 3.2 indicates, most of the time, the total cost of bribes is smaller than the value of policy change for the chief executive. However, there are some draws for which the value of $Y > \theta_E$. Therefore,

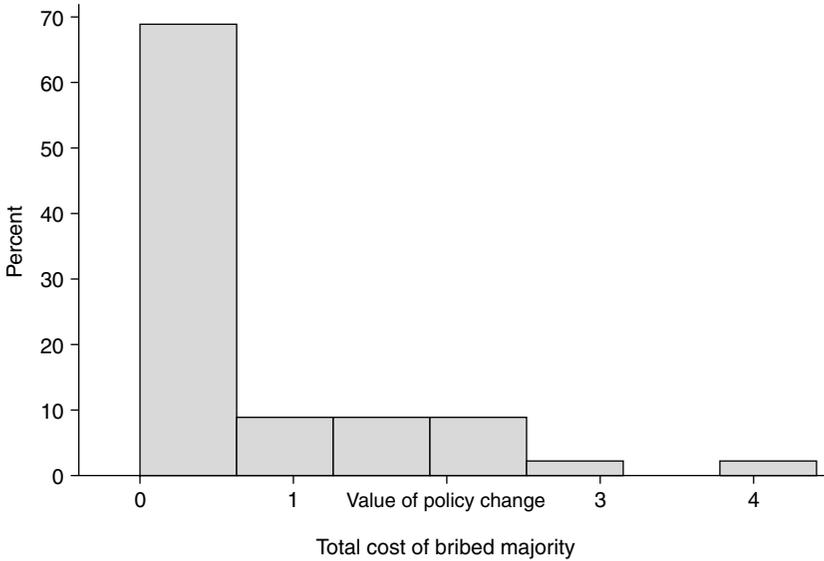


Figure 3.2. Simulated cost of winning coalition.

Notes: This figure presents a graphical representation of the simulated cost of a winning coalition. These values correspond to those in column 12 of Table 3.2. In most cases, the total cost of bribes is smaller than 2, the value of policy change for the chief executive. However, there are some draws for which the total cost of securing additional votes exceeds the value of policy change.

as the model predicts, when chief executives cannot identify the policy preferences of legislators’ principals, they may suffer defeats.

3.4 CONCLUDING REMARKS

As discussed in the previous chapter, the existing literature does not provide us with a good explanation of why chief executives suffer legislative defeats. Most models are inadequate on two counts. First, they are unrealistic in their predictions that executive-initiated bills are never defeated. Second, they often neglect the role of cross-pressured legislators.

The model introduced in this chapter explains the main puzzle posed in this book: Even if a proposer has no intention to be defeated, his initiatives can fail at the legislative stage. The results suggest that it takes very little uncertainty for a chief executive to miscalculate his legislative support. The analysis also reveals that when a bill commands too much opposition (and thus the total cost of a bribed winning coalition is too high), chief

executives will be unwilling or unable to pay the necessary compensations to achieve their preferred policies.

Beyond explaining why executive-initiated bills sometimes fail, the model also provides new insights into statutory policy making. My findings indicate that: (1) cross-partisan minimum-winning coalitions would occur only under a very restrictive set of circumstances when no bribes are paid; (2) without bribes, minimum-winning coalitions composed exclusively of government legislators would occur only under an even more restrictive set of circumstances; (3) equilibrium vote-buying behavior does not result in supermajority coalitions.

The model's predictions are borne out by the outcomes generated using simulated data. Moving beyond the simulations, the model also yields several important implications with regard to the variation in chief executives' legislative passage rates that are observed in the real world. I examine this variation using data from fifty two countries in the period between 1946 and 2008 in Part II of the book.