

# EITM and Comparative Politics

## Morning Session

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# Inference without Theoretical Models

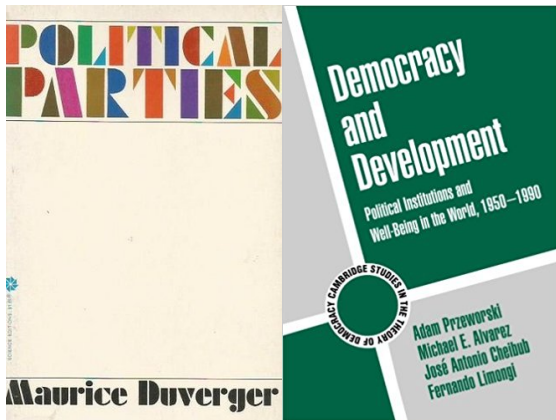
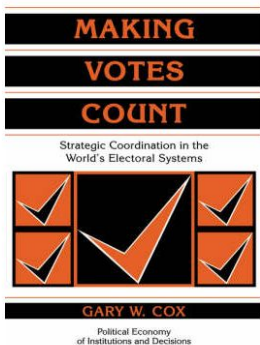


Figure 1: Examples

# Theory makes its way!



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## Democracy as an equilibrium

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**Abstract.** Observation shows that while democracy is fragile in poor countries, it is impregnable in developed ones. To explain this pattern, I develop a model in which political parties propose redistributions of incomes, observe the result of an election, and decide whether to comply with the outcome or to launch a struggle for dictatorship. Democracy prevails in developed societies because too much is at stake in turning against it. More income can be redistributed in developed than in poor countries without threatening democracy. Limits on redistribution arise endogenously, so that constitutions are not necessary for democracy to endure. A democratic culture characterizes the equilibrium.

Figure 2: Examples

# Theoretical Models without Empirics

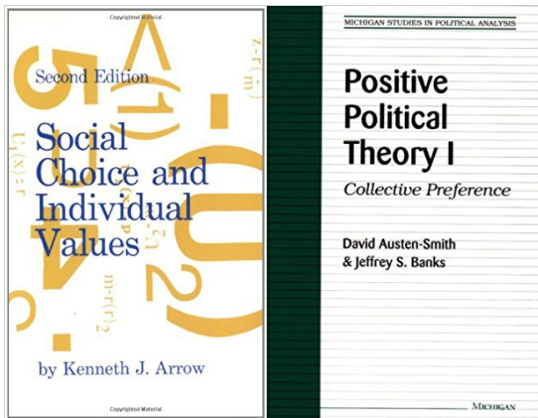


Figure 3: Examples

# Theoretically Rooted Empirical Studies

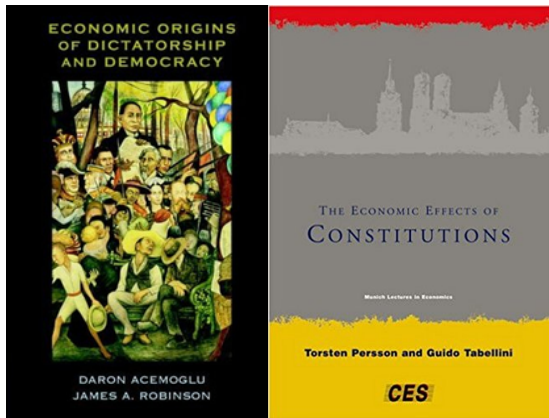


Figure 4: Examples

# EITM Approach

We are in debt to both traditions in comparative politics; but we are going to focus on the second approach.

Speculation is the soul of social sciences; but how can we increase the quality of speculation?

- ▶ Interesting topic: insufficient
- ▶  $N \text{ problems} = \infty$
- ▶ For any problem,  $N \text{ theories} = \infty$

Value of contribution increases with value of **simplification**.

Example: Borges' map.

# Model-Building

Models are theoretical exercises of abstraction.

- ▶ Ability to **abstract** from reality (ignore details in order to focus on most important elements of problem)
- ▶ Aptitude for making inferences within an abstract model (deductive logic).
- ▶ Competence at **evaluating** a model (logical consistency; empirical check).
- ▶ Familiarity with common/canonical models (two seemingly different models might actually be the same!)

## Model-Building (cont.)

In practical terms, the following steps are usually followed:

1. Observe some facts
2. Speculate about processes/mechanisms that might have produced such results
3. Deduce other results (implications/consequences/predictions) from the model.
4. Ask yourself whether these other implications are true (alternative models).



## Model-Building (cont.)

What makes a *good* model?

- ▶ **Simplicity.** A model that has a small number of assumptions is more attractive than one having a large number of them.
- ▶ **Fertility.** A good model will produce a relatively large number of interesting implications per assumption.
- ▶ **Unpredictability.** Attractive models usually generate unexpected implications that are surprising, and not immediately obvious from the assumptions.

# Models and Empirical Analysis

Models can be tested using statistics and econometrics.

- ▶ *Reduced form*: statistical relationship between observed variables (e.g. without reference to the specific details of the model).
- ▶ *Structural estimation*: use underlying model to identify parameters of interest – including unobservable parameters that help describe behavior at a deep level.

A linear regression will be a reduced-form of some true structural model (e.g. how much Fort Knox should spend on protection?).

# Canonical Models for Political Analysis

In a world where all individuals have identical goals, there would be no disagreement over policies.

- ▶ In the real world, though, policies are often chosen by a collective decision-making mechanism that must balance conflicting interests.
- ▶ From an analytical point of view, we can thus conceive “politics” as the process by which groups of individuals with conflicting goals reconcile these goals with each other.

Models: *Mechanisms* by which these conflicts are resolved.

## Canonical Models for Political Analysis (cont.)

Social-choice theoretic results illuminate the difficulties inherent in accommodating multiple goals within a single decision framework.

- ▶ Arrow's theorem: Suppose there are at least three alternatives. If an aggregation rule with unrestricted domain is transitive, weakly Paretian and independent of irrelevant alternatives, then it is dictatorial.
- ▶ Gibbard-Satterthwaite theorem: If the range of a strategy-proof collective choice function contains at least three alternatives, then it is dictatorial.

Welcome to politics!

## Canonical Models for Political Analysis (cont.)

Focus on two broad mechanisms used to aggregate individual preferences into a collective choice

- ▶ Voting (Median Voter Theorem)
- ▶ Bargaining (Divide the Dollar)

Two simple representations that allow us to capture distributive nature of politics.

# Median Voter Theorem

Suppose that a group uses majority vote to reach a collective decision.

- ▶ *Condorcet's paradox* famously illustrates a problem stemming from majority rule in which pairwise voting over three or more alternatives can lead to intransitive (or cyclic) outcomes.
- ▶ A possible “solution” to the problem of indeterminacy under majority rule is to impose restrictions on preferences.

Such restriction leads to the **median voter theorem** (Black 1948).

## Median Voter Theorem (cont.)

Two **key** assumptions:

- ▶ Voters' preferences are defined on a single dimension.
- ▶ Each voter's preferences are single-peaked in that one dimension.

Intuitively, a single-peaked profile is one in which the set of alternatives can be ordered along a left-right scale in such a way that each individual has a unique most preferred alternative (or *ideal point* and the individual's ranking of other alternatives falls as one moves away from her ideal point.

## Median Voter Theorem (cont.)

Suppose that a set  $Q$  of outcomes is linearly ordered as  $q^1 < q^2 < \dots < q^k$ .

- ▶ *MVT*: under majority rule, if the number of voters is odd, then there is a unique winning outcome, equal to the ideal point of the individual whose ideal point constitutes the median (with respect to  $Q$ ) of the set of ideal points.

Proof: Denote by  $q_i^*$  the ideal point of individual  $i$ , and order individuals by increasing ideal points:  $q_1^* \leq q_2^* \leq \dots q_n^*$ . As  $N$  is odd, the number  $m = \frac{(N+1)}{2}$  is an integer, and  $q_m^*$  is the median ideal point. The individuals whose ideal point is not greater than  $q_m^*$  form a strict majority; and those whose ideal point is not smaller than  $q_m^*$  form another majority. Therefore,  $q_m^*$  is the unique winner!



# Party Competition

We can now use the MVT to examine a simple model of party competition (Downs)

- ▶ A set of voters  $y \in Y$  (each voter's *type* may be thought as a vector of traits which characterizes her preferences).
- ▶ A set of policies  $t \in T$ , where  $T$  is a real interval (i.e. unidimensional issue space).
- ▶ Every voter has a preference order over policies,  $v : T \rightarrow \mathbf{R}$
- ▶ There 2 political parties,  $i = \{1, 2\}$ .
- ▶ Party  $i$  has a payoff function  $\Pi^i : T \times T \rightarrow \mathbf{R}$ .

We assume that voting takes place under majority rule and that everybody votes.

## Party Competition (cont.)

A political equilibrium is a policy pair  $(t_1^*, t_2^*)$  such that:

- ▶  $\forall t \in T, \Pi^1(t_1^*, t_2^*) \geq \Pi^1(t_1, t_2^*)$
- ▶  $\forall t \in T, \Pi^2(t_1^*, t_2^*) \geq \Pi^2(t_1^*, t_2)$

We assume that both parties know the distribution of voter types, and their utility function  $v$

- ▶ Let  $\rho(t_i, t_{-i})$  be the fraction of voters who will vote for  $t_i$  when facing a choice between  $t_i$  and  $t_{-i}$ . This fraction is either more than, equal to, or less than one-half.

## Party Competition (cont.)

Denote by  $\pi_i(t_i, t_{-i})$  the probability that party  $i$  wins the election, then:

- ▶  $\pi_i(t_i, t_{-i}) = 1$  if  $\rho > \frac{1}{2}$
- ▶  $\pi_i(t_i, t_{-i}) = \frac{1}{2}$  if  $\rho = \frac{1}{2}$
- ▶  $\pi_i(t_i, t_{-i}) = 0$  if  $\rho < \frac{1}{2}$

We assume that in the case of a tie, a fair coin is tossed to determine the winner.

## Party Competition (cont.)

The Downs model is a special case of political equilibrium in which:

- ▶  $\Pi^1(t_1, t_2) = \pi_1(t_1, t_2)$
- ▶  $\Pi^2(t_1, t_2) = 1 - \pi_1(t_1, t_2)$

That is, we assume that each party wants to maximize its probability of winning.

- ▶ Let  $t^* \in T$  be a strict *Condorcet Winner* (the alternative that obtains a majority of the vote in every pairwise contest against the other alternatives).

Then, the policy pair  $(t^*, t^*)$  is the unique Downsian equilibrium.

## Party Competition (cont.)

Proof:

- 1 Existence.  $\Pi_i(t^*, t^*) = \pi_i(t^*, t^*) = \frac{1}{2}$  Is there an alternative  $t$  such that  $\pi(t, t^*) > \frac{1}{2}$ ? No.

So, if either party deviates, its probability of victory falls to 0.

- 2 Uniqueness. Suppose  $(t_1^*, t_2^*)$  is a plausible equilibrium where  $t_1^* \neq t^* \neq t_2^*$ . Both parties cannot win for sure at  $(t_1^*, t_2^*)$ .

- ▶ Suppose  $\pi_1(t_1^*, t_2^*) < 1$ , then party 1 should deviate to  $t^*$  so that  $\pi_1(t^*, t_2^*) = 1$
- ▶ Suppose  $t_1^* = t^*$  and  $t_2^* \neq t^*$ , then  $\Pi^2(t_1^*, t_2^*) = 0$ . But, by switching to  $t^*$ ,  $\pi_2(t_1^*, t^*) = \frac{1}{2}$ . So, party 2 should deviate.

Cool! But, remember that this result holds as long as  $v$  is continuous on its arguments, and single-peaked (in  $t$ ) for all  $y \in Y$ .

## Application: Income Redistribution

Consider redistribution of income in a world where all affected individuals are included in the decision-making process.

- ▶ The relevant heterogeneity across individuals is in their income levels.
- ▶ The analysis should illustrate the **amount** of redistribution that emerges from the political process.

More concretely, the application is how tax policy is determined through political competition.

# Full Redistribution

Let's characterize now the *Downsian* equilibrium in a one-dimensional type space where  $y \in Y$  is income.

- ▶ The tax rate is drawn from an interval  $t \in [0, 1]$ .
- ▶ Suppose that revenues (from proportional taxation) are redistributed equally on a *per capita* basis.
- ▶ Every voter has a preference over policies, represented by  $v(t, y)$ .

As before, we assume that the number of voters  $n \in N$  is odd; and that everybody votes.

## Full Redistribution (cont.)

The utility of individual  $i$  is given by her after-tax income:

$$v^i(t, y) = (1 - t)y^i + t\bar{y},$$

$$\text{where } \bar{y} = \frac{1}{n} \sum_{i=1}^n y^i.$$

The effect of a change in the tax rate  $t$  on  $i$ 's welfare is given by:

$$\frac{\partial v_i}{\partial t} = -y^i + \bar{y}.$$

Notice that if  $y^i > \bar{y}$ , then  $\frac{\partial v_i}{\partial t} < 0$ ; so  $t^* = 0$ . But if  $y^i < \bar{y}$ , then  $\frac{\partial v_i}{\partial t} > 0$ ; so  $t^* = 1$ . Therefore, there are two ideal tax rates for this polity: zero and one.



## Full Redistribution (cont.)

Denote by  $y^m$  the individual with the median income.

Suppose that  $y^m < \bar{y}$  (as it is always the case in actual societies)

Then, the *Downsian* equilibrium will be  $t^* = 1$ .

That is, there will be complete redistribution of income toward the mean!

## Redistribution without Full Confiscation

Suppose now that, to make the model more realistic, we assume that taxation is **not** costless.

Technically  $c(t)$ , and  $c'(\cdot) > 0$ ;  $c''(\cdot) \geq 0$ .

We also assume that  $c'(0) = 0$ ;  $c'(1) = 1$ .

To make things simpler, we can set  $c = \frac{1}{2}t^2$  (so  $\frac{dc}{dt} = t$ ).

- ▶ The *per capita* transfer is now:  $\frac{1}{n}(\sum_{i=1}^n ty^i - c(t)n\bar{y})$ , or  $(t - (\frac{t^2}{2}))\bar{y}$ .
- ▶ And, individual  $i$ 's utility is given by:  
 $v^i(t, y) = (1 - t)y^i + (t - (\frac{t^2}{2}))\bar{y}$ .

## Redistribution without Full Confiscation (cont.)

How does  $v$  change as  $t$  changes?

$$\frac{\partial v_i}{\partial t} = -y^i + \bar{y} - t\bar{y}.$$

- The F.O.C. for a maximum with respect to  $t$  is that

$$-y^i + \bar{y} - t\bar{y} = 0,$$

and the second-order condition is that  $-\bar{y} \leq 0$ .

The second-order condition is satisfied, so it follows that the optimal value of  $t$  is  $t^* = 1 - \frac{y^i}{\bar{y}}$

If  $y^i \geq \bar{y}$ , then  $t^* = 0$  (we rule out “negative” taxes).

If  $y^i < \bar{y}$ , then  $t^* > 0$ . But, less than  $t^* = 1$  because of  $c(t)$ !

## Redistribution without Full Confiscation (cont.)

The intuitive result is that rich people prefer lower tax rates and less redistribution than poor people.

- ▶ To derive this comparative statics result formally, assume that  $t^i > 0$  and find the optimal tax rate of individual  $i$  as a function of her income  $t(y^i)$ .
- ▶ Note that  $v(t, y)$  is a function of both  $t$  and  $y$  (for each different value of  $y$  there will typically be a different optimal choice of  $t$ ).
- ▶ So, we can define the optimal value function,  
$$M(y) = f(t(y), y).$$

The function  $M$  tells us what the optimized value of  $v$  is for different income levels  $y$ .

## Redistribution without Full Confiscation (cont.)

The optimal choice function  $t(y)$  must satisfy the condition

$$\frac{\partial v(t(y), y)}{\partial t} \equiv 0.$$

Differentiating both sides of this identity,

$$\frac{\partial^2 v(t(y), y)}{\partial t^2} \frac{dt(y)}{dy} + \frac{\partial^2 v(t(y), y)}{\partial t \partial y} \equiv 0.$$

Solving for  $\frac{dt(y)}{dy}$ , we have

$$\frac{dt(y)}{dy} = - \frac{\frac{\partial^2 v(t(y), y)}{\partial t \partial y}}{\frac{\partial^2 v(t(y), y)}{\partial t^2}} = - \frac{1}{\bar{y}} < 0$$

We know that the denominator of this expression is negative due to the second-order conditions for maximization.

So, as income increases, the voter's optimal tax rate decreases.

# Inequality and Redistribution

We can now examine an even simpler model in which there are just two income levels (Acemoglu-Robinson 2006)

- ▶ Two types,  $y^p < y^r$ .
- ▶ Total population is normalized to 1. A fraction  $(1 - \delta) > \frac{1}{2}$  has income  $y^p$  (the fraction  $\delta$  has income  $y^r$ ).
- ▶ We denote mean income by  $\bar{y}$ .
- ▶ To examine the effect of inequality, consider  $\theta$  as the share of total income accruing to the rich; hence we have:

$$y^p = \frac{(1-\theta)\bar{y}}{1-\delta} \text{ and } y^r = \frac{\theta\bar{y}}{\delta}$$

An increase of *theta* represents an increase in inequality.

## Inequality and Redistribution (cont.)

An income distribution where  $y^p < \bar{y} < y^r$  implies that:

$$\frac{(1-\theta)\bar{y}}{1-\delta} < \frac{\theta\bar{y}}{\delta}; \text{ or } \theta > \delta.$$

Politics determines  $t \geq 0$ , the proceeds of which are redistributed lump sum to everyone.

- ▶ As before, the government's budget constraint is  $T = (t - \frac{t^2}{2})\bar{y}$ .
- ▶ The individual with income  $y^p$ 's ideal policy is  $t^p = 1 - \frac{y^p}{\bar{y}}$ , or

$$t^p = 1 - \frac{\frac{(1-\theta)\bar{y}}{1-\delta}}{\bar{y}}.$$

## Inequality and Redistribution (cont.)

With a little bit of algebra, we can simplify the previous expression:

$$t^p = \frac{\theta - \delta}{1 - \delta}.$$

Remember that we normalized the population to 1 (so  $\delta < 1$ ), and we stipulated that  $\theta > \delta$ . That is, both terms are positive.

► Therefore, we can sign  $t^p$ .

Once again, we can use the expression above to find some comparative statics results.



## Inequality and Redistribution (cont.)

In this case, we want to know how  $t^P$  changes as inequality ( $\theta$ ) changes.

- Recall that the sign of  $\frac{dt^P}{d\theta}$  will depend on the second partial of  $v(t(\theta), \theta)$ :

$$\frac{dt^P}{d\theta} = -\left(\frac{\frac{\bar{y}}{1-\delta}}{-\bar{y}}\right) = -\left(\frac{-1}{1-\delta}\right) > 0.$$

Therefore, the greater the inequality, the higher the tax rate.

# Bargaining as an Extensive Form Game

In many situations redistribution is not decided by voting; but rather parties bargain over how to divide a “pie.”

- ▶ Real bargaining is a very complicated affair: sometimes individuals can make take-it-or-leave-it offers; sometimes they make offers simultaneously; and others they just shout at each other.
- ▶ Formalizing bargaining requires that we define a game that will capture certain aspects of bargaining.

We will analyze one very general form of bargaining: the players alternate offers sequentially.

# Divide the Dollar

Lets start first with the classical divide the dollar game:

- ▶ Suppose that two players split a pie of size  $C$  they both value.
- ▶ For simplicity, let  $C = 1$ .
- ▶ Player 1 proposes a division  $(x_1, x_2)$  of the pie, where  $x_1 + x_2 = 1$  and  $0 \leq x_i \leq 1$ , for  $i = 1, 2$ .
- ▶ Player 2, in turn, either accepts this division, in which case she receives  $x_2$  and player 1 receives  $x_1$ , or she rejects it, in which case neither player receives any pie.

## Divide the Dollar (cont.)

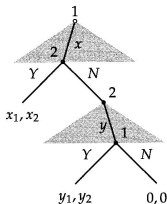
The unique sub-game perfect equilibrium of this game is the following:

- ▶ Player 1 proposes the division  $(1, 0)$ , and player 2 accepts the offer.
- ▶ Player 2 cannot do better because her only alternative to the acceptance of player 1's proposal is rejection, which yields no pie.

Suppose now that player 2 has the option of making a counter-proposal after rejecting player 1's proposal, which player 1 may accept or reject.

## Divide the Dollar (cont.)

This situation can be represented with the following game tree:



- ▶ The triangles represent the continuum of possible proposals. Notice that in this version of the game, player 1 is powerless; her proposal at the start of the game is irrelevant.

## Divide the Dollar (cont.)

- ▶ Every sub-game following player 2's rejection of a proposal of player 1 is a variant of the divide the dollar game in which player 2 moves first.
- ▶ Thus, every such sub-game has a unique sub-game perfect equilibrium in which player 2 offers nothing to player 1, and player 1 accepts.
- ▶ Using Backward Induction, player 2's optimal action after any offer  $(x_1, x_2)$  of player 1 with  $x_2 < 1$  is rejection.

Therefore, in every SGPE player 2 obtains all the pie.

## Divide the Dollar (cont.)

In the extension of this game in which the players alternate offers over many periods, a similar result holds:

- ▶ In every SGPE, the player who makes the offer in the last period obtains all the pie.
- ▶ This result, though, hinges on the assumption that players do not care about the timing of the agreement.

However, it may be more realistic to assume that players prefer to reach a bargain sooner than later.

## Divide the Dollar: Discounting

We can model the “effect” of time in the following way:

- ▶ Suppose the players alternate proposals, one per “period.”
- ▶ When a game has many rounds of play, we need to decide whether discounting is appropriate.
- ▶ If delay has an effect on the payoffs, then player  $i$  will evaluate an outcome in which she receives all of the pie after  $t$  periods as equivalent to an outcome in which she immediately receives the fraction  $\delta_i^t$  of the pie.

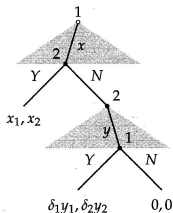
If, for example,  $C = 1$ , then given a discount factor  $0 < \delta_i < 1$ , the total value of the pie for player  $i$  in the first period is 1; the value is  $\delta$  in the second,  $\delta^2$  in the third, and so on and so forth.



## Divide the Dollar: Discounting (cont.)

Consider first a game in which two periods are possible: if player 2 rejects player 1's initial proposal she may make a counter-proposal which, if rejected by player 1, ends the game with a payoff of 0 for each player.

The game would look like this:



## Divide the Dollar: Discounting (cont.)

We can find the SGPE of this game using backward induction:

- ▶ In the second period, if it is reached, player 2 should offer  $y_1 = 0$  to player 1, keeping  $y_2 = 1$  for herself, and player 1 would accept.
- ▶ In the first period, though, player 1 could offer player 2  $\delta_2$ , keeping  $(1 - \delta_2)$  for herself, and player 2 would accept (she could receive a greater share by refusing, but that greater share would arrive later and be discounted).
- ▶ Hence, the game has a unique SGPE in which player 1 proposes  $(1 - \delta_2, \delta_2)$  and player 2 accepts.

## Divide the Dollar: Discounting (cont.)

We will now extend this game by allowing the players to alternate proposals over many periods rather than two.

- ▶ Suppose we want to model a three-period bargaining game with alternating offers:
- ▶ In period 3, if it is reached, player 1 would offer 0 to player 2, keeping 1 for herself.
- ▶ In period 2, player 2 could offer player 1 a share  $\delta_1$ , keeping  $(1 - \delta_1)$  for herself.

By the same token, in the first period, player 1 could offer player 2 a share  $\delta_2(1 - \delta_1)$  and keep the rest,  $1 - \delta_2(1 - \delta_1)$  for herself.

## Divide the Dollar: Discounting (cont.)

The following table summarizes the sequential payoffs for  $T = 3$ :

Round	1's Share	2's Share	$u_1(\cdot)$	$u_2(\cdot)$
T-2	$1 - \delta_2(1 - \delta_1)$	$\delta_2(1 - \delta_1)$	$1 - \delta_2(1 - \delta_1)$	$\delta_2(1 - \delta_1)$
T-1	$\delta_1$	$1 - \delta_1$	$\delta_1^2$	$\delta_2(1 - \delta_1)$
T	1	0	$\delta_1^2$	0
Rejected	0	0	0	0

The game has a unique SGPE in which player 1 offers the amount  $\delta_2(1 - \delta_1)$  to player 2 at the beginning of the game, and she accepts.

## Alternating Offers Over Infinite Time

Lets look now at what would happen if we allow the players to bargain forever.

- ▶ The first two periods of the game look like the 2-period version, except that player 1's rejection of an offer in the second period does not leads to the end of the game (with payoffs  $(0,0)$ ), but to a sub-game in which the first move is a proposal of player 1.
- ▶ The structure of this sub-game is the same as the structure of the whole game.

This implies that player 1 always makes the same offer in every round.

## Alternating Offers Over Infinite Time (cont.)

Let the players have discount factors of  $\delta_1$  and  $\delta_2$ , which are not necessarily equal, but are strictly positive and no greater than 1.

- ▶ Let  $M$  be player 1's optimal offer, and consider the game starting at  $t$ :
- ▶ Player 1 is sure to get no more than  $M$ . The trick is to find a way besides  $M$  to represent the maximum that player 1 can obtain.

Consider the offer made by player 2 at  $t - 1$ . Player 1 will accept any offer which gives her more than the discounted value of  $M$  received one period later.

## Alternating Offers Over Infinite Time (cont.)

- ▶ So, player 2 can make an offer  $\delta_1 M$  to player 1, retaining  $1 - \delta_1 M$  for herself.
- ▶ At  $t - 2$ , player 1 knows that player 2 will turn down any offer less than the discounted value of the minimum player 2 can look forward to receiving at  $t - 1$ .
- ▶ Player 1 cannot offer any less than  $\delta_2(1 - \delta_1 M)$  to player 2, and retain for herself any more than  $1 - \delta_2(1 - \delta_1 M)$  at  $t - 2$ .
- ▶ This just follows from the logic of the 3-round bargaining model: player 1 should offer  $1 - \delta_2(1 - \delta_1 M)$  two rounds earlier.

## Alternating Offers Over Infinite Time (cont.)

Now, we have an expression for “the maximum” which player 1 can receive:  $M = 1 - \delta_2(1 - \delta_1 M)$ .

Solving for  $M$ , we obtain,

$$M = 1 - \delta_2 + \delta_1 \delta_2 M,$$

$$M - \delta_1 \delta_2 M = 1 - \delta_2,$$

$$M(1 - \delta_1 \delta_2) = 1 - \delta_2,$$

$$M = \frac{1 - \delta_2}{1 - \delta_1 \delta_2}.$$



## Alternating Offers Over Infinite Time (cont.)

- ▶ We can repeat the argument using  $m$ , the minimum of player 1's share, and we will get an expression for  $m$  that looks exactly like the one we got for  $M$ .
- ▶ So, this is an equilibrium outcome. And, it is unique!
- ▶ In equilibrium, player 1 offers a partition

$$\frac{1 - \delta_2}{1 - \delta_1 \delta_2},$$

and player 2 accepts it, receiving

$$\frac{\delta_1(1 - \delta_2)}{1 - \delta_1 \delta_2}.$$

## Discussion: Properties of SGPE

- ▶ Efficiency: Player 2 accepts player 1's first offer, so that an agreement is reached immediately; no resources are wasted in delay.
- ▶ Changes in Patience: For a given value of  $\delta_2$ , the equilibrium payoff of player 1 increases as  $\delta_1$  approaches 1 (Player 1 is more patient). Symmetrically, fixing the patience of player 1, player 2's share increases as she becomes more patient.

## Discussion: Properties of SGPE (cont.)

- Firs-Mover Advantage: If  $\delta_1 = \delta_2 = \delta$ , then the only asymmetry in the game is that player 1 moves first. In that case, her payoff would be:

$$\frac{1 - \delta}{1 - \delta^2} = \frac{1}{1 + \delta},$$

which, for  $\delta \neq 1$ , is  $> \frac{1}{2}$ .

## Application: Public Finance

Consider now a simple model of public finance (Persson, Roland, and Tabellini 2000).

- ▶ Society is composed by 3 distinct groups of citizens, denoted by  $i = 1, 2, 3$ .
- ▶ Preferences of a member of group  $i$  in period  $j$  are given by

$$u_j^i = \sum_{t=j}^{\infty} \delta^{(t-j)} U^i(\mathbf{q}_t),$$

where  $\delta < 1$  is a discount factor,  $\mathbf{q}_t$  is a vector of policies at  $t$ , and  $U^i$  is the per-period utility function.

# Public Finance

Individual  $i$  utility function is assumed to be quasi-linear in the consumption of private and public goods:

$$U^i(\mathbf{q}_t) = c_t^i + H(g_t) = 1 - \tau_t + r_t^i + H(q_t),$$

where  $\tau_t$  is a common tax rate,  $r_t^i$  is a transfer to group  $i$ , and  $g_t$  is the supply of a Samuelsonian public good.

- ▶ Assumption I: Public good is valuable to citizens ( $\frac{dH}{dg}(0) > 1$ ), but exhibits marginal decreasing returns (i.e.  $H(g) = \log(g)$ ).
- ▶ Assumption II: Nondistortionary taxes

Public policy vector  $\mathbf{q}$  has the following nonnegative components:

$$\mathbf{q}_t = [\tau_t, g_t, \{r_t^i\}, \{s_t^l\}],$$

where  $s_t^l$  denotes a diversion of funds benefiting legislator  $l$ .

## Public Finance (cont.)

The public policy vector in period  $t$  must satisfy the government budget constraint

$$3\tau_t = \sum_i r_t^i + \sum_l s_t^l + g_t \equiv r_t + s_t + q_t,$$

where  $r_t$  and  $s_t$  denote aggregate redistributive expenditures.

- ▶ What policy would a benevolent social planner choose in this setting?
- ▶ First, the planner would set  $s_t^l = 0$ .
- ▶ Second, it is always efficient to have  $r_t^i = 0$  (if utility of private consumption is concave)

## Public Finance (cont.)

The planner would thus set  $g_t$  so as to maximize

$$\sum_i U^i(\mathbf{q}_t) = 3[1 - \tau_t + H(g_t)] = 3[1 - \frac{g_t}{3} + H(g_t)].$$

- ▶ The F.O.C. for a maximum with respect to  $g_t$  is that

$$-1 + 3 \frac{dH}{dg_t} = 0$$

- ▶ The second-order condition is satisfied ( $H(g_t)$  is concave).

So, it follows that the optimal policy is  $\frac{dH}{dg_t} = \frac{1}{3}$ : the marginal aggregate benefit is equal to the marginal social cost (with  $H(g)=\log(g)$ ,  $g_t = 3$ ).

## Public Finance (cont.)

Which public policy would a Leviathan policy maker choose?

- ▶ Without any constraints, the planner would choose  $\tau_t = 1$ ,  $s_t = 3$ ; and  $g_t = r_t = 0$ .

We can now use these two “benchmarks” to examine how different settings affect the provision of public good, the allocation of tax revenues, and redistribution.

- ▶ Single agenda-setter
- ▶ Two agenda-setters
- ▶ Coalition government



## Single Agenda-Setter

Consider a simple legislature where each group  $i$  is represented by exactly one legislator, so that  $i = l = 1, 2, 3$ . Each legislator wants to maximize  $s_t^l$  subject to a reelection constraint.

- ▶ Nature randomly selects an agenda setter  $a$  among the three legislators.
- ▶ Legislator  $a$  proposes a public policy  $\mathbf{q}_t$ .
- ▶ The legislature votes on the proposal. If a majority (at least two legislators) supports it, proposal is implemented.

Otherwise, a default policy is implemented, with  $\tau = s^l = \sigma > 0$  and  $g = r^i = 0$ .

## Single Agenda-Setter (cont.)

Consider districts  $m, n \neq a$ .

- ▶ Denote  $W$  the expected equilibrium continuation value for each legislator before nature has selected the agenda setter.
- ▶ The agenda-setter will seek the support of only one more legislator.
- ▶ Moreover, in equilibrium, the agenda setter will offer  $r^{-a} = 0$ .

Recall, that, without any constraints, a *Leviathan* would choose  $\tau_t = 1$ ,  $s_t = 3$ ; and  $g_t = r_t = 0$ . But, the agenda-setter now needs to “buy” and additional vote and may care about reelection.

## Single Agenda-Setter (cont.)

Suppose the agenda-setter does not care about reelection.

- ▶ Then, he/she can “buy” an additional vote at the reservation value  $\sigma$  and keep  $3 - \sigma$  for himself/herself.

Let's consider now the effect of the “electoral” constraint.

- ▶ The agenda-setter will seek reappointment if  $s^a + \delta W \geq 3 - \sigma$
- ▶ A “Yes” vote from  $m$  will now cost  $s^m + \delta W \geq \sigma$ .

Therefore,  $s = s^a + s^m \geq 3 - 2\delta W$ .

## Single Agenda-Setter (cont.)

To maximize the utility of voters in his /her district  $a$  solves:

$$\max[r + 1 - \tau + H(g)]$$

subject to

$$3(\tau - 1) + 2\delta W \geq r + g$$

The solution to this optimization problem implies:

- ▶  $\tau = 1$
- ▶  $r = 2\delta W - g$
- ▶  $s = 3 - 2\delta W$
- ▶  $g = 2\delta W - r$  or  $g = 2\delta W$  if  $r = 0$ .

## Single Agenda-Setter (cont.)

In equilibrium, everyone gets reelected. We thus have

$$s = 3W(1 - \delta)$$

$$W = \frac{3 - 2\delta W}{3} + \delta W$$

$$W = \frac{1}{[1 - (\frac{\delta}{3})]}$$

Replacing  $W$  we get:

- ▶  $s = 3 \frac{1-\delta}{[1-(\frac{\delta}{3})]}$
- ▶  $r = \frac{2\delta}{[1-(\frac{\delta}{3})]} - g$
- ▶  $g = \frac{2\delta}{[1-(\frac{\delta}{3})]} - r$  or  $g = \frac{2\delta}{[1-(\frac{\delta}{3})]}$  if  $r = 0$ .

# Single Agenda-Setter: Implications

Three “political failures” (relative to the socially optimal policy):

- ▶ Some spending is wasteful ( $s^L > 0$ )
- ▶ Public goods are underprovided
  - ▶ For example, with  $H(g) = \log(g)$ , then  $g^L = 1 < g^* = 3$ .
- ▶ A politically powerful minority may obtain net redistribution to itself at the expense of rest of voters.

## Two Agenda-Setters

As before, consider a legislature where each group  $i$  is represented by exactly one legislator, so that  $i = l = 1, 2, 3$ .

- ▶ Nature randomly selects two agenda setters: one,  $a_\tau$  for taxes and another one,  $a_g$  for spending.
- ▶ Legislator  $a_\tau$  proposes a tax rate  $\tau$ . If at least two legislators supports it, proposal is implemented. Otherwise, a default tax rate  $\tau = \sigma < 1$  is enacted.
- ▶ Legislator  $a_g$  proposes  $[g, \{s^i\}, \{r^i\}]$  subject to the budget constraint  $r + s + g \leq 3\tau$ .

If at least two legislators are in favor, the policy is implemented. Otherwise, a default policy with  $g = 0$ ,  $r^i = 0$  and  $s^i = \tau$  is adopted.

## Two Agenda-Setters (cont.)

The “spending” part of the game is similar to the one we analyzed before, except that  $\tau$  is taken as given. So,

- ▶  $s^a \geq 2\tau - \delta W$
- ▶  $s^m = \tau - \delta W$
- ▶  $s = 3\tau - 2\delta W$
- ▶  $r = 2\delta W - g$
- ▶  $g = 2\delta W - r$  or  $g = 2\delta W$  if  $r = 0$ .

Now, by assumption  $a_\tau \neq a_g$ ; therefore neither  $a_\tau$  nor the voters he/she represents are residual claimants for higher taxes.



## Two Agenda-Setters (cont.)

By assumption  $a_\tau \neq a_g$ ; therefore neither  $a_\tau$  nor the voters he/she represents are residual claimants for higher taxes.

- ▶ But,  $a_\tau$  may be included as a “partner” in the *spending* minimum winning coalition with probability  $\frac{1}{2}$ .

Suppose that he/she sets  $\tau = 1$ . The expected payoff would be:

$$\frac{s^m}{2} + \delta W \geq \frac{1}{2} \text{ or } \frac{\tau - \delta W}{2} + \delta W \geq \frac{1}{2}$$

- ▶ Solving for  $\tau$ , we obtain  $\tau \geq 1 - \delta W$ .

So, the maximum incentive-compatible tax rate that  $a_\tau$  can propose is  $\tau = 1 - \delta W$ .

- ▶ This tax rate will be supported by the other legislator, too. Voting no causes  $\tau = \sigma < 1$  and no reelection (in that case  $\delta W = 0$ , so legislator is better off voting yes).

## Two Agenda-Setters (cont.)

As before, equilibrium, everyone gets reelected. We thus have

$$W = \frac{\tau}{[1 - (\frac{\delta}{3})]}$$

Replacing  $W$  we get:

- ▶  $\tau = \frac{1 - (\frac{\delta}{3})}{1 + (\frac{2\delta}{3})} < 1$
- ▶  $s = 3 \frac{1 - \delta}{[1 + (\frac{2\delta}{3})]} < 3 \frac{1 - \delta}{[1 - (\frac{\delta}{3})]}$
- ▶  $r = \frac{2\delta}{[1 + (\frac{2\delta}{3})]} - g$
- ▶  $g = \frac{2\delta}{[1 + (\frac{2\delta}{3})]} - r$  or  $g = \frac{2\delta}{[1 + (\frac{2\delta}{3})]}$  if  $r = 0$ .

## Two Agenda-Setters: Implications

We can compare this outcome with that in the legislature with a single agenda-setter.

- ▶ The government raises less taxes.
- ▶ Less resources are wasted ( $s$  is lower).
- ▶ The overall amount of public goods is the same or smaller (if  $g = 0$ ).
- ▶ For same the same level of public goods, less money is spent on redistribution.

When decision-making authority is split between different decision-makers (who still need to make joint decisions), voters can exploit the conflict of interest among policy makers and hold them more accountable.

## Two Agenda-Setters: Implications (cont.)

This result, however, holds up as long as some assumptions remain.

- ▶ It is crucial that decisions over the size and the *allocation* of the budget are kept separate.
- ▶ It is also important to assume that collusive agreements between  $a_T$  and  $a_G$  cannot be enforced.

One should keep these caveats in mind when thinking about empirics.

# Majority Coalition Government

We can think of this setup as a stylized representation of a parliamentary government.

- ▶ Nature picks *two* legislators as members of a majority coalition constituting the “government.”
- ▶ Each coalition partner has a veto right.
- ▶ If veto is exercised, a government crisis follows.

After a crisis, a new agenda-setter is picked at random and the decision-making rules entail a single-agenda setter.

## Majority Coalition Government (cont.)

This game has a continuum of equilibria such that:

- ▶  $\tau = 1 > \frac{1 - (\frac{\delta}{3})}{1 + (\frac{2\delta}{3})}$
- ▶  $s = 3 \frac{1 - \delta}{[1 - (\frac{\delta}{3})]} > 3 \frac{1 - \delta}{[1 + (\frac{2\delta}{3})]}$
- ▶  $r = \frac{2\delta}{[1 - (\frac{\delta}{3})]} - g \geq 0$
- ▶  $g \leq \bar{g}$ , with  $\bar{g}$  defined by  $\frac{dH}{dg}(\bar{g}) = \frac{1}{2}$ .

Higher benefits for one set of voters are associated with lower benefits for the other group (hence the multiplicity of equilibria).

# Majority Coalition Government: Implications

What are the outcomes associated with this type of government?

- ▶ Equilibrium taxes are higher than with two agenda-setters.
- ▶  $s$  is higher: in comparison with the two agenda-setters regime, there is more scope for collusion among coalition partners.
- ▶ If  $r > 0$ , redistribution goes to a majority.
- ▶ In addition, if  $r > 0$ , public-good provision must be jointly optimal for the two groups of voters in the majority ( $2 \frac{dH}{dg}(\bar{g}) = 1$ ).

# Comparative Politics

Let the majority coalition government represent *parliamentarism*, and the two agenda-setters *presidentialism*.

- ▶ Then, we can calculate voters' welfare,  $E(u^{parl}) - E(u^{pres})$ , as:

$$\frac{1}{1-\delta} \left( \{ [H(g^{parl}) - \frac{g^{parl}}{3}] - [H(g^{pres}) - \frac{g^{pres}}{3}] \} - \frac{\delta(1-\delta)}{[1 - (\frac{\delta}{3})][1 + (\frac{2\delta}{3})]} \right)$$

- ▶ The first term inside the large parentheses is always positive and captures the welfare effect of public goods.
- ▶ The second one is always negative, and captures the effect of higher waste (and higher associated taxes).

So parliamentarism is better for voters if public goods are very valuable or if the political agency problem is small (as  $\delta \approx 1$ ).



# Comparative Politics: Empirical Evidence

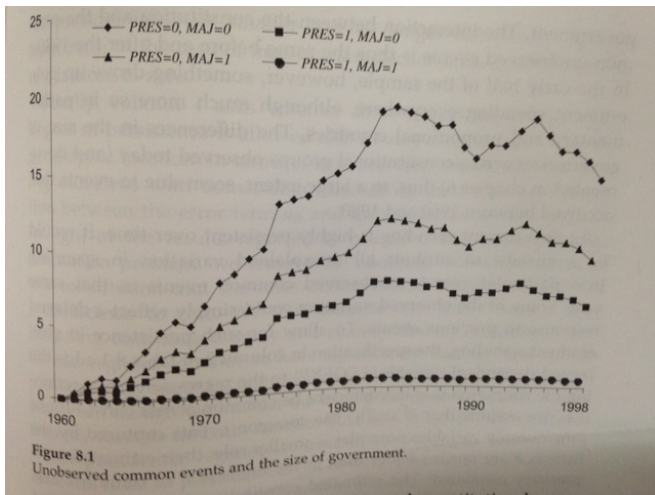


Figure 5: Source: Persson and Tabellini (2005)